Beyond the Standard Lore of the SZ effect

S. Colafrancesco

INAF - Osservatorio Astronomico di Roma
Via Frascati 33, I-00040 Monteporzio, Italy
Email: Sergio.Colafrancesco@mporzio.astro.it

Abstract

Multi-frequency (X-ray, optical and radio) observations of galaxy clusters indicate that the atmospheres of these cosmic structures consist of a complex structure of thermal (hot and warm) and non-thermal (with different origin and spectra) distribution of electrons (and protons) which is, therefore, far from its modelling as a single, thermal electronic gas. This evidence requires to go beyond the simple, standard lore of the SZ effect. This task is challenging for both the theoretical aspects of their modelling and for the experimental goals to be achieved, but it will return a large amount of physical information by using the SZ effect as a unique tool for astro-particle and cosmology.

Key words: Cosmology, CMB, Dark Matter, galaxy clusters, galaxies

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1 The SZ effect: the standard lore

The Sunyaev-Zel’dovich effect (hereafter SZE, Zel’dovich & Sunyaev 1969, Sunyaev & Zel’dovich 1972, 1980) produces distortions of the CMB spectrum by means of the Compton scattering of CMB photons off the energetic electrons which are present in the atmosphere of cosmic structures, like clusters of galaxies and galaxies (see Rephaeli 1995, Birkinshaw 1999, Colafrancesco 2004a for reviews). Such a scattering is proportional to the energy density of the electron population and produces a systematic shift of the CMB photons from the Rayleigh-Jeans (RJ) to the Wien side of the spectrum. In this respect, it is a powerful probe of the physical conditions of electronic plasmas in astrophysical and cosmological context.

The standard description of the non-coherent Compton scattering of an isotropic Planckian radiation field by a non-relativistic Maxwellian electron population – like the one constituting the hot (with temperature $T_e \sim 10^7 - 10^8$ K), optically thin (with density $n_e \sim 10^{-3} - 10^{-2}$ cm$^{-3}$, and size $R \sim$ Mpc) intracluster...
(IC) medium – can be obtained by means of the solution of the Kompaneets (1957) equation. As such, the origin of the SZE can be considered as a fall-out effect of the cold war: in fact, the Compton scattering Fokker-Planck equation for a population of scattered photons was first derived by A.S. Kompaneets in the early 1950 and then remained classified due to nuclear bomb research until 1956 (see, e.g., Goncharov 1996); it was finally published in 1957 (Kompaneets 1957). In 1969, Ya.B. Zel’dovich and R. Sunyaev (1969) derived the SZ effect applying the Kompaneets equation to the case of the Compton scattering of CMB photons off the thermal population of electrons confined in the potential wells of galaxy clusters, i.e. the intra-cluster medium (ICM).

The change in the spectral intensity of the CMB seen in the direction of a galaxy cluster as due to the scattering of CMB photons by a thermal electron distribution can be written as

$$\Delta I_{\text{th}} = 2 \frac{(k_BT_0)^3}{(hc)^2} y_{\text{th}} g(x),$$

where $x = h\nu/k_BT_0$ is the a-dimensional frequency and the spectral features of the effect are contained in the function $g(x)$ which reads as

$$g_{\text{non-rel}}(x) = \frac{x^4e^x}{(e^x-1)^2} \left[ x\frac{e^x + 1}{e^x - 1} - 4 \right]$$

in the non-relativistic limit. This function (see Fig.1) is zero at the frequency

![Fig. 1. The function $g(x)$ in eq.(2) (solid line) is compared with the function $\tilde{g}(x)$ for thermal electron populations with $k_BT_e = 10$ (dot-dashed), 5 (dashes), 3 (long dashes) and 1 (dotted) keV, respectively.](image-url)
$x_0 = 3.83$ (or $\nu = 217$ GHz for a value of the CMB temperature $T_0 = 2.726$ K), negative at $x < x_0$ (in the RJ side) and positive at $x > x_0$ (in the Wien side). The Comptonization parameter, $y_{th}$, due to the thermal SZE reads as

$$y_{th} = \frac{\sigma_T}{m_e c^2} \int d\ell \ n_e k_B T_e \ ,$$

(3)

where $n_e$ and $T_e$ are the electron density and temperature of the IC gas, respectively; $\sigma_T$ is the Thomson cross section, valid in the limit $T_e \gg T_0$, $k_B$ is the Boltzmann constant and $m_e c^2$ is the rest mass energy of the electron. The Comptonization parameter $y_{th}$ is proportional to the integral along the line of sight $\ell$ of the kinetic energy density of the IC gas, $\epsilon_{th} \approx n_e k_B T_e$, or equivalently the kinetic pressure $^1$ that we define here as $P_{th} = n_e k_B T_e$. Hence, eq.(3) can be written as

$$y_{th} = \frac{\sigma_T}{m_e c^2} \int d\ell \ P_{th} \ ,$$

(4)

where the relevant dependence from the total kinetic pressure, $P_{th}$, of the electrons along the line of sight $\ell$ appears. Since the previous description of the thermal SZE is obtained under the Kompaneets approximation and in the single scattering regime of the photon frequency redistribution function, it only provides an approximation of the SZE for low electronic temperatures ($k_B T_e \lesssim 3$ keV) and low values of the optical depth

$$\tau = \sigma_T \int d\ell n_e \ ,$$

(5)

which is usually $\lesssim 10^{-3}$ in galaxy clusters.

