The results of an extensive flavour physics analysis in the Littlest Higgs model with T-parity are presented. In particular, mixing and CP-violation in $K^0 - \bar{K}^0$ and $B^0_d,s - \bar{B}^0_d,s$ systems have been studied. Several scenarios defined by the values of mirror quark masses and the parameters of the new flavour mixing matrix $V_{Hd}$ have been considered. A very interesting scenario is identified, where the well-known kaon constraints are satisfied, the $\sin 2\beta$ and $\Delta M_s$ issues are improved w.r.t. the Standard Model and visible effects could be soon found in $A_{SL}$ and $S_{\psi \phi}$.

Keywords: Beyond Standard Model; CP violation.

1. The Littlest Higgs Model

The Standard Model (SM) is in excellent agreement with the results of particle physics experiments, in particular with the electroweak (ew) precision measurements, thus suggesting that the SM cutoff scale is at least as large as 10 TeV. Having such a relatively high cutoff, however, the SM requires unsatisfactory fine-tuning to yield a correct ($\approx 10^2$ GeV) scale for the Higgs mass, being the Higgs boson a fundamental scalar with one-loop quadratically divergent corrections to its squared mass. This "little hierarchy problem" has been one of the main motivations to elaborate models of physics beyond the SM. While Supersymmetry is at present the leading candidate, different proposals have been formulated more recently. Among them, Little Higgs models play an important role, being perturbatively computable up to about 10 TeV and with a rather small number of parameters.

In Little Higgs models, the Higgs is interpreted as a Nambu-Goldstone boson (NGB) corresponding to a spontaneously broken global symmetry, thus explaining its lightness. An exact NGB, however, would have only derivative interactions. Gauge and Yukawa interactions of the Higgs have to be incorporated. This can be done without generating quadratically divergent one-loop contributions to the Higgs mass, through the so-called collective symmetry breaking. The collective symmetry breaking (SB) has the peculiarity of generating the Higgs mass only when two or more couplings in the Lagrangian are non-vanishing. This mechanism is diagrammatically realized through the cancellation of the SM quadratic divergences by the contributions of new particles with masses around 1 TeV.

The most economical, in matter content, Little Higgs model is the Littlest Higgs (LH) model, where the global group $SU(5)$ is spontaneously broken into $SO(5)$ at the scale $f \approx \mathcal{O}(1 \text{ TeV})$ and the ew sector of the SM is embedded in an $SU(5)/SO(5)$ nonlinear sigma model. Gauge and Yukawa Higgs interactions are introduced by gauging the subgroup of $SU(5)$: $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$, with gauge couplings respectively equal to $g_1, g'_1, g_2, g'_2$. The key feature for the realization of collective SB is that the two gauge factors commute with a different $SU(3)$ global symmetry subgroup of $SU(5)$, implying that neither of the gauge factors alone can generate a potential for the Higgs. Consequently, quadratic corrections
to the squared Higgs mass involve two couplings and cannot appear at one-loop. In the LH model, the new particles appearing at the TeV scales are the heavy gauge bosons \((W_H^\pm, Z_H, A_H)\) the heavy top \((T)\) and the scalar triplet \(\Phi\).

In the LH model, significant corrections to EW observables come from tree-level heavy gauge boson contributions and the triplet vacuum expectation value (vev) which breaks the custodial SU(2) symmetry. Consequently, EW precision tests are satisfied for values of the NP scale \(f \geq 2 - 3 \text{ TeV}\), too large to solve the little hierarchy problem.

Motivated by reconciling the LH model with EW observables, Cheng and Low\(^{10}\) proposed to enlarge the symmetry structure of the theory by introducing a discrete symmetry called T-parity. T-parity acts as an automorphism which exchanges the [SU(2) x SU(1)]\(_1\) and [SU(2) x SU(1)]\(_2\) gauge factors. The invariance of the theory under this automorphism implies \(g_1 = g_2\) and \(g_1' = g_2'\). Furthermore, in studied observables, T-parity explicitly forbids the tree-level contributions of heavy gauge bosons and the interactions that induced the triplet vev. The custodial SU(2) symmetry is restored and the compatibility with EW precision data is obtained already for smaller values of the NP scale, \(f \geq 500\) GeV\(^{3,4}\). Another important consequence is that particle fields are T-even or T-odd under T-parity. T-even states are the SM particles and the heavy top \(T_+\), T-odd states, instead, are the heavy gauge bosons \(W_H^\pm, Z_H, A_H\), the scalar triplet \(\Phi\) and additional particles required by T-parity: the odd heavy top \(T_-\) and the so-called mirror fermions, i.e., fermions corresponding to the SM ones but with opposite T-parity and \(O(1\text{ TeV})\) mass. Mirror fermions are characterized by new flavour interactions with SM fermions and heavy gauge bosons, which involve in the quark sector two new unitary mixing matrices analogous to the Cabibbo-Kobayashi-Maskawa (CKM) matrix \(V_{CKM}\).

