Quantum correlations are a signature of nonclassical light generation. The optical parametric oscillator (OPO) is the best-known and most widely used source of such correlations. Squeezing in the intensity difference of the twin beams it produces above-threshold (signal and idler) \[\text{BF} 1\] reached the record value of \(-9.7\) dB \[\text{BF} 2\]. Bi-partite continuous variable (CV) entanglement in this system, which requires the observation of phase anti-correlations as well, was only demonstrated very recently \[\text{BF} 3, 4, 5\]. The parametric process involves three fields, yet the pump field is typically treated as a classical quantity. As an exception, quantum properties of the pump were first measured by Kasai et al. \[\text{BF} 6\]. This arises a natural question: are there quantum correlations between all three fields? An affirmative answer has been recently given by Villar et al. \[\text{BF} 7\], who investigated the problem theoretically. Here, we provide the first experimental affirmative answer by observing triple correlations between quadratures of pump, signal, and idler fields.

Beyond the demonstration of nonclassical light features, one should notice that all three fields have different frequencies. Indeed, quantum correlations exist between quadratures of pump, signal, and idler fields. The above-threshold OPO produces, in general, nondegenerate twin beams, as we show below.

Quantum correlations between bright pump, signal, and idler beams produced by an optical parametric oscillator, all with different frequencies, are experimentally demonstrated. We show that the degree of entanglement between signal and idler fields is improved by using information of pump fluctuations. This is the first observation of three-color optical quantum correlations.

PACS numbers:

Quantum correlations should exist between the phase quadratures of the pump field and the sum of phase quadratures of signal and idler fields as a direct consequence of energy conservation \[\omega_0 = \omega_1 + \omega_2\]. Indeed, by relating frequency fluctuations to phase fluctuations, we obtain \[\delta \phi_0 = \delta \phi_1 + \delta \phi_2\]. Indices \(j \in \{0, 1, 2\}\) refer to pump, signal, and idler fields, respectively. The quadratures are defined through the field annihilation operators \(\hat{a}_j = \exp(i \hat{\phi}_j) (\hat{p}_j + i \hat{q}_j)\), where \(\hat{\phi}_j\) is chosen so that \[\langle \hat{q}_j \rangle = 0\]. When the OPO is detuned from exact triple resonance, this phase-phase correlation, \[C_{\hat{q}_0 \hat{q}_+} = \langle \delta \hat{q}_0 \delta \hat{q}_+ \rangle\], is partially transferred to an amplitude-phase correlation, \[C_{\hat{p}_0 \hat{q}_+} = \langle \delta \hat{p}_0 \delta \hat{q}_+ \rangle\], owing to phase noise to amplitude noise conversion \[\text{BF} 10\] inside the OPO cavity \[\hat{q}_+ \equiv (\hat{q}_1 + \hat{q}_2)/\sqrt{2}\]. Our experiment is designed to measure joint fluctuations of a combination of \(\hat{q}_+\) and \(\hat{p}_0\) and compare them to the shot noise level, which defines the standard quantum limit (SQL). This will enable us to improve the bipartite entanglement of the twin beams.

Twin beam entanglement is proven by violation of an inequality derived by Duan et al. \[\text{BF} 11\] and Simon \[\text{BF} 12\]. Van Loock and Furusawa \[\text{BF} 13\] generalized it to include a third field:

\[
\Delta^2 \hat{p}_- + \Delta^2 (\hat{q}_+ - \alpha_0 \hat{p}_0) \geq 2, \quad \text{with} \quad \alpha_0 = \frac{C_{\hat{p}_0 \hat{q}_+}}{\Delta^2 \hat{p}_0} .
\]

Here \(\alpha_0\) is a parameter chosen to minimize the left-hand side of the expression above and \(\hat{p}_- = (\hat{p}_1 + \hat{p}_2)/\sqrt{2}\). Each term \(\Delta^2 \hat{p}_-\) and \(\Delta^2 (\hat{q}_+ - \alpha_0 \hat{p}_0)\) is normalized to the SQL. Our attention will be focused on the corrected phase sum noise (second term above), which can be rewritten as

\[
\Delta^2 \hat{q}_+ \equiv \Delta^2 \hat{q}_+ - \beta_0, \quad \text{with} \quad \beta_0 = \frac{C_{\hat{p}_0 \hat{q}_+}^2}{\Delta^2 \hat{p}_0}.
\]