The existence of many high-temperature X-ray clusters (see Arnaud 2005 for a recent review) with $k_B T_e$ up to $\sim 17$ keV (see, e.g., Tucker et al. 1998, Liang et al. 2002) which correspond to $k_B T_e/m_e c^2 \approx 3.3 \cdot 10^{-2}$, requires to take into account the appropriate relativistic corrections in the calculation of their thermal SZE (see Rephaeli 1995, Birkinshaw 1999 and references therein). Analytical and numerical expressions for the SZE in the relativistic case have been considered by various authors using both analytical and Monte Carlo techniques (see, e.g., Stebbins 1997; Itoh et al. 1998; Challinor & Lasenby 1998; see also Birkinshaw 1999 and references therein). Some of these calculations (out of the Monte Carlo simulations) are still approximate since they have been

$^1$ Pressure in a fluid may be considered to be a measure of energy per unit volume or energy density. According the kinetic theory of ideal gases - see Halliday, Resnick & Walker 2000 - the gas pressure can be defined as the average momentum transfer per unit area per unit time due to collisions between a confined gas and its boundary. Using Newton’s second law, this pressure can be shown to be given by one third of the average kinetic energy of particles in the gas $\langle \epsilon_{th} \rangle = 3k_B T_e n_e$. 

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carried out in the limits of i) single scattering of CMB photons against the IC gas electrons; ii) diffusion limit in which the use of the Kompaneets equation is justified. The results of Itoh et al. (1998) based on a generalised Kompaneets equation and by direct integration of the Boltzmann collision term, can be regarded as exact in the framework of the single scattering approximation. Analytical fitting formulae of such derivation are available (Nozawa et al. 2000; Itoh et al. 2002) and offer a detailed description of the thermal SZE for \( k_B T_e \lesssim 15 \) keV, while Monte Carlo simulations describe more correctly the thermal SZE even for \( k_B T_e \gtrsim 20 \) keV (see, e.g., Challinor and Lasenby 1998). Numerical solutions for the thermal SZE which are valid for generic values of \( \tau \) and \( T_e \) have been given by Dolgov et al. (2001) based on an analytical reduction of the collision integral which is contained in the Boltzmann-like collision equation. Sazonov and Sunyaev (2000) presented a derivation of the monochromatic redistribution function in the mildly relativistic limit which considers also quantum effects and the use of the Klein-Nishina cross-section which reproduces, in the limit \( h\nu \ll k_B T_e \), the results of Fargion et al. (1997). However, they still consider only the single Compton scattering limit and the relativistic corrections up to some intermediate order due to low-energy photons and relativistic electrons. Itoh et al. (2001) also presented a calculation of the thermal SZE which considers the contribution from multiple scattering in the relativistic limit, and Colafrancesco et al. (2003) derived a general form of the SZE valid in the full relativistic regime, for generic values of \( \tau \) and \( T_e \) and for multiple scattering regimes.

Another general assumption which is made in the calculation of the SZE is the use of a single population of thermal electrons (i.e., the hot ICM). This assumption is based on the evidence that the ICM is mainly constituted by thermal electrons (and protons) which are responsible for the thermal bremsstrahlung X-ray emission observed in clusters (see Sarazin 1988 for a review). A further assumption is to use the electronic temperature \( T_e \) as a measure of the average energy per particle, a condition which is not ensured in plasma undergoing fast, non-equilibrium processes which may yield \( T_e \neq T_p \).

The study of the thermal SZE, caused by the random scatterings of the thermal (isotropically distributed) electrons, is complemented in the standard description by a kinematic (Doppler) SZE, \( \text{SZ}_{\text{kin}} \), which appears when the cluster has a whole has a finite (peculiar) velocity \( V_r \) in the CMB frame. The expressions for the intensity and temperature changes due to the \( \text{SZ}_{\text{kin}} \) can be obtained, assuming that the \( \text{SZ}_{\text{th}} \) and \( \text{SZ}_{\text{kin}} \) are separable, by a simple relativistic transformation (see, e.g., Rephaeli 1995 for a review) which yields

\[
\Delta I_{\text{kin}}(x) = -2 \frac{k_B T_{\text{CMB}}^3 V_r}{(hc)^2} \frac{2}{\tau} \frac{x^4 e^x}{(e^x - 1)^2},
\]

where the line-of-sight peculiar velocity \( V_r \) is positive (negative) for a receding (approaching) cluster. At variance with the temperature change due to the
thermal SZE (see eqs. 1 and 2), the \( \Delta T_{\text{kin}} = -T_0 \left( \frac{V_r}{c} \right) \tau \) is independent of frequency.

**The standard lore of the SZE: simple physics and cosmology**

Simple astrophysical and cosmological results can be obtained from the study of the standard (both thermal and kinematic) SZE (see Birkinshaw 2003).

As for cluster physics, the SZE can be used to study:

- the integrated SZE, which provides information on the total thermal energy content and the total electron content;
- the spatial SZ structures: these are not as sensitive as the available X-ray data, and also need for IC gas temperature estimates;
- the mass structures and their relationship to gravitational lensing derived structures;
- the radial peculiar velocity of galaxy clusters via the \( \Delta T_{\text{kin}} \).

As for cosmology, the SZE can provide information on:

- cosmological parameters, like the cluster-based Hubble diagram or the distribution of the cluster counts vs. redshift, which can be used to probe \( \Omega_m \), with a minor sensitivity to \( \Omega_\Lambda \) (see Carlstrom et al. 2002);
- cluster evolution physics, i.e. the evolution of cluster atmospheres, the evolution of their radial velocity distributions and the evolution of their baryon fraction;
- the evolution of \( T_{\text{CMB}} \) with redshift (see, e.g., Battistelli et al. 2002).

We must keep in mind, however, that two basic working approximations are assumed in such use of the standard SZE: i) the diffusion limit (i.e., the single scattering approximation valid for \( \tau \ll 1 \)); ii) a single electron population (i.e., the population of thermal hot electrons which is confined in the cluster atmosphere and emits in the X-rays by bremsstrahlung.

There are several pieces of evidence and physical arguments that require to go beyond this standard lore of the SZE.

### 2 The SZE: more than basics.

The electronic distribution of the atmospheres of galaxy clusters is neither simple nor unique. There are, in fact, three matter components in clusters that can provide different sources of electrons: baryons, Dark Matter, relativistic plasmas.

While the SZE from the baryonic, hot plasma component has been described
in the previous Sect. 1, we will provide here the basic information on other non-thermal electronic components residing in cluster atmospheres.