They are \(V_{Hd}\) and \(V_{Hu}\), respectively involved when the SM quark is of down- or up-type, and satisfying \(V_{Hd}^\dagger V_{Hd} = V_{CKM}\). The Littlest Higgs model with T-parity (LHT) does not belong to the Minimal Flavour Violation (MFV) class of models, where the CKM matrix is the only source of flavour and CP-violation. The LHT peculiarities are the rather small number of new particles and parameters (the SB scale \(f\), the parameter \(x_L\) describing \(T_+\) mass and interactions, the mirror fermion masses and \(V_{Hd}\) parameters), the absence of new operators in addition to the SM ones and, at the same time, the possibility to have visible effects in flavour observables.

2. LHT Flavour Analysis

Several studies of flavour physics in the LH model without T-parity have been performed in the last three years\(^{10}\). Without T-parity, mirror fermions and new sources of flavour and CP-violation are absent, the LH model is a MFV model and NP contributions result to be very small.

More recently, the first flavour physics analysis in the LHT model has been performed\(^{8}\), where the mass differences \(\Delta M_K, \Delta M_{d,s}, \Delta M_D\) and the CP-violation parameter \(\varepsilon_K\) have been studied. We have confirmed\(^{11}\) their analytic expressions and performed a wider phenomenological analysis in the LHT model, including also the width differences \(\Delta \Gamma_{d,s}\), the radiative decays \(B \to X_{s,d}\gamma\) and the CP-violating asymmetries \(A_{CP}(B_d \to \psi K_S), A_{CP}(B_s \to \psi\phi)\) and \(A_{SL}^{d,s}\). Two interesting issues considered in our analysis are the possible discrepancy between the values of \(\sin2\beta\) following directly from \(A_{CP}(B_d \to \psi K_S)\) and indirectly from the usual analysis of the unitarity triangle\(^{12,13}\), and the recent measurement of the angle \(\phi\) in the LHT model, which is different from the SM prediction. The recent measurement of the angle \(\phi\) in the LHT model, which is different from the SM prediction, also suggests that the parameter \(\varepsilon_K\) should be smaller than its SM value. The first flavour physics analysis in the LHT model has been performed\(^{8}\), where the mass differences \(\Delta M_K, \Delta M_{d,s}, \Delta M_D\) and the CP-violation parameter \(\varepsilon_K\) have been studied. We have confirmed\(^{11}\) their analytic expressions and performed a wider phenomenological analysis in the LHT model, including also the width differences \(\Delta \Gamma_{d,s}\), the radiative decays \(B \to X_{s,d}\gamma\) and the CP-violating asymmetries \(A_{CP}(B_d \to \psi K_S), A_{CP}(B_s \to \psi\phi)\) and \(A_{SL}^{d,s}\). Two interesting issues considered in our analysis are the possible discrepancy between the values of \(\sin2\beta\) following directly from \(A_{CP}(B_d \to \psi K_S)\) and indirectly from the usual analysis of the unitarity triangle\(^{12,13}\), and the recent measurement of the angle \(\phi\) in the LHT model, which is different from the SM prediction. The recent measurement of the angle \(\phi\) in the LHT model, which is different from the SM prediction, also suggests that the parameter \(\varepsilon_K\) should be smaller than its SM value.
measurement of $\Delta M_s$ by the CDF and D0 collaborations, that although close to the SM value is slightly smaller than expected. An important result of our study is that in the LHT model $\Delta M_s$ can be smaller than in the SM, as experimentally indicated, and that, for the same range of parameters, the “sin 2$\beta$” discrepancy can be cured and visible effects in observables unknown or with still large uncertainties like $A_{CP}(B_s \to \psi \phi)$ and $A_{SL}^d$ can be found.

In our LHT flavour analysis we have considered several scenarios for the structure of the $V_{td}$ matrix and the mass spectrum of mirror fermions in order to gain a global view over possible signatures of mirror fermions and T-even contributions. In all these scenarios the two additional phases of $V_{td}$, whose impact is numerically small, have been set to zero. The CKM parameters entering the analysis have been taken from tree level decays only, where NP effects can be neglected. In order to simplify the numerical analysis we have set all non-perturbative parameters to their central values, while allowing $\Delta M_K$, $\varepsilon_K$, $\Delta M_d$, $\Delta M_s$, $\Delta M_s/\Delta M_d$ and $S_{\psi K}$ to differ from their experimental values by $\pm 50\%$, $\pm 40\%$, $\pm 40\%$, $\pm 20\%$ and $\pm 8\%$, respectively. This rather conservative choice guarantees that interesting effects are not missed. In scenarios 3 – 5, described below, the parameters $f$ and $x_L$ have been fixed to $f = 1$ TeV and $x_L = 0.5$ in accordance with electroweak precision tests. The main features of the five studied scenarios are described below.

