Warped Supersymmetry Breaking

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Abstract: We address the size of supersymmetry-breaking effects within higher-dimensional settings where the observable sector resides deep within a strongly warped region, with supersymmetry breaking not necessarily localized in that region. Our particular interest is in how the supersymmetry-breaking scale seen by the observable sector depends on this warping. We obtain this dependence in two ways: by computing within the microscopic (string) theory supersymmetry-breaking masses in supermultiplets; and by investigating how warping gets encoded into masses within the low-energy 4D effective theory. We find that the lightest gravitino mode can have mass much less than the straightforward estimate from the mass shift of the unwarped zero mode. This lightest Kaluza-Klein excitation plays the role of the supersymmetric partner of the graviton and has a warped mass $m_{3/2} \propto e^A$, with $e^A$ the warp factor, and controls the size of the soft SUSY breaking terms. We formulate the conditions required for the existence of a description in terms of a 4D SUGRA formulation, or in terms of 4D SUGRA together with soft-breaking terms, and describe in particular situations where neither exist for some non-supersymmetric compactifications. We suggest that some effects of warping are captured by a linear $A$ dependence in the Kähler potential. We outline some implications of our results for the KKL T scenario of moduli stabilization with broken SUSY.
1. Introduction

Understanding how supersymmetry breaks has been a Holy Grail for string theorists for decades, because it is likely a crucial prerequisite for understanding string theory’s low-energy predictions. Like the search for the Grail this has proven to be an elusive quest, complicated as it is by related issues of modulus stabilization. Considerable progress has come recently, however, with the recognition that Type IIB string vacua can stabilize many moduli in the presence of fluxes [1, 2].
The remarkable properties of the warped compactifications to which these studies lead open up potentially interesting new possibilities for constructing phenomenologically attractive string vacua, both for applications to particle physics [3] and to cosmology [4]. They do so for several reasons. First, by providing a plausible setting in which all moduli may be fixed [2, 5] they provide a concrete laboratory within which to compute how supersymmetry breaks. This potentially represents a great leap forward since it allows one to deal with a serious drawback of previous calculations of supersymmetry-breaking effects for string vacua (for a review see [6]). Because these earlier calculations did not construct the potential which stabilized the various moduli, they could not determine the values of the moduli. In particular, they could not address whether or not supersymmetry was restored at a minimum of the moduli potential, as is very often the case in practice.

The second reason these warped compactifications have been so intriguing for phenomenology is that they can contain strongly-warped regions ('throats') within which the energies of localized states can be strongly suppressed by gravitational redshift. This possibility is very interesting since the energy gained by localization can dominate the energy cost due to the gradients which the localization requires. This can dramatically change the kinds of states which dominate the low-energy world, with localized states often being preferred over those which spread to fill all of the compact dimensions. Among the consequences of this observation is the possibility of having new ways to obtain large hierarchies of scale, such as by having all Standard Model degrees of freedom localized in a region of strong warping [7, 8, 9, 10]. It also opens up new ways for energy to be efficiently channelled into such throats, potentially providing new ways to think about reheating during early-universe cosmology [11, 12].

For all of these reasons there is considerable motivation for understanding how supersymmetry breaks in strongly-warped compactifications, with particular interest in supersymmetry breaking by bulk fluxes such as arises within Type IIB vacua. For phenomenological purposes what is of most interest is an understanding of the scale of supersymmetry breaking for the observable (Standard Model) sector residing deep down a warped throat, given various choices for how supersymmetry might be broken throughout the bulk (for related work in this direction see [13]).

Our presentation comes in two parts. In the next section, 2, we provide a generic semi-quantitative discussion of the scales which arise in supersymmetry-breaking compactifications with warped extra dimensions. Although some estimates and explicit calculations of supersymmetry breaking scales have been made [14], these apparently overestimate the size of the effective supersymmetry breaking scale in a strongly-warped regime, for reasons we describe. In particular, we give insights on how the gravitino mass undergoes warping suppression, illustrating the phenomenon with the aid of a scalar field toy model. We provide a criterion for when to expect the low-energy 4D effective theory to be described by a 4D supergravity, by a supergravity supplemented by soft-breaking terms, or by a generic non-supersymmetric field theory. Section 3 then follows with simple calculable examples of flux-induced mass terms for D3 and D7 branes, together with a discussion of an approach to summarizing the warp-
ing dependence of the mass in the low-energy 4D effective field theory. We describe aspects of phenomenology in $\S$4 and then close in $\S$5 with a short summary and some concluding remarks.

2. Mass scales and effective descriptions

We now turn to a qualitative description of the scales which arise when higher-dimensional supergravities are compactified on strongly warped geometries.\footnote{For some earlier general discussion of scales see [14]. For some discussion of the problems that arise in trying to derive an N=1 SUGRA potential in the context of non-trivial warping see [15]; for discussion of a prescription to derive the potential and other aspects of the effective theory see [16].} With a view to phenomenological applications, our interest is in particular on the scales and the effective field theory which are relevant to the 4D dynamics of those particles which are localized deep within the strongly warped region, with supersymmetry broken in some other sector, possibly due to fluxes (or other supersymmetry-breaking effects).