Many galaxy clusters contain – in addition to the thermal IC gas – a population of relativistic electrons which produce a diffuse radio emission (radio halos and/or relics) via synchrotron radiation in a magnetized ICM (see, e.g., Govoni & Feretti 2004 for a review). The electrons which are responsible for the radio halo emission have energies \( E_e \approx 16 \text{GeV} B^{-1/2}_\mu \left(\nu / \text{GHz}\right)^{1/2} > \text{a few GeV} \) in order to reproduce the main properties of the observed radio halos (see, e.g., Blasi & Colafrancesco 1999; Colafrancesco & Mele 2001, Colafrancesco et al. 2006a and references therein). The origin of such relativistic electrons is not certain and models of bottom-up production (i.e., re-accelerated by IC turbulence, see e.g. Brunetti 2003 for a review) or top-down origin (i.e., secondarily produced by Dark Matter WIMP annihilation, see e.g. Colafrancesco et al. 2006a,b) can fit the observed radio-halo features.

The presence of Extreme UV/soft X-ray excesses (Lieu et al. 1996, Kaastra et al. 2002; Bowyer 2000) and of an hard X-ray excess (Fusco-Femiano 2004; Rephaeli 2004; Kaastra et al. 1999) in a few nearby clusters indicate the presence of an additional population of secondary relativistic electrons (see Bowyer et al. 2004, Marchegiani, Perola & Colafrancesco 2006) or a combination of warm (reproducing the EUV excess, Lieu et al. 2000) and (quasi-)thermal (reproducing the hard X-ray excess by bremsstrahlung, see Wolfe & Melia 2006, Dogiel et al. 2006) populations of distinct origins.

The further evidence for new physical phenomena occurring in the cluster atmospheres – e.g., non-thermal heating in the cluster cores (see, e.g., Colafrancesco, Dar & DeRujula 2004 and references therein), AGN and radio-
galaxy feedback (Siemiginowska et al. 2005), intra-cluster cavities (McNamara et al. 2005) and radio bubbles (Birzan et al. 2004) filled with relativistic non-thermal electrons, multi-scale magnetic fields (see, e.g., Govoni & Feretti 2004) – imply the presence of additional electronic components with peculiar spectral and spatial characteristics (see Fig.2).

Finally, there are strong physical arguments indicating that viable Dark Matter candidate annihilation can produce copious amounts of secondary electrons with a spatial distribution which, in massive clusters like Coma, is strictly related to that of the original DM (see Colafrancesco et al. 2006a). It is interesting to note, in this context, that the spectral distribution of DM-produced secondary electrons carries information on the mass and the physical composition of the original DM particles.

In conclusion, the cluster electronic atmosphere is a complex combination of thermal (hot and warm) and non-thermal (quasi-thermal due to stochastic acceleration, relativistic due to DM annihilation and/or to AGN injection) distributions with different energy spectra and spatial distributions (see, e.g., Colafrancesco 2006b for a review). Each one of the electron populations which reside in the cluster atmosphere inevitably produces a distinct SZE with peculiar spectral and spatial features.

The description of the non-thermal SZE produced by a single electron population with a non-thermal spectrum has been attempted by various authors (McKinnon et al. 1991; Birkinshaw 1999; Ensslin & Kaiser 2000). Several limits to the non-thermal SZE are available in the literature (see, e.g., Birkinshaw 1999 for a review) from observations of galaxy clusters which contain powerful radio halo sources (such as Coma and A2163) or radio galaxies (such as A426), but only a few detailed analysis of the results (in terms of putting limits to the non-thermal SZE) have been possible so far (see Colafrancesco et al. 2003, Colafrancesco 2004a). The problem of detecting the non-thermal SZE in radio-halo clusters is likely to be severe because of the associated synchrotron radio emission, which could contaminate at low radio frequencies the small negative signal produced by the SZE. At higher frequencies there is in principle more chance to detect the non-thermal SZE, but even here there are likely to be difficulties in separating the SZE from the flat-spectrum component of the synchrotron emission (see Birkinshaw 1999). In addition, Colafrancesco (2004a) noticed that dust obscuration does not allow any detection of the SZ signal from clusters at frequencies $\gtrsim 600$ GHz.

From the theoretical point of view, preliminary calculations (Birkinshaw 1999, Ensslin & Kaiser 2000) of the non-thermal SZE have been carried out in the diffusion approximation ($\tau \ll 1$), in the limit of single scattering and for a single non-thermal population of electrons. Matters are significantly more complicated if the full relativistic formalism is used. However, this is necessary, since many galaxy clusters show extended radio halos and the electrons which produce the diffuse synchrotron radio emission are certainly highly relativistic so that the use of the Kompaneets approximation is invalid. Moreover, the co-spatial presence of thermal and non-thermal electrons renders the single
scattering approximation and the single population approach unreasonable, so that the treatment of multiple scattering among different electronic populations coexisting in the same cluster atmosphere is necessary to describe correctly the overall SZE. In this context, a complete and general derivation of the SZE produced by a general distribution of non-thermal electrons in the full-relativistic description, with multiple scatterings and also with different families of electrons co-spatially distributed has been provided by Colafrancesco et al. (2003). We will refer to this approach in our discussion aimed to probe the origin of every particle family using a single technique: the SZE.

Fig. 3. The function $g(x)$ is shown as a function of the adimensional frequency $x$ for different electronic populations residing in the cluster atmosphere: thermal with $k_B T_e = 8.2$ keV (blue); warm with $k_B T_e = 1$ keV (cyan); secondary electrons from DM annihilation with $M_\chi = 20$ GeV (red); relativistic electrons which fit the Coma radio halo integrated spectrum (yellow). Also the kinematic SZE with a negative peculiar velocity (green) is shown for comparison. The amplitudes of the various curves have been artificially re-normalized to highlight their frequency dependence.

