Recent analysis of the isospin diffusion data from heavy-ion collisions based on an isospin- and momentum-dependent transport model with in-medium nucleon-nucleon cross sections has led to the extraction of a value of $L = 88 \pm 25$ MeV for the slope of the nuclear symmetry energy at saturation density. This imposes stringent constraints on both the parameters in the Skyrme effective interactions and the neutron skin thickness of heavy nuclei. Among the 21 sets of Skyrme interactions commonly used in nuclear structure studies, the 4 sets SIV, SV, $G_\sigma$, and $R_\sigma$ are found to give $L$ values that are consistent with the extracted one. Further study on the correlations between the thickness of the neutron skin in finite nuclei and the nuclear matter symmetry energy in the Skyrme Hartree-Fock approach leads to predicted thickness of the neutron skin of $0.22 \pm 0.04$ fm for $^{208}$Pb, $0.29 \pm 0.04$ fm for $^{132}$Sn, and $0.22 \pm 0.04$ fm for $^{124}$Sn.

1. Introduction

The study of the equation of state (EOS) of isospin asymmetric nuclear matter, especially the nuclear symmetry energy, is currently an active field of research in nuclear physics. Although the nuclear symmetry energy at normal nuclear matter density is known to be around 30 MeV from the empirical liquid-drop mass formula, its values at other densities, especially at supranormal densities, are poorly known. Advances in radioactive nuclear beam...
facilities provide, however, the possibility to pin down the density dependence of the nuclear symmetry energy in heavy ion collisions induced by these nuclei. Indeed, significant progress has recently been made in extracting the information on the density dependence of nuclear symmetry energy from the isospin diffusion data in heavy-ion collisions at NSCL/MSU. Using an isospin- and momentum-dependent IBUU04 transport model with in-medium nucleon-nucleon (NN) cross sections, the isospin diffusion data were found to be consistent with a relatively soft nuclear symmetry energy at subnormal density.

Information on the density dependence of the nuclear symmetry energy can also be obtained from the thickness of the neutron skin in heavy nuclei as the latter is strongly correlated with the slope $L$ of the nuclear matter symmetry energy at saturation density. Because of the large uncertainties in measured neutron skin thickness of heavy nuclei, this has not been possible. Instead, studies have been carried out to use the extracted nuclear symmetry energy from the isospin diffusion data to constrain the neutron skin thickness of heavy nuclei. Using the Hartree-Fock approximation with parameters fitted to the phenomenological EOS that was used in the IBUU04 transport model to describe the isospin diffusion data from NSCL/MSU, it was found that a neutron skin thickness of less than 0.15 fm for $^{208}$Pb was incompatible with the isospin diffusion data.

In the present talk, we report our recent work on constraining the Skyrme effective interactions and the neutron skin thickness of heavy nuclei using the isospin diffusion data from heavy ion collisions. Using the value of $L$ obtained from the extracted density dependence of the nuclear symmetry energy from the isospin diffusion data, we have been able to limit the allowed parameter sets for the Skyrme interaction. Also, studying the correlation between the density dependence of the nuclear symmetry energy and the thickness of the neutron skin in a number of nuclei within the framework of the Skyrme Hartree-Fock approach further allows us to obtain stringent constraints on the neutron skin thickness of the nuclei $^{208}$Pb, $^{132}$Sn, and $^{124}$Sn.

### 2. Nuclear symmetry energy and the Skyrme interaction

The nuclear symmetry energy $E_{\text{sym}}(\rho)$ at nuclear density $\rho$ can be expanded around the nuclear matter saturation density $\rho_0$ as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2,$$

where $L$ and $K_{\text{sym}}$ are the slope and curvature of the nuclear symmetry energy at $\rho_0$, i.e.,

$$L = 3\rho_0 \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \bigg|_{\rho = \rho_0}, \quad K_{\text{sym}} = 9\rho_0^2 \frac{\partial^2 E_{\text{sym}}(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0}.$$
The $L$ and $K_{\text{sym}}$ characterize the density dependence of the nuclear symmetry energy around normal nuclear matter density, and thus provide important information on the properties of nuclear symmetry energy at both high and low densities.