The experimental setup is sketched in Fig. 1. The triply resonant type-II OPO is pumped by a frequency doubled diode-pumped Nd:YAG laser (Innolight Diabolo) at 532 nm. This laser is first transmitted through...
a filter cavity (bandwidth of 2.4 MHz) prior to injection in the OPO, which removes all classical noise for analysis frequencies above 15 MHz. It is important to have a shot noise-limited pump, since excess pump phase noise is converted to excess noise in \( \hat{q}_+ \), thus hindering twin beam entanglement [14]. The nonlinear crystal is a 10 mm long Potassium Titanyl Phosphate (KTP) from Litton. The OPO cavity input mirror is flat, directly coated on one crystal surface, with 97% reflection at 532 nm and high reflectivity (R>99.8%) at 1064 nm. The other crystal surface is anti-reflection coated for both wavelengths (R<3% at 532 nm and R<0.25% at 1064 nm). The spherical output mirror is a high reflector for 532 nm (R>99.8%) and partial reflector for 1064 nm, R=96%, with a curvature radius of 25 mm. The OPO cavity bandwidth for 1064 nm is 50 MHz and the threshold power is 12 mW. Orthogonally polarized signal and idler beams are separated by a polarizing beam splitter (PBS). In order to measure their quadrature noise, each beam is reflected off an analysis optical cavity, which converts phase noise to amplitude noise as a function of its detuning [13][14]. The twin beams are finally detected on high quantum efficiency (> 93%) photodiodes (Epi- taxx ETX300). Both analysis cavities have bandwidths of 14(1) MHz. Overall detection efficiency is \( \eta = 80\% \). Signal and idler optical frequencies differ by approximately 0.35 THz, corresponding to \( \Delta \lambda = 1.3 \) nm in wavelength. Photocurrents are recorded as a function of time as both cavities are synchronously scanned. At the same time, amplitude fluctuations of the reflected pump beam (extracted by means of a PBS and a Faraday Rotator) are recorded by another photodetector (EG\&G FND100, quantum efficiency 60%), with an overall detection efficiency of \( \eta_0 = 45\% \) (we are currently working to improve this value by using higher quantum efficiency photodiodes). Noise power spectra are obtained by direct demodulation of the photocurrents. Each photocurrent is electronically mixed with the same sinusoidal reference at the analysis frequency \( \nu = 27 \) MHz and the low frequency beat signal is sampled at a 600 kHz repetition rate by an analog-to-digital (A/D) card connected to a computer. Variances of these fluctuations are then calculated by taking groups of 1000 points, and finally normalized to the SQL.

Typical noise spectra of sum and difference of twin beam quadratures are presented in Fig. 2 as functions of analysis cavities’ detuning relative to their bandwidth, \( \Delta \). Phase noise \( \Delta^2 \hat{q}_\pm \) is measured for \( \Delta = 0.5 \) and also (partially) for \( \Delta \equiv 1.4 \). Far off \(|\Delta| \geq 2.5 \) and on exact resonance (\( \Delta = 0 \)) the amplitude noise \( \Delta^2 \hat{p}_\pm \) is measured. The squeezed difference of amplitude quadratures \( \Delta^2 \hat{p}_- = 0.53(2) \) and the shot noise limited sum of phase quadratures \( \Delta^2 \hat{q}_+ = 0.99(2) \) suffice to demonstrate bipartite entanglement, since \( \Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.52(3) < 2 \).

Quantum correlations between pump and downconverted fields are demonstrated in the full line curve. It shows that \( \Delta^2 \hat{q}_- \) can be reduced by using information from the pump beam amplitude. When corrected by \( \beta_0 = 0.13(3) \), the shot noise limited sum of phases \( \Delta^2 \hat{q}_+ \) becomes squeezed: \( \Delta^2 \hat{q}_+ = 0.86(2) \). The generalized criterion of Eq. 1 assumes the improved value \( \Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.39(3) < 2 \).

The behaviors of \( \beta_0, \Delta^2 \hat{q}_+ \), and \( \Delta^2 \hat{q}_+ \) are presented in Fig. 3 as functions of pump power relative to threshold, \( \sigma \). Each experimental point was taken from curves similar to those of Fig. 2. The solid lines were calculated from the standard linearized OPO theory [14]. As described in Ref. [17] the theory has to be corrected to include extra noise acquired by the intracavity pump field and which is not related to the parametric process. We
model this by simply adding noise to the input pump field. This noise depends on the OPO cavity detuning for the pump field \( \Delta' \). Since in our present experiment we do not have precise control or knowledge of OPO cavity detunings for pump, \( \Delta'_0 \), and downconverted fields, \( \Delta' \) (all normalized to the OPO cavity bandwidth for the twin beams), these are used as free parameters to fit the data of Fig. 1. Furthermore, the excess phase noise added to the input pump, \( S_{q_0}^{in} \), which is deduced from independent measurements of the reflected pump beam for \( \Delta'_0 = 0 \) and \( \sigma = 0 \) (\( S_{q^0}^{in} \approx 23 \)), also has to be adjusted for the nonzero detunings.

We verify that \( \Delta^2 q_+ \) is squeezed close to threshold and its noise increases as the pump power is increased, crossing the shot noise value at \( \sigma \approx 1.2 \). Its behavior was studied in detail in Ref. 17. The correction term \( \beta_0 \) is always nonzero, varying from \( \beta_0 \approx 0.10 \) to \( \beta_0 \approx 0.23 \) for increasing \( \sigma \). This is in agreement with the theoretical prediction of Ref. 2 since the degree of triple correlations should be maximum close to \( \sigma = 1.5 \), where all fields have approximately the same intensity. In particular, for \( \sigma \leq 1.3 \), the correlation between \( \hat{p}_0 \) and \( \hat{q}_+ \) reveals or increases the squeezing value in \( \Delta^2 q_+ \), attesting its quantum nature. Better control of the detunings should be maximum close to \( \sigma = 0 \), \( q_0 \) and \( q_+ \) should be strongly correlated.

In summary, twin beam entanglement produced by an above-threshold OPO can be improved by using the quantum correlations with the pump beam demonstrated here. To our knowledge, this is the first experimental demonstration of three-color quantum correlations. They can be used, for instance, to increase the fidelity of quantum information distribution. The measurement of triple optical quantum correlations is a necessary first step en route to the observation of three-color entanglement in the above-threshold OPO.

Acknowledgments

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, through Instituto do Milênio de Informação Quântica).

References