3 The SZE: a generalized description

The generalized expression for the SZE which is valid in the Thomson limit ($\gamma h\nu \ll m_e c^2$ in the electron rest frame) for a generic electron population, in the full relativistic treatment and includes also the effects of multiple scatterings and the combination with other electron populations has been derived by Colafrancesco et al. (2003) and we will refer to this paper for technical details. Such derivation is has the advantage to describe both thermal and non-thermal SZE using a unique formalism in terms of a generalized Compton parameter $y$ and of a spectral function $\tilde{g}(x)$. 
According to these results, the spectral distortion observable in the direction of a galaxy cluster can be written as

\[ \Delta I(x) = 2 \left( \frac{k_B T_{CMB}}{hc} \right)^3 y \tilde{g}(x) , \]  

(7)

where the generalized Comptonization parameter \( y \) is given by

\[ y = \frac{\sigma_T}{m_e c^2} \int P_e d\ell , \]  

(8)

in terms of the pressure \( P_e \) contributed by the specific electron distribution within the cluster. The function \( \tilde{g}(x) \) for the considered electron population can be written, in its most general form, as

\[ \tilde{g}(x) = \frac{m_e c^2}{(k_B T_e)} \left\{ \frac{1}{\tau} \left[ \int_{-\infty}^{+\infty} i_0(xe^{-s})P(s)ds - i_0(x) \right] \right\} , \]  

(9)

in terms of the photon redistribution function \( P(s) \) and of the undistorted CMB spectrum \( i_0(x) = 2(k_B T_{CMB})^3/(hc)^2 \times x^3/(e^x - 1) \). The quantity

\[ \langle k_B T_e \rangle \equiv \frac{\sigma_T}{\tau} \int P_e d\ell = \int_0^\infty dp f_e(p) \frac{1}{3} pv(p)m_e c \]  

(10)

(see Colafrancesco et al. 2003) is the analogous of the average temperature for a thermal electron population (in this case \( \langle k_B T_e \rangle = k_B T_e \) obtains, in fact). The photon redistribution function \( P(s) = \int dp f_e(p)P_s(s;p) \) with \( s = \ln(\nu'/\nu) \), in terms of the CMB photon frequency increase factor \( \nu'/\nu \), contains the crucial dependence on the electron momentum distribution \( f_e(p) \), where \( p \) is normalized to \( m_e c \). Here \( P_s(s;p) \) is the mono-energetic frequency redistribution function (see Colafrancesco et al. 2003 for details). For a thermal electron population in the non-relativistic limit with momentum distribution \( f_{e,th} \propto p^2 e^{\eta \sqrt{T + p^2}} \) and \( \eta = m_e c^2/k_B T_e \), the pressure writes as \( P_{th} = n_e k_B T_e \) and it is easy from eq.(8) to re-obtain the Compton parameter

\[ y_{th} = \frac{\sigma_T}{m_e c^2} \int n_e k_B T_e d\ell = \frac{\tau k_B T_e}{m_e c^2} \]  

(11)

(we consider here, for simplicity, an isothermal cluster with electronic temperature \( T_e \)). The relativistically correct expression of the function \( \tilde{g}(x) \) for the
same thermal population of electrons writes, at first order in $\tau$, as:

$$\tilde{g}(x) = \frac{\Delta i}{y_{rh}} = \frac{\tau [j_1 - j_0]}{\tau \frac{P_1}{m_e c^2}} = \frac{m_e c^2}{k_B T_e} [j_1 - j_0], \quad (12)$$

where $j_i \equiv J_i \left( \frac{hc}{2(k_B T_0)} \right)^3$ and the functions $J_i$ are given in eq. (18) of Colafrancesco et al. (2003). Along the same line, it is possible to write the expression of $\tilde{g}(x)$ up to higher orders in $\tau$ (see Colafrancesco et al. 2003). The expression of $\tilde{g}(x)$ approximated at first order in $\tau$ (as given by eq. 12) is the one to compare directly with the expression of $g(x)$ obtained from the Kompaneets (1957) equation, since both are evaluated under the assumption of single electron-photon scattering. Fig. 1 shows how the function $\tilde{g}(x)$ tends to $g(x)$ for lower and lower IC gas temperatures $T_e$. This confirms that the distorted spectrum obtained from the Kompaneets equation is the non-relativistic limit of the exact spectrum. It is worth to notice that while in the non-relativistic case it is possible to separate the spectral dependence of the effect [which is contained in the function $g(x)$] from the dependence on the cluster parameters [which are contained in Compton parameter $y$], this is no longer valid in the relativistic case in which the function $J_1$ depends itself also on the cluster parameters. Specifically, for a thermal electron distribution, $J_1$ depends non-linearly from the electron temperature $T_e$ through the frequency redistribution function $P_1(s)$ (see Colafrancesco et al. 2003 for details). This means that, even at first order in $\tau$, the spectral shape $\tilde{g}(x)$ of the SZE depends on the cluster parameters, and mainly from the electron pressure $P_e$. This result is valid for any general electron distribution.

In order to generalize the derivation of the SZE to any arbitrary non-thermal electron distribution, it is necessary to use the most general expressions of $P_e$ and $f_e(p)$ in eqs. (8) and (9). For example, the general expressions for the pressure of a relativistic non-thermal distribution of electrons with a power-law spectrum, $n_{e,rel} = AE^{-\alpha}$, is the following:

$$P_e = n_{e,rel} \int_0^\infty dp f_e(p) \frac{1}{3} pv(p) m_e c = \frac{n_{e,rel} m_e c^2 (\alpha - 1)}{6 [p^{1-\alpha}]^{p_1}} \left[ B_{\frac{1}{1+p^2}} \left( \frac{\alpha - 2}{2}, \frac{3 - \alpha}{2} \right) \right]^{p_1} \quad (13)$$