**Scenario 1:** Mirror fermions are degenerate in mass, therefore only the T-even sector contributes. This is the MFV limit of the LHT model where a new phase curing the sin 2$\beta$ discrepancy cannot appear and $(\Delta M_s)_{LHT} \geq (\Delta M_s)_{SM}$, disfavored by the CDF measurement is found.

**Scenario 2:** A scan over non-degenerate mirror fermion masses is performed, while $V_{td}$ has the same structure of the CKM matrix ($V_{td} = V_{CKM}$). Similarly to the previous scenario, no improvements concerning the sin 2$\beta$ and $\Delta M_s$ issues are achieved.

**Scenario 3:** Mirror fermion masses are $m_{H1} = 400$ GeV, $m_{H2} = 500$ GeV, $m_{H3} = 600$ GeV, while for $V_{td}$ an arbitrary structure is allowed. The freedom in $V_{td}$ allows to soften the sin 2$\beta$ discrepancy, while for $\Delta M_s$ no improvement is found.

**Scenario 4:** This is the most interesting scenario where the $\Delta M_s$ and sin 2$\beta$ issues can be improved and, simultaneously, visible departures from the SM and MFV can be obtained, mainly in $A_{SL}^d$ and $A_{CP}(B_s \to \psi \phi)$. Here $m_{H1} \approx m_{H2} \approx 500$ GeV, $m_{H3} = 1000$ GeV, $1/\sqrt{2} \leq s_{12}^d \leq 0.99$, $5 \cdot 10^{-5} \leq s_{23}^d \leq 2 \cdot 10^{-4}$, $4 \cdot 10^{-2} \leq s_{13}^d \leq 0.6$ and the phase $\delta_{13}^d$ is arbitrary. The latter sines and phase are those describing $V_{td}$ in a parameterization similar to the usual $V_{CKM}$’s one. The very different hierarchy of $V_{td}$ w.r.t. $V_{CKM}$, with a large complex phase in the $(V_{td})_{32}$ element assures large CP-violating effects in the $B_d^0 - \bar{B}_d^0$ system. Moreover, $\Delta M_s$ can be smaller than its SM value and interesting effects in the $B_d^0 - \bar{B}_d^0$ system are found. The effects in the kaon system are unimportant, being the first two mirror fermion masses almost degenerate, as required by the $\Delta M_K$ and $\varepsilon_K$ constraints.

**Scenario 5:** In all the previous scenarios the SM solution ($71^\circ \pm 16^\circ$) has been considered for the angle $\gamma$ such that only small departures from the SM in the $B_d^0 - \bar{B}_d^0$ system can be consistent with the data. In this last scenario, instead, we have assumed the second solution $\gamma = -109^\circ \pm 16^\circ$. Moreover, $m_{H1} = 500$ GeV, $m_{H2} = 450$ GeV, $m_{H3} = 1000$ GeV, $5 \cdot 10^{-5} \leq s_{12}^d \leq 0.015$, $2 \cdot 10^{-2} \leq s_{23}^d \leq 4 \cdot 10^{-2}$, $0.2 \leq s_{13}^d \leq 0.5$ and the phase $\delta_{13}^d$ is arbitrary. We have found that with this $V_{td}$ hierarchy, inverted relative to the CKM one and also different from that in scenario 4, a rough consistency with the existing data can be obtained. In spite of that, the combined measurements of $A_{SL}^d$ and $\cos 2\beta$ and...
the indirect experimental estimate of $A_{SL}^s$ make this scenario very unlikely.

In conclusion, we stress that in the interesting scenario 4, significant enhancements of the CP-asymmetries $A_{CP}(B_s \rightarrow \psi \phi)$ (≈ 5 – 10 times larger than in the SM) and $A_{SL}^s$ (≈ 10 – 20 times larger than in the SM) are possible as shown in Fig.1 while satisfying all existing constraints, in particular from the $B \rightarrow X_s \gamma$ decay, as shown in Fig.2. In order to improve the LHT flavour analysis the next step is certainly the inclusion of rare decays in both $K$ and $B_{d,s}$ meson systems. This study has been just performed finding that visible effects in observables accessible to future experiments, mainly in $K$ physics, are possible.

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