In the standard Skyrme Hartree-Fock approach,$^{46,47,49,50,39}$ the interaction is taken to be zero-range and density- and momentum-dependent with its parameters fitted to the binding energies and charge radii of a large number of nuclei in the periodic table. For infinite nuclear matter, the nuclear symmetry energy from the Skyrme interaction can be expressed as$^{49,50}$

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} - \frac{1}{8} t_0 (2x_0 + 1) \rho - \frac{1}{48} t_3 (2x_3 + 1) \rho^{\sigma+1}$$

$$+ \frac{1}{24} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[-3t_1 x_1 + (4 + 5x_2) t_2\right] \rho^{5/3}, \quad (3)$$

where the $\sigma$, $t_0 - t_3$, and $x_0 - x_3$ are Skyrme interaction parameters.

Fig. 1. (Color online) Density dependence of the nuclear symmetry energy $E_{\text{sym}}(\rho)$ for 21 sets of Skyrme interaction parameters (solid lines) as well as the MDI interaction with $x = -1$ (open squares) and 0 (solid squares). Fig. 1 displays the density dependence of $E_{\text{sym}}(\rho)$ for 21 sets of Skyrme interaction parameters, i.e., SKM, SKM*, RATP, SI, SII, SIII, SIV, SV, SVI, E, $E_\sigma$, G$_\sigma$, R$_\sigma$, Z, $Z_\sigma$, $Z'_\sigma$, T, T3, SkX, SkXce, and SkXm. Values of the parameters in these Skyrme interactions can be found in Refs. $^{46,47,48}$. For comparison, we also show in Fig. 1 results from the phenomenological parametrization of the momentum-dependent nuclear mean-field potential based on the Gogny effective interaction $^{51}$, i.e., the MDI interactions with $x = -1$ (open squares) and 0 (solid squares), where different $x$ values correspond to different density dependence of the nuclear symmetry energy but keep other properties of the nuclear EOS the same $^{37}$. By comparing the isospin diffusion data from NSCL/MSU with results from the IBUU04 transport model using in-medium NN cross sections, these interactions have been shown
to give, respectively, the upper and lower bounds for the stiffness of the nuclear symmetry energy. It is seen from Fig. 1 that the density dependence of the symmetry energy varies drastically among different interactions. Although the values of $E_{\text{sym}}(\rho_0)$ are all in the small range of 26-35 MeV, the values of $L$ and $K_{\text{sym}}$ are in a much larger range of $-50$-$100$ MeV and $-700$-$50$ MeV, respectively.

3. Constraining symmetry energy from isospin diffusion data in heavy ion collisions

Nuclear symmetry energy is known to affect the diffusion of isospin in intermediate-energy heavy ion collisions. Experimentally, the degree of isospin diffusion between the projectile nucleus $A$ and the target nucleus $B$ can be studied via the quantity $R_i$,\(^\text{52,36}\)

\[ R_i = \frac{2X^{A+B} - X^{A+A} - X^{B+B}}{X^{A+A} - X^{B+B}} , \]

where $X$ is any isospin-sensitive observable. By construction, the value of $R_i$ is 1 ($-1$) for symmetric $A + A$ ($B + B$) reaction. If isospin equilibrium is reached during the collision as a result of isospin diffusion, the value of $R_i$ is about zero. In the NSCL/MSU experiments with $A = ^{124}\text{Sn}$ and $B = ^{112}\text{Sn}$ at a beam energy of 50 MeV/nucleon and an impact parameter about 6 fm, the isospin asymmetry of the projectile-like residue was used as the isospin tracer $X$.\(^\text{36}\) Using an isospin- and momentum-dependent IBUU04 transport model with free-space experimental NN cross sections, the dependence of $R_i$ on the nuclear symmetry energy was studied from the average isospin asymmetry of the projectile-like residue that was calculated from nucleons with local densities higher than $\rho_0/20$ and velocities larger than $1/2$ the beam velocity in the center-of-mass frame.\(^\text{37}\) Comparing theoretical results with experimental data has allowed us to extract a nuclear symmetry energy of $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$. Including also medium-dependent NN cross sections, which are important for isospin-dependent observables, the isospin diffusion data leads to an even softer nuclear symmetry energy of $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^\gamma$ with $\gamma \approx 0.7$.\(^\text{38}\)