(see, e.g., Ensslin & Kaiser 2000, Colafrancesco et al. 2003), where $B_x(a, b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$ is the incomplete Beta function. For a relativistic population of electrons the relation $\epsilon_e = 3 \cdot P_e$ holds. It is clear from eq. (13) that the pressure (and energy density) of the electron population depends mainly from the value of the minimum momentum $p_1$ for spectra with slopes $\alpha > 2$. For an electron population with a double power-law (or more complex) spectrum, analogous results can be obtained (see Colafrancesco et al. 2003).
At first order in $\tau$ the function $\tilde{g}(x)$ of the non-thermal SZE writes as

$$\tilde{g}(x) = \frac{\Delta i}{y_{\text{non-th}}} = \frac{\tau[j_1 - j_0]}{m_e c^2 \int P_e d\ell} \equiv \frac{m_e c^2}{(k_B T_e)}[j_1 - j_0],$$

and the Comptonization parameter $y_{\text{non-th}}$ writes, as a function of the quantity $\langle k_B T_e \rangle$ and of the optical depth $\tau$, as

$$y_{\text{non-th}} = \frac{\sigma_T}{m_e c^2} \int P_{\text{rel}} d\ell = \sigma_T \frac{\langle k_B T_e \rangle}{m_e c^2} \int n_{e,\text{rel}} d\ell = \frac{\langle k_B T_e \rangle}{m_e c^2} \tau.$$

The spectral function of the SZE evaluated for various electron spectra is shown in Fig. 3. It is clear that the different nature of the electron distribution reflects directly in the different spectral shape of the relative SZE. Thus, the SZE is a powerful probe of the energy spectrum of the various electronic populations residing in the atmospheres of cosmic structures. We will describe in the following the relevance of the SZE as a tool to study several different aspects of the astro-particle physics of cosmic structures in the universe.

### 3.1 The SZE and cosmic rays in galaxy clusters

We know that high-energy and relativistic particles exist in galaxy clusters through their diffuse radio-halo emission and their hard X-ray emission, but we do not know yet their origin. There are three viable scenarios for the production of high-energy particles in the atmospheres of galaxy clusters: i) direct acceleration or stochastic re-acceleration; ii) injection by AGNs or other compact objects; iii) Dark Matter annihilation.

**Testing the acceleration history of cosmic rays.** One of the most favourite scenarios is that high-energy particles (cosmic rays) are energized by stochastic acceleration due to e.g., cluster turbulence, merging shocks or other injection/acceleration mechanisms acting coherently on the large ($\sim$ Mpc) scale of the cluster atmospheres. Specific models of turbulent re-acceleration producing a high-energy tail up to relativistic energies (e.g., Brunetti et al. 2003) and stochastic acceleration of thermal particles producing a quasi-thermal tail (Dogiel et al. 2006) have been worked out. Whatever is the mechanism that accelerate particles, thus producing non-thermal or quasi-thermal spectral tails, it should be extremely efficient to beat the quite fast competing thermalization process for such high-energy particles (see, Wolfe & Melia 2006). As a result, the viable model produce rather different non-thermal or quasi-thermal electron spectra (see an example in Fig. 4) whose nature can be constrained by studying the associated SZE.

If the acceleration is so efficient to produce a high-E tail, the SZE attributed to
the relativistic electrons that produce the cluster radio-halo spectra observed in the 30 MHz - 5 GHz range is expected to be completely negative at all the frequencies relevant for the SZ experiments ($\nu \sim 20 - 350$ GHz) and with an amplitude of a few % of the thermal SZE in the cluster (see Colafrancesco et al. 2003, Colafrancesco 2004a, Shimon & Rephaeli 2003) which renders its detection quite challenging.

On the other hand, the SZE effect produced by quasi-thermal electrons stochastically accelerated in the cluster environment is expected to produce a detectable CMB temperature decrement $\Delta T \sim 40 - 50$ $\mu$K at low frequencies (Dogiel et al. 2006, see also Fig.4) that could prove or set relevant constraints to this scenario as well as to the efficiency of thermalisation and acceleration processes.

**Testing the content and the energetics of cluster cavities.** Cavities with diameters ranging from a few to a few hundreds of kpc have been observed by Chandra in the X-ray emission maps of several galaxy clusters and groups (see, e.g., Birzan et al. 2004, McNamara et al. 2005). These cavities are supposed to contain high-energy plasma with a non-thermal spectrum $n_{e,\text{rel}} \propto E_e^{-\alpha}$ with a typical index $\alpha \sim 2.5$. While the properties of these cavities and of the relativistic plasma they contain is usually studied by combining high-resolution X-ray and radio maps, we have proposed, as an alternative strategy, to study the consequences of the Compton scattering between the high-energy electrons filling the cavities and the CMB photon field (i.e. the SZE) whose amplitude, spectral and spatial features depend on the overall pressure and energetics of the relativistic plasma in the cavities (see Colafrancesco 2005a for the specific case of the cluster MS0735+7421). At frequencies $x \sim 2.5$ there is the maximum amplitude of the non-thermal SZE from the cavity (for
Fig. 5. Left. The geometry of the cavities in the cluster MS0735.6+7421 is shown. The two cavities have a radius \( \approx 100 \) kpc and are located at a distance of \( \approx 125 \) kpc and \( \approx 170 \) kpc from the central radio galaxy (whose position is indicated by a cross) along the axis represented in the picture. Right. The spectrum of the overall SZE from the cluster MS0735.6+7421 has been computed at a projected radius of \( \approx 125 \) kpc from the cluster center where the los passes through the center of cavity A. We show the thermal SZE (blue), the non-thermal SZE from the cavity (black) and the total SZE (red). The plotted curves are for different values of the lowest electron momentum: \( p_1 = 200 \) (dot-dashes), \( p_1 = 20 \) (dashes), \( p_1 = 10 \) (dots) and \( p_1 = 2 \) (solid). The non-thermal SZE is normalized to the cavity pressure \( P = 6 \times 10^{-11} \) erg cm\(^{-3}\) and, in this respect, it must be considered as a lower limit of the true SZE coming from the cavity.

