Thick brane solution with two scalar fields

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A new 5D thick brane solution is presented. We conjecture that the deduced thick brane is a plane defect in a bulk gauge condensate.

1. Introduction

In recent years there has been a revived interest in theories having a larger number of spatial dimensions than the three that are observed. In contrast to the original Kaluza-Klein theories of extra dimensions, the recent development of extra dimensional theories allow the extra dimensions to be large and even infinite in size (in the original Kaluza-Klein theories the extra dimensions were curled up or compactified to the experimentally unobservable small size of the Planck length: $10^{-33}$ cm). These new extra dimensional theories have opened up new avenues to explaining some of the open questions in particle physics (the hierarchy problem, nature of the electro-weak symmetry breaking, explanation of the family structure) and astrophysics (the nature of dark matter, the nature of dark energy). In addition they predict new experimentally measurable phenomena in high precision gravity experiments, particle accelerators, and in astronomical observations.

We consider 5D gravity + two interacting fields.\textsuperscript{1} The key for the existence of a regular solution here is that the scalar fields potential have to have local and global

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minima, and at infinity the scalar fields tend to a local but *not* to global minimum.

The 5D metric is

$$ds^2 = a(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \quad (1)$$

The Lagrangian for the scalar fields $\phi$ and $\chi$ is

$$\mathcal{L} = \frac{1}{2}(\nabla A \phi \nabla^A \phi + \frac{1}{2} \nabla A \nabla^A \chi - V(\phi, \chi)), \quad (2)$$

where $A = 0, 1, 2, 3, 5$. The potential $V(\phi, \chi)$ is

$$V(\phi, \chi) = \frac{\lambda_1}{4} (\phi^2 - m_1^2)^2 + \frac{\lambda_2}{4} (\chi^2 - m_2^2)^2 + \phi^2 \chi^2 - V_0, \quad (3)$$

where $V_0$ is a constant which can be considered as a 5D cosmological constant $\Lambda$.

The profile of the potential $V(\phi, \chi)$ is presented in Fig. 1. The corresponding field equations are

$$\frac{a''}{a} - \frac{a''}{a^2} = -\frac{1}{2} (\phi'^2 + \chi'^2), \quad (4)$$

$$\frac{a''}{a^2} = \frac{1}{8} \left[ \phi'^2 + \chi'^2 - \frac{\lambda_1}{2} (\phi^2 - m_1^2)^2 - \frac{\lambda_2}{2} (\chi^2 - m_2^2)^2 - \phi^2 \chi^2 + 2V_0 \right]. \quad (5)$$

$$\phi'' + 4 \frac{a'}{a} \phi' = \phi \left[ \chi^2 + \lambda_1 \left( \phi^2 - m_1^2 \right) \right], \quad (6)$$

$$\chi'' + 4 \frac{a'}{a} \chi' = \chi \left[ \phi^2 + \lambda_2 \left( \chi^2 - m_2^2 \right) \right]. \quad (7)$$

The boundary conditions are

$$a(0) = a_0, \ a'(0) = 0, \ \phi(0) = \phi_0, \ \phi'(0) = 0, \ \chi(0) = \chi_0, \ \chi'(0) = 0. \quad (8)$$

The boundary condition $\phi(0) = \phi_0$ and Eq. (5) give us the following constraint

$$V_0 = \frac{\lambda_1}{4} (\phi_0^2 - m_1^2)^2 + \frac{\lambda_2}{4} (\chi_0^2 - m_2^2)^2 + \frac{1}{2} \phi_0^2 \chi_0^2, \quad (9)$$

Fig. 1. The profile of the potential $V(\phi, \chi)$. 

![](image.png)

The potential $V(\phi, \chi)$. 

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This means that the constant $V_0$ is not an arbitrary constant but it is defined by the $Z_2$–symmetry of the thick brane.

The mathematical problem for solving these equations is that not for all values of the parameters $m_{1,2}$ a regular solution exists. Thus the problem of finding a regular solution of these equations is a non-linear eigenvalue problem for the parameters $m_{1,2}$ and the eigenfunctions $\phi, \chi$. In Fig. 2 we present the regular solution which describes the thick brane solution for the 5D gravity. The dimensionless energy density is presented in Fig. 3. The thickness of the brane depends on all parameters which are included in the equations: $\lambda_{1,2}, \phi(0), \chi(0)$.

2. Main features of the presented solution

- At the infinity the scalar fields tends to a local minimum not to global.
- The first item leads to the fact that the solution is topologically trivial.
- Mathematically the solution of Einstein-scalar fields equations is a non-linear eigenvalue problem.
- There are arguments (2) that these scalar fields present non-perturbatively quantized SU(3) gauge field. In this case the brane world is a plain defect in 5D spacetime filled with a gauge condensate.

More background material can be found in Refs. 3 and 4.

References

1. V. Dzhunushaliev, “Thick brane solution in the presence of two interacting scalar fields”, gr-qc/0603020.