We start with a general description of the scales and effective theories which can arise within strongly warped compactifications, before returning later to describe their relevance for the scale of supersymmetry breaking as seen by observers localized in the warped regions.

2.1 Mass scales in warped throats

Since many of the issues which arise for supersymmetry breaking in warped environments are generic to the kinematics of warping, we start here by summarizing some general properties of warped compactifications, together with examples from IIB compactifications.

2.1.1 KK excitations and scales

It is useful to begin with a reminder of the mass scales which arise within unwarped string compactifications, having a product-space vacuum configuration

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n,$$

(2.1)

together with configurations for the other bosonic supergravity fields. Our conventions are to use 10D indices $M,N,\ldots = 0,\ldots,9$ for spacetime; 4D indices $\mu,\nu,\ldots = 0,\ldots,3$ for the observed (noncompact) dimensions; and 6D indices $m,n,\ldots = 4,\ldots,9$ for the hidden (compact) dimensions.

For simplicity we restrict ourselves to geometries, $g_{mn}$, whose volume and curvatures are all characterized by a single scale: $M_{KK} = 1/L$. Control over semiclassical calculations typically requires both a small dilaton, $g_s = e^\phi \ll 1$, and small curvatures, $\alpha'/L^2 \ll 1$, and so $M_{KK} \ll M_s$ where $M_s^2 = 1/\alpha'$ denotes the string scale. At scales below $M_s$ massive string modes can be integrated out, leaving an effective-field-theory description in terms of a higher-dimensional supergravity.
When higher-dimensional fields are dimensionally reduced on such a space their linearized fluctuations about this background are expanded in a complete set of Kaluza Klein (KK) eigenmodes, e.g.

$$\delta \phi(x, y) = \sum_k \varphi_k(x) u_k(y), \quad (2.2)$$

with mode functions $u_k(y)$ chosen as eigenfunctions of various differential operators arising from the extra-dimensional kinetic operators, $\Delta u_k = \lambda_k u_k$. The differential operator is chosen so that the resulting 4D fields, $\varphi_k(x)$, satisfy the appropriate field equations for a particle having a mass $m_k$, which is computable in terms of the corresponding eigenvalue $\lambda_k$. This produces a tower of states having masses ranging between 0 and $\infty$ (for marginally stable vacua), which are split in mass by $\Delta m_k \sim O(M_{KK})$.

The effective field theory describing energies smaller than $M_{KK}$ is a four dimensional one, because there is insufficient energy to excite KK modes which can probe the extra dimensions. Conversely, although it is possible to think of the effective theory above $M_{KK}$ as a complicated 4D theory involving KK modes and strong couplings, it is better to think of it as being higher dimensional for several reasons, not least of which being that it is much simpler to do so. Moreover, it is also true that the KK modes have masses reaching right up to the UV cutoff, and so there is never a parametrically wide gap between the UV cutoff and the mass of the heaviest mode. Furthermore, the 4D picture obscures higher-dimensional spacetime symmetries, like general covariance and supersymmetry, whose presence is crucial to the consistency of the theory.

For warped configurations the main difference is that the vacuum metric takes the more general form

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n, \quad (2.3)$$

where the warp factor $e^{A(y)}$ varies in the extra dimensions. The physical interpretation of the warp factor can be found by considering the energy of a test particle having mass $m$ and proper velocity $u^M$. If the particle is stationary at a specific point, $y_0$, in the extra dimensions then the normalization condition, $u^2 = -1$, implies $u^M = \delta^M_0 e^{-A(y_0)}$. The four-dimensional energy of such a particle is then $E = -m \xi^M u^M = e^{2A(y_0)} u_0 m = e^{A(y_0)} m$, where $\xi^M = \delta^M_0$ is the timelike Killing vector field corresponding to time translation. We see that the energy of such a test particle is highly suppressed in strongly warped regions where $e^A \ll 1$.

The KK reduction of fluctuations about such a space takes a form similar to eq. (2.2), but with mode functions which diagonalize different differential operators and satisfy different normalization conditions than in the unwarped case. For instance, a higher-dimensional scalar fluctuation of ten dimensional mass $m_{10}$, satisfies the equation

$$(\Box_{10} - m_{10}^2) \delta \phi = \left[ e^{-2A} \eta^{\mu\nu} \partial_{\mu} \partial_{\nu} + \frac{e^{-4A}}{\sqrt{g}} \partial_m \left( \sqrt{g} e^{4A} g^{mn} \partial_n \right) - m_{10}^2 \right] \delta \phi = 0. \quad (2.4)$$

Four-dimensional KK masses are given by eigenvalues of the operator

$$\Delta = \frac{e^{-2A}}{\sqrt{g}} \partial_m (\sqrt{g} e^{4A} g^{mn} \partial_n) - e^{2A} m_{10}^2, \quad (2.5)$$
rather than operator $\Delta_0 = (1/\sqrt{g})\partial_m(\sqrt{g}g^{mn}\partial_n) - m_{10}^2$ which would have been used in the absence of warping. Notice one effect of warping is to convert the 10D mass $m_{10}$ into a potential $e^{2A(y)}m_{10}^2$ for the wavefunctions of the KK excitations. For a wavefunction localized in a region of large warping, with the local warp factor $e^A \ll 1$, one can expect a redshifted four dimensional mass $m \sim m_{10}e^A$ in accord with the preceding discussion.