a given electron spectrum and pressure). This produces a bump in the spatial distribution of the overall SZE at the cavity location with the addition of a negative SZE signal to the thermal SZE of the cluster (see Colafrancesco 2005a, Pfrommer et al. 2005). At \( x > x_{0,\text{th}}(P_{\text{th}}) \) we have the opposite effect but with smaller amplitudes: a depression in the SZE at the cavity location with the addition of a negative SZE signal to the positive thermal SZE. We emphasize that SZ observations at the frequency \( x = x_{0,\text{th}}(P_{\text{th}}) \) (which is \( \approx 3.87 \) for a \( k_B T = 5 \) keV cluster) provide a unique tool to probe the overall energetics, the pressure and the spatial extent of the non-thermal plasma contained in giant cavities, an observation which is rich in information and complementary to those obtained by X-ray and radio observations of cluster cavities. At this frequency, in fact, the overall SZE from the cluster reveals only the Compton scattering of the electrons residing in the cavities without the presence of the intense thermal SZE observable at lower and higher frequencies. Hence, the SZE from a cluster containing cavities (like the case of MS0735.6+7421) shows up uncontaminated at frequencies \( \sim 220 \) GHz: it is less extended than the overall cluster SZE because it is only emerging from the cavity regions and it is also well separable because the cavities are well defined in both X-rays and SZ images. We also emphasize, in addition, that the observation of the zero of the
non-thermal SZE in the cavities (which is found at high frequencies, depending on the value of \( p_1 \) or equivalently on the value of the pressure in the cavity) provides a definite way to determine uniquely the total pressure and hence the nature of the electron population within the cavity, an evidence which adds crucial, complementary information to the X-ray and radio analysis. A plausible source of bias to these observations could be provided by a possibly relevant kinematic SZE due to the cluster peculiar velocity (see Colafrancesco et al. 2003, Pfrommer et al. 2005 for a discussion). However, the SZE from the giant cavities in a cluster like MS0735.6+7421 can be effectively studied at frequency \( x = x_{0,th}(P_{th}) \approx 3.87 \) where it is not affected by the thermal SZE and it is only marginally affected by a possible kinematic SZE even at a level of a few hundreds km/s (see Colafrancesco 2005a). Such studies show more generally that the combination of high spatial resolution and high–sensitivity SZ observations with X-ray and radio data will definitely shed light on the morphology, on the physical structure, on the dynamics and the origin of these recently discovered non-thermal features of galaxy clusters. These studies will be also relevant to determine the impact of specific events of the nature of cavities on the use of SZ and X-ray clusters as probes for cosmology and for the large scale structure of the universe.

Testing relativistic electrons in powerful radio-galaxy jets/lobes The SZE produced in the jets of isolated powerful radio-galaxies is, similarly to the case of cluster cavities, completely non-thermal and it is, contrary to the case of cluster cavities, not contaminated by the surrounding ICM. Colafrancesco (2006a) showed that high-sensitivity and high-resolution SZE observations (like those achievable with ALMA) can also provide relevant information on the spectrum and on the pressure and energetic structure of the jets/lobes of powerful radio-galaxies.

3.2 SZE and the nature of Dark Matter

Dark Matter (DM) annihilations in the halo of galaxies and galaxy clusters have relevant astrophysical implications, even for SZE observations. Galaxy clusters and dark (dwarf) galaxies are, in fact, gravitationally dominated by Cold Dark Matter for which the leading candidate is the lightest supersymmetric (SUSY) particle, plausibly the neutralino \( \chi \). Experimental and theoretical considerations for having a cosmologically relevant neutralino DM lead to bound its mass \( M_\chi \) in the range between a few GeV to a few hundreds of GeV (see discussion in Colafrancesco et al. 2006a). The decays of neutralino annihilation products (fermions, bosons, etc.) yield, among other particles, energetic electrons and positrons up to energies comparable to the neutralino mass. Colafrancesco (2004b) proposed to explore the consequences of the Compton scattering between the secondary electrons produced from...
the WIMP annihilation in massive DM halos, like galaxy clusters and dwarf galaxies, and the CMB photon field, i.e. the DM-induced SZE which has specific spectral and spatial features. This is an inevitable consequence of the presence and of the nature of DM in large-scale structures. The analysis of

![Fig. 6. Left. A simple model which shows the basic astrophysical mechanisms underlying the search for the nature of DM particles ($\chi$) through the emission features occurring in large-scale structures (e.g., galaxy clusters and galaxies). These mechanisms are, among others: $\gamma$-ray emission from $n^0 \rightarrow \gamma + \gamma$, relativistic bremsstrahlung of secondary $e^{\pm}$ and ICS of CMB photons by secondary $e^{\pm}$; X-ray/UV emission due to non-thermal bremsstrahlung and ICS of background photons by secondary $e^{\pm}$; synchrotron emission by secondary $e^{\pm}$ diffusing in the intra-cluster magnetic field; $SZ_{DM}$ (ICS of CMB photons by secondary $e^{\pm}$) effect. Right. The overall SZE in Coma produced by the combination of various electron populations: thermal hot gas with $k_B T_e = 8.2$ keV and $\tau = 4.9 \cdot 10^{-3}$ (solid blue curve) which best fits the available SZ data (DePetris et al. 2002); relativistic electrons which best fit the radio-halo spectrum (yellow curve) provide a small additional SZE (Colafrancesco et al. 2003); warm gas with $k_B T_e \approx 0.1$ keV and $n \approx 10^{-3}$ cm$^{-3}$ (cyan curve) provides a small SZE due to its low pressure; DM produced secondary electrons with $M_\chi = 10$ (black dotted curve), 20 GeV (black solid curve) and 30 GeV (dashed solid curve). A pure-gaugino $\chi$ reference model is assumed in the computations. The relative overall SZE is shown as the dotted, solid and dashed red curves, respectively. A zero peculiar velocity of Coma is assumed consistently with the available limits. SZ data are from OVRO (magenta), WMAP (cyan) and MITO (blue).]