In Fig. 2, we show the degree of the isospin diffusion $1 - R_i$ as a function of $L$ obtained from the IBUU04 transport model with in-medium NN cross sections and the mean-field potential based on the MDI interactions. The shaded band in Fig. 2 indicates the data from NSCL/MSU.\(^\text{36}\) It is seen that the strength of isospin diffusion $1 - R_i$ decreases monotonically with decreasing value of $x$ or increasing value of $L$. This is expected as the parameter $L$ reflects the difference in the pressures on neutrons and protons. From comparison of the theoretical results with the data, we can clearly exclude the MDI interaction with $x = 1$ and $x = -2$ as they give either too large or too small a value for $1 - R_i$ compared to that of data. The range of $x$ or $L$ values that give values of $1 - R_i$ falling within the band of experimental values could in principle be determined in our model by detailed calculations. Instead, we determine this schematically by using the results from the four $x$ values. For
the centroid value of $L$, it is obtained from the interception of the line connecting the theoretical results at $x = -1$ and 0 with the central value of $1 - R_i$ data in Fig. 2, i.e., $L = 88$ MeV. The upper limit of $L = 113$ MeV is estimated from the interception of the line connecting the upper error bars of the theoretical results at $x = -1$ and $-2$ with the lower limit of the data band of $1 - R_i$. Similarly, the lower limit of $L = 65$ MeV is estimated from the interception of the line connecting the lower error bars of the theoretical results at $x = 0$ and $-1$ with the upper limit of the data band of $1 - R_i$. This leads to an extracted value of $L = 88 \pm 25$ MeV as shown by the solid square with error bar in Fig. 2.

The extracted value of $L = 88 \pm 25$ MeV gives a rather stringent constraint on the density dependence of the nuclear symmetry energy and thus puts strong limitations on the nuclear effective interactions as well. For the Skyrme effective interactions shown in Fig. 1, for instance, all of those lie beyond $x = 0$ and $x = -1$ in the sub-saturation region are not consistent with the extracted value of $L$. Actually, we note that only 4 sets of Skyrme interactions, i.e., SIV, SV, $G_\sigma$, and $R_\sigma$, in the 21 sets of Skyrme interactions considered here have nuclear symmetry energies that are consistent with the extracted $L$ value.

4. Correlations between neutron skin thickness of finite nuclei and the nuclear symmetry energy at saturation density

Also affected by the density dependence of nuclear symmetry energy is the neutron skin thickness $S$ of a nucleus, which is defined as the difference between the root-mean-square radii $\sqrt{\langle r_n^2 \rangle}$ of neutrons and $\sqrt{\langle r_p^2 \rangle}$ of protons, i.e.,

$$S = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}.$$ (5)
In particular, $S$ is sensitive to the slope parameter $L$ of the nuclear symmetry energy at normal nuclear matter density. Using above 21 sets of Skyrme interaction parameters, we have evaluated the neutron skin thickness of $^{208}\text{Pb}$ with $L$, $K_{\text{sym}}$, and $E_{\text{sym}}(\rho_0)$. It is seen from Fig. 3(a) that there exists an approximate linear correlation between $S$ and $L$. The correlations of $S$ with $K_{\text{sym}}$ and $E_{\text{sym}}(\rho_0)$ are less strong and even exhibit some irregular behavior. The solid line in Fig. 3(a) is a linear fit to the correlation between $S$ and $L$ and is given by the following expression:

$$S^{^{208}\text{Pb}} = (0.1066 \pm 0.0019) + (0.00133 \pm 3.76 \times 10^{-5}) \times L,$$

or

$$L = (-78.5 \pm 3.2) + (740.4 \pm 20.9) \times S^{^{208}\text{Pb}},$$

where the units of $L$ and $S$ are MeV and fm, respectively. Therefore, if the value for either $S^{^{208}\text{Pb}}$ or $L$ is known, the value for the other can be determined.