In type IIB string theory, it is possible to write explicit solutions for warped compactifications \[2\]. We shall use these solutions as our laboratory to study the effects of warped throats on supersymmetry breaking. First, we review some features of these compactifications which shall be relevant for our discussion.

In these constructions, the geometry takes the form
\[
d_{10}^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A} \tilde{g}_{mn}(y) dy^m dy^n, \tag{2.6}
\]
where $\tilde{g}_{mn}$ is a metric on Calabi-Yau of fiducial volume $\tilde{V} = (\alpha')^3$. The warp factor satisfies the equation of motion
\[
-\tilde{\nabla}^2 e^{-4A} = \frac{G_{mnp}\tilde{G}^{mnp}}{12\text{Im} \tau} + 2\kappa_3^2 T_3 \rho_{3 \text{loc}} \tag{2.7}
\]
where $\tau = C_0 + ie^{-\phi}$ is the axio-dilaton, $G_3 = F_3 - \tau H_3$ is the complex three-form field strength, the tilde indicates indices raised with $\tilde{g}^{mn}$, and $\rho_{3 \text{loc}}$ represents localized sources of D3-brane charge. Given a particular solution $e^{-4A_0}$ of (2.7), we can always find a family of solutions \[16\] with parameter $c$,
\[
e^{-4A} = e^{-4A_0} + c. \tag{2.8}
\]
One convenient choice is to take the particular solution to be orthogonal to the zero mode $c$,
\[
\int d^6 y \sqrt{\tilde{g}} e^{-4A_0} = 0, \tag{2.9}
\]
which emphasizes that $e^{-4A_0}$ is not everywhere positive. This agrees with the statement that this quantity may become negative in regions of string-size around negative tension objects, where the supergravity approximation fails.

For large $c$, in most parts of the manifold, the warp factor is approximately constant and the background resembles that of a standard Calabi-Yau compactification. This Calabi-Yau like region is often referred to as the bulk. In this case $c$ sets the scale of the metric over much of the bulk and so controls the overall size of the compactification, with
\[
c \approx \frac{V^{2/3}}{\alpha'^2} \equiv \mathcal{V}^{2/3}. \tag{2.10}
\]
Here $V$ is the volume of the internal metric and $\mathcal{V}$ the dimensionless volume relative to the string scale.

For later use, we also note the form of the metric associated with the four-dimensional Einstein frame (which is related to (2.6) by rescaling the noncompact dimensions):
\[
d_{10}^2 = \lambda[e^{-4A_0} + c]^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + [e^{-4A_0} + c]^{1/2} \tilde{g}_{mn}(y) dy^m dy^n, \tag{2.11}
\]
with
\[ \lambda = \frac{\hat{V}}{\int d^6y \sqrt{g} (c + e^{-4A_0})}. \] (2.12)

With the choice (2.9) this simplifies to
\[ \lambda = \frac{1}{c} \approx \mathcal{V}^{-2/3}. \] (2.13)

For generality we often quote results for arbitrary \( \lambda \), bearing in mind the specialization to Einstein frame through (2.12).

In addition to the bulk, small regions where the variation of the warp factor is large can arise from typical values of the flux quantum numbers, as shown in [2]. For instance, if \( M \) units of R-R flux and \( K \) units of NS-NS flux wrap the A and B cycles of a conifold locus respectively, the warp factor \( e^{-4A} \) becomes large and attains a finite maximum value
\[ e^{-4A_m} \approx e^{8\pi K / 3M g_s}. \] (2.14)

The relative redshift \( \Omega \) between points on the manifold is given by the ratio of \( e^A \) at the two points. Since the warp factor in the bulk is \( e^A \approx c^{-1/4} \), (2.14) corresponds to a redshift
\[ \Omega \simeq \left( \frac{e^{-4A_m} + c}{e^{-1/4}} \right)^{-1/4} \] (2.15)
relative to the bulk. Note that this depends on the volume of the compactification through its dependence on \( c \). In the regime
\[ c \ll e^{-4A_m} \] (2.16)
the redshift factor \( \Omega \) is large and we find \( \Omega \approx e^{A_m c^{1/4}} \). The region of large warping is then referred to as a throat. The condition (2.16) moreover implies that the geometry (2.6) in the throat region is largely independent of \( c \) and so also of the overall volume of the compactification. On the other hand, for
\[ c \gg e^{-4A_m} \] (2.17)
the redshift (2.13) tends to unity and the geometry is that of a usual Calabi-Yau compactification.\(^2\)

The fluxes generically introduce masses for complex structure moduli and the dilaton [2]. The mass spectrum of the excitations of the dilaton (which can be thought of as representative of modes that acquire flux-induced masses) was studied in detail in [17]. This involved analyzing the linearized equations of motion [16] for the dilaton about the backgrounds of [2]. These equations have the structure of the kind of eigenvalue problem described above