the DM induced SZE in galaxy clusters provides a probe for the presence and for the nature of DM in cosmic structures which is complementary to those obtainable through a multifrequency analysis, from radio to gamma-rays (see Fig.6 and Colafrancesco et al. 2006a). The available SZ observations on the Coma cluster (see Fig.6 and Colafrancesco 2004b) can already set a lower limit to the neutralino mass of $M_\chi \gtrsim 17 - 20$ GeV ($M_\chi \gtrsim 13$ GeV at 90 % c.l. with the adopted value of $\langle \sigma v \rangle_0 = 3 \cdot 10^{-27}$ cm$^{-3}$s$^{-1}/\Omega_\chi h^2$ with $\Omega_\chi h^2 = 0.116$), which is consistent with the limits set by accelerators (e.g., Belanger et al. 2003). The $SZ_{DM}$ signal does not strongly depend on the assumed DM density.
profile at intermediate angular distances from the cluster center and on the DM clumpiness since \( y_{\text{DM}} = \left( \frac{\sigma_{\text{T}}}{m_e c^2} \right) \int d\ell P_{\text{e,DM}} \), the integral of the total DM-produced secondary electron pressure, \( P_{\text{e,DM}} \), along the line of sight. The presence of a substantial SZ\(_{\text{DM}}\) effect is likely to dominate the overall SZ signal at frequencies \( x \gtrsim 3.8 - 4.5 \) providing a negative total SZE (see Fig. 6). It is, however, necessary to stress that in such frequency range there are other possible contributions to the SZE, like the kinematic effect and the non-thermal SZE which could provide additional biases (see, e.g., Colafrancesco et al. 2003). Nonetheless, the peculiar spectral shape of the SZ\(_{\text{DM}}\) effect is quite different from that of the kinematic SZE and of the thermal SZE and this allows to disentangle it from the overall SZ signal. An appropriate multifrequency analysis of the overall SZE based on observations performed on a wide spectral range (from the radio to the sub-mm region) is required, in principle, to separate the various SZ contributions and to provide an estimate of the DM induced SZE. In fact, simultaneous SZ observations at \( \sim 150 \) GHz (where the SZ\(_{\text{DM}}\) deepens the minimum with respect to the dominant thermal SZE), at \( \sim 220 \) GHz (where the SZ\(_{\text{DM}}\) dominates the overall SZE and produces a negative signal instead of the expected \( \approx \) null signal) and at \( \gtrsim 250 \) GHz (where the still negative SZ\(_{\text{DM}}\) decreases the overall SZE with respect to the dominant thermal SZE) coupled with X-ray observations which determine the gas distribution within the cluster (and hence the associated dominant thermal SZE) and lensing data (which determine the shape of the DM-dominated potential wells) can separate the SZ\(_{\text{DM}}\) from the overall SZ signal, and consequently, set constraints on the neutralino mass. Observations of the radio-halo emission in the cluster can provide an estimate of the cosmic-ray electron population and consequently an estimate of the associated non-thermal SZE (which is usually quite small and with a different spectral shape at high frequencies, see e.g., Colafrancesco et al. 2003). A particularly good case in which the SZ\(_{DM}\) could be revealed is that of the cluster 1E0657-556 (see Fig. 2) where the SZ\(_{DM}\) signal is peaked on the DM clumps, and is well separated (by several arcmin) from the thermal SZE concentrated on the X-ray emitting IC gas location and from the non-thermal SZE associated with the shock. We stress here that the SZ\(_{DM}\) signal is the only one remaining at frequencies \( x = x_{\text{o,th}} \) at the DM clump locations (see Colafrancesco et al. 2006c). Because the amplitude of the SZ\(_{DM}\) effect increases with decreasing values of \( M_\chi \), the high-sensitivity SZ experiments have, hence, the possibility to set reliable constraints to the nature, amount and spatial distribution of DM in galaxy clusters.

While the SZ\(_{DM}\) effect in galaxy clusters could be contaminated by other possible sources of SZE (thermal from both hot and warm gas, non-thermal from cosmic rays, relativistic electrons injected by radio-galaxies and AGNs, kinematic due to the cluster peculiar motion), the Comptonization of the CMB photons by secondary electrons produced by DM annihilation in a ”pure” DM halo (i.e., a dark galaxy) produce a SZ\(_{DM}\) which is un-contaminated and therefore carries secure information on the DM nature. Colafrancesco (2004b,
2005b) proposed that this is the case to be expected in a dwarf galaxy like Draco where no material other than DM and stars are present. In this respect, the possible detection of the $\text{SZ}_{\text{DM}}$ from nearby dwarf galaxies could either provide extremely clear and strong constraints on the DM nature or even detect their indirect signals. Detailed analysis on the SZE expected from Draco (Culverhouse, Evans & Colafrancesco 2006, Colafrancesco, Profumo & Ullio 2006b) revealed that the $\text{SZ}_{\text{DM}}$ signal is however quite tiny, and typically requires long duration observations with the future sensitive SZ experiments like ALMA. However, future SZ experiment with sub-$\mu$K sensitivity (and even moderate angular resolution) could be able to detect direct signals from DM annihilation in the most favourable supersymmetric scenarios.

### 3.3 SZE and magnetic fields

There are also non-standard aspects of the widely-used thermal SZE that need to be carefully and specifically addressed. In particular, the presence of an intra-cluster magnetic field can produce relevant changes to the thermal SZE in galaxy clusters (see, e.g., Colafrancesco & Giordano 2006a for details). We know, in fact, that magnetic fields exist in clusters of galaxies for several reasons: first, in many galaxy clusters we observe the synchrotron radio-halo emission produced by relativistic electrons spiraling along magnetic field lines; second, the Faraday rotation of linearly polarized radio emission traversing the ICM proves directly and independently the existence of intracluster magnetic fields (see, e.g., Carilli & Taylor 2002, Govoni & Feretti 2004 for reviews). Other estimates of the magnetic field strength on the cluster wide scale come from the combination of synchrotron radio and inverse Compton detections in the hard X-rays (e.g., Colafrancesco, Marchegiani & Perola 2005), from the study of cold fronts and from numerical simulations (see, e.g., Govoni & Feretti 2004). This evidence provides indication on the wide-scale B-field which is at the level of a few tens up to several $\mu$G (and in some cases up to $\sim 10 \mu$G, as in Coma) with the larger values being attained by the most massive systems. The presence of a wide-scale magnetic field implies modification on the thermal and density structure of the IC gas by acting on both the magnetic virial theorem (MVT, from which the overall IC gas temperature is derived, see Colafrancesco & Giordano 2006b)

$$ 2U + U_B + W = 0 , $$

(16)