It is of interest to see if there are also correlations between the neutron skin thickness of other neutron-rich nuclei and the nuclear symmetry energy. Fig. 4 shows the same correlations as in Fig. 3 but for the neutron-rich nuclei $^{132}\text{Sn}$, $^{124}\text{Sn}$, and $^{48}\text{Ca}$. For the heavier $^{132}\text{Sn}$ and $^{124}\text{Sn}$, we obtain a similar conclusion as for $^{208}\text{Pb}$, namely, $S$ exhibits an approximate linear correlation with $L$ but weaker correlations with $K_{\text{sym}}$ and $E_{\text{sym}}(\rho_0)$. For the lighter $^{48}\text{Ca}$, on the other hand, all the correlations become weaker than those of heavier nuclei. The neutron skin thickness of heavy nuclei is thus better correlated with the density dependence of the nuclear symmetry energy. As in Eq. (6) and (7), a linear fit to the correlation between $S$ and $L$ can also be obtained for $^{132}\text{Sn}$ and $^{124}\text{Sn}$, and the corresponding expressions are

$$S^{^{132}\text{Sn}} = (0.1694 \pm 0.0025) + (0.0014 \pm 5.12 \times 10^{-5}) \times L,$$
Table 1. Linear correlation coefficients $C_l$ with $L$, $K_{\text{sym}}$ and $E_{\text{sym}}(\rho_0)$ for $^{208}\text{Pb}$, $^{132}\text{Sn}$, $^{124}\text{Sn}$, and $^{48}\text{Ca}$ from 21 sets of Skyrme interaction parameters.

<table>
<thead>
<tr>
<th></th>
<th>$^{208}\text{Pb}$</th>
<th>$^{132}\text{Sn}$</th>
<th>$^{124}\text{Sn}$</th>
<th>$^{48}\text{Ca}$</th>
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<tr>
<td>$S-L$</td>
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<td>98.75</td>
<td>93.66</td>
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<td>87.89</td>
<td>85.74</td>
<td>85.77</td>
<td>81.01</td>
</tr>
</tbody>
</table>

$L = (-117.1 \pm 5.4) + (695.1 \pm 25.3) \times S(^{132}\text{Sn})$, \hspace{1cm} (9)

and

$S(^{124}\text{Sn}) = (0.1255 \pm 0.0020) + (0.0011 \pm 4.05 \times 10^{-5}) \times L$, \hspace{1cm} (10)

$L = (-110.1 \pm 5.2) + (882.6 \pm 32.3) \times S(^{124}\text{Sn})$, \hspace{1cm} (11)

Similar linear relations between $S$ and $L$ are also expected for other heavy nuclei. This is not surprising as detailed discussions in Refs. \cite{39,40,41,42,43,44} have shown that the thickness of the neutron skin in heavy nuclei is determined by the pressure difference between neutrons and protons, which is proportional to the parameter $L$.

To give a quantitative estimate of above discussed correlations, we define the following linear correlation coefficient $C_l$:

$$ C_l = \sqrt{1 - q / t}, \hspace{1cm} (12) $$

where

$$ q = \sum_{i=1}^{n} [y_i - (A + Bx_i)]^2, \hspace{0.5cm} t = \sum_{i=1}^{n} (y_i - \bar{y})^2, \hspace{0.5cm} \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i . \hspace{1cm} (13) $$

In the above, $A$ and $B$ are the linear regression coefficients, $(x_i, y_i)$ are the sample points, and $n$ is the number of sample points. The linear correlation coefficient $C_l$
measures the degree of linear correlation, and $C_l = 1$ corresponds to an ideal linear correlation. Table 1 gives the linear correlation coefficient $C_l$ for the correlation of $S$ with $L$, $K_{sym}$ and $E_{sym}(\rho_0)$ for $^{208}$Pb, $^{132}$Sn, $^{124}$Sn, and $^{48}$Ca shown in Figs. 3 and 4 for different Skyrme interactions. It is seen that these correlations become weaker with decreasing nucleus mass, and a strong linear correlation only exists between the $S$ and $L$ for the heavier nuclei $^{208}$Pb, $^{132}$Sn, and $^{124}$Sn. Therefore, the neutron skin thickness of these nuclei can be extracted once the slope parameter $L$ of the nuclear symmetry energy at saturation density is known.

