Remarks on Yang-Mills Theory

Tai Tsun Wu¹

Gordon McKay Laboratory, Harvard University, Cambridge, MA 02138

and

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

The Yang-Mills theory is the most important development in physics in the second half of this century. Some of the highlights are briefly summarized.

¹Presented at Symmetries and Reflections, a Symposium in honor of Professor C. N. Yang, May 22, 1999. Work supported in part by the United States Department of Energy under Grant No. DE-FG02-84ER40158.

During my first year as a graduate student at Harvard, Professor Yang came to give a Monday Physics Colloquium. It was on the newly conceived Yang-Mills theory. Since I had not yet learned quantum field theory, I was not able to follow his discussions. It is my understanding that in 1954 Professor Yang gave just two seminars on this new theory, the first one at the Institute for Advanced Study and this was the second one.

If I remember correctly, Professor Yang gave this Colloquium in May of 1954. If this is correct, then it was almost exactly forty-five years ago. It was the first time I met Professor Yang.

Shortly after carrying out this work, Yang and Mills gave a summary at the Washington meeting of the American Physical Society [1] and also submitted their famous paper for publication in the Physical Review [2]. The Abstract of their talk is reproduced as Fig. 1; it took place on April 30, 1954, with Professor Freeman Dyson presiding.

The title and abstract of Reference [2] is shown in Fig. 2, and some of the basic equations from this paper are shown in Fig. 3. There are a number of remarkable features in this paper besides the fundamental concept of non-Abelian gauge fields; let me mention just the following two.

First, after the Introduction section, Gauge Transformation is discussed before the Field Equations, as seen from Fig. 3. This is the authors' way of emphasizing the importance of gauge transformation.

Secondly, near the end of this paper, just after their Fig. 1 which shows the self-interaction of the Yang-Mills b field, there is a paragraph discussing its mass. This paragraph is reproduced as Fig. 4. We shall return to this prophetic paragraph later.

Generalization to other Lie groups is straightforward. Therefore, the Yang-Mills theory gives a principle for writing down the interactions of elementary particles.

It is natural to ask the question: Which Lie groups are appropriate for physics? When Yang and Mills wrote their papers, there was no way for them to prefer one Lie group over another. This important question was answered in the 1960's by Glashow, Weinberg and Salam [3] for electroweak interactions and in 1973 by Fritzsch, Gell-Mann and Leutwyler [4] for strong interactions. The answers are $SU(2) \times U(1)$ for electroweak and $SU(3)$ for strong interactions.

Aside from this important answer about the choice of Lie groups, some of the developments, both theoretical and experimental, of the Yang-Mills theory are shown in Fig. 5. For example, twenty years after 1954, the global formulation of Yang-Mills theory was accomplished [5]. This makes connection not only with non-integrable phase factors but also with fiber bundles. The resulting translation between the gauge-field and the fiber-bundle terminologies is reproduced in Table I. The group $SU(2)$ is readily replaced by other non-Abelian Lie groups.

On the experimental side, twenty-five years after 1954, the Yang-Mills particle was observed for the first time. It was the one for strong interactions, and this feat was accomplished by the TASSO Collaboration at DESY [6]. Nearly thirty years after 1954, the Yang-Mills particle for electroweak interactions was observed for the first time by the UA(1) Collaboration at CERN [7]. These pioneering experimental observations completed the list of all the vector bosons of the standard model [3, 4].

Let me return to the interesting paragraph of the Yang-Mills paper shown in Fig. 4. When Professor Yang gave the seminar on this new theory at the Institute for Advanced Study in February of 1954, Pauli repeatedly asked the question: What is the mass of this field B_μ ?

No definitive answer was possible at that time, and by now it is clear why. The Yang-Mills particle for strong interactions, now often called the gluon, is massless, while the Yang-Mills particles Z and W for electroweak interactions are massive, about $80\text{--}91\text{ GeV}/c^2$. Therefore both possibilities, massless and massive, can happen for Yang-Mills particles and actually do. This is the reason why the question raised by Pauli could not have been answered at that time, and also why the paragraph of Fig. 4 can only be called prophetic.

In summary, since 1954 particle physicists have been learning about and developing the Yang-Mills theory, which permeates throughout particle physics. The Yang-Mills theory is the most important and deepest development in physics for the second half of the twentieth century.

I would like to thank the Theory Division of CERN for its kind hospitality.

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