(here $U$ is the kinetic energy, $W$ is the potential energy and $U_B$ is the magnetic energy of the system), and the hydrostatic equilibrium condition (HE, from which the radial profile of the IC gas density is derived, see Colafrancesco &
Fig. 7. The modifications of the spectral (left) and spatial distribution (right) of the thermal SZE for a $k_B T_e = 5$ keV cluster induced by a magnetic field $B(r) \propto B_s \rho_g^{0.9}$ for values $B_s = 0$ (solid), 0.5 (dotted), 1 (dot-dashes) and 3 (dashes) $\mu$G. Figure from Colafrancesco & Giordano (2006a).

Giordano 2006c)

\[
\frac{dP_g}{dr} + \frac{dP_B}{dr} = -\frac{GM(\leq r)}{r^2} \rho_g,
\]

(17)

where $\rho_g \equiv \rho_g(r, B)$ is the density of the thermal IC gas and $P_g(r, B)$ is its pressure, with both quantities depending in general on the magnetic field $B$. As a consequence, the cluster thermal SZE (see eqs. 1-4 and 7-11) is certainly influenced by the presence of an intracluster magnetic field, as shown in Fig.7. For idealized isothermal clusters, the magnetic SZE is reduced w.r.t. the unmagnetized case by two separate effects: i) the decrease in the cluster temperature as derived by the MVT (see eq.16) for increasing values of $B$; and ii) the decrease of the central IC gas density profile according to the HE condition (see eq.17) for increasing values of $B$. The variations of the thermal SZE for magnetized clusters is larger for lower-mass systems because in these structures the pressure provided by the magnetic field is of larger relative importance w.r.t. to the thermal pressure of the IC gas settling in the potential wells provided by Dark Matter. The increasing observational evidence, the refinements of numerical simulations and the theoretical expectations for the presence of magnetic fields in galaxy clusters render a revision of the standard description of the thermal SZE necessary to describe both single SZ observations and SZ scaling-law analyses in a self-consistent astrophysical and cosmological framework (see discussion in Colafrancesco & Giordano, 2006a).
4 Epilogue

The most recent achievements in multi-frequency (radio thru X-rays) precision observations of galaxies and galaxy clusters provided a wealth of detailed and, in some cases, unexpected physical information on the structure, physical state and composition of the atmospheres of these cosmic structures. Radio-halo non-thermal emission, hard–X-ray quasi-thermal excess emission, cluster cavities and radio bubbles, absence of strong radiative cooling in cool-cores, AGN’s feedback, Dark Matter distribution and dynamics, appear to be relevant ingredients for a detailed description of cluster structure and evolution: these processes put the standard description of the cluster atmosphere (as a single, thermal IC gas) to the ropes. Therefore, the theoretical modelling of galaxy cluster atmospheres evolved in the last decade up to a level of high-detail, complex physical description that requires to take consistently into account several electronic components (of thermal, non-thermal and relativistic nature), the effects of magnetic field, the feedback (heating, particle injection, magnetic field compression, etc.) produced by AGNs, radio-galaxies and blazars, the evolution and the interaction of relativistic plasma bubbles, the complex interplay of cooling and (non-thermal) heating processes in cool cores, the effects of a possible Dark Matter annihilation, the non-trivial physical conditions of the IC gas at cluster boundaries and its transition into the cosmic-web IGM distribution on larger scales.

As a consequence, the standard lore of the SZE is no longer viable in cosmic structures on large scales, like galaxy clusters. The simple SZ physics is no longer representative of the actual observational status; in this sense, it cannot provide neither reliable cluster physics nor an adequate cosmological use. It is therefore inevitable to go beyond the standard lore of the SZE. This provides, nonetheless, a way to use SZE as a single technique to efficiently study the leptonic structure of clusters/galaxy atmospheres and thus obtain information on the density, entropy, pressure and energetics of the electrons, the presence and the spectra of different electron populations, their (possible) equilibrium conditions.

The SZ signals expected from the non-thermal plasma, from DM annihilation and from cluster cavities and radio bubbles in addition to the thermal SZE and its modifications due to the inclusion of the intra-cluster magnetic field physics are usually in the range from a few $\mu$K to a few tens of $\mu$K. This is the reason why they have not yet clearly discovered in the past and ongoing SZ experiments. These additional SZ components are however present in the overall SZ signal observable from many galaxy clusters, and therefore they enrich the available physical information contained in the SZE data. Such signals could be disentangled from the thermal SZE by the future experiments with $\mathcal{O}(\mu K)$ sensitivity and sub-arcmin resolution. Nonetheless, they constitute a bias of complex spatial and spectral nature for those experiments with $\gtrsim 10\mu K$ sensitivity (like WMAP and PLANCK). In each case, their theoretical study and
simulation analysis is mandatory for any astrophysical and cosmological use to be reliably carried on. Such a program requires a definite technological effort which is directed towards high sensitivity (sub- or \( \sim \mu K \) level) and high spatial resolution (\( \sim \) arcsec–arcmin level) together with a wide-band continuum spectral coverage obtainable from space experiments (see Masi et al. 2006). This goal is definitely at the frontier of the present technology and is therefore a challenge for the future SZE experiments but it will be, nonetheless, able to open the door to the exploration of fundamental (astro-particle) issues like the nature of Dark Matter, the origin and the distribution of cosmic rays and magnetic fields in large-scale structures and other relevant questions which are on the discussion table of modern astrophysics and cosmology.

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