5. Predictions on the neutron skin thickness of heavy nuclei

The extracted $L$ value from the isospin diffusion data in heavy ion collisions allows us to determine from Eqs. (6), (8), and (10), respectively, a neutron skin thickness of $0.22 \pm 0.04$ fm for $^{208}$Pb, $0.29 \pm 0.04$ fm for $^{132}$Sn, and $0.22 \pm 0.04$ fm for $^{124}$Sn. Experimentally, great efforts were devoted to measure the thickness of the neutron skin in heavy nuclei $^{54,55}$, and a recent review can be found in Ref. $^{56}$. The data for the neutron skin thickness of $^{208}$Pb indicate a large uncertainty, i.e., $0.1-0.28$ fm. Our results for the neutron skin thickness of $^{208}$Pb are thus consistent with present data but give a much stronger constraint. A large uncertainty is also found experimentally in the neutron skin thickness of $^{124}$Sn, i.e., its value varies from 0.1 fm to 0.3 fm depending on the experimental method. The proposed experiment of parity-violating electron scattering from $^{208}$Pb at the Jefferson Laboratory is expected to give another independent and more accurate measurement of its neutron skin thickness (within 0.05 fm), thus providing improved constraints on the density dependence of the nuclear symmetry energy $^{57,58}$.

Most recently, an accurately calibrated relativistic parametrization based on the relativistic mean-field theory has been introduced to study the neutron skin thickness of finite nuclei $^{59}$. This parametrization can describe simultaneously the ground state properties of finite nuclei and their monopole and dipole resonances. Using this parametrization, the authors predicted a neutron skin thickness of 0.21 fm in $^{208}$Pb, 0.27 fm in $^{132}$Sn, and 0.19 fm in $^{124}$Sn $^{59,60}$. These predictions are in surprisingly good agreement with our results constrained by the isospin diffusion data from heavy-ion collisions.

6. Summary

Both the structure of nuclear surface and the dynamics of isospin equilibration in heavy ion collisions are affected by the density dependence of nuclear symmetry energy. From the most recent analysis of the isospin diffusion data in heavy-ion collisions using an isospin- and momentum-dependent transport model with in-medium NN cross sections, a value $L = 88 \pm 25$ MeV has been extracted for the slope of the nuclear symmetry energy at saturation density. This relatively constrained value imposes strong constraints on the parameters in the Skyrme effective interactions. Among the 21 sets of commonly used Skyrme parameters, only SIV, SV, $G_{\sigma}$, and
Rσ have L values that are consistent with the extracted one. We have also studied the correlation between the neutron skin thickness of finite nuclei and the nuclear symmetry energy within the framework of the Skyrme Hartree-Fock model. As in previous studies, we have found a strong linear correlation between the neutron skin thickness of heavy nuclei and the slope L of the nuclear matter symmetry energy at saturation density. This correlation provides stringent constraints on both the density dependence of the nuclear symmetry energy and the thickness of the neutron skin in heavy nuclei. The extracted L value from the isospin diffusion data then leads to predicted neutron skin thickness of 0.22 ± 0.04 fm for 208Pb, 0.29 ± 0.04 fm for 132Sn, and 0.22 ± 0.04 fm for 124Sn. Our work has thus demonstrated that information obtained from studying isospin dynamics in heavy ion collisions can be very useful for understanding the structure of nuclei.

Acknowledgments

The work was supported in part by the National Natural Science Foundation of China under Grant No. 10575071, MOE of China under project NCET-05-0392, Shanghai Rising-Star Program under Grant No. 06QA14024 (LWC), the US National Science Foundation under Grant No. PHY-0457265 and the Welch Foundation under Grant No. A-1358 (CMK), as well as the US National Science Foundation under Grant No. PHY-0354572 and PHY-0456890 and the NASA-Arkansas Space Grants Consortium Award ASU15154 (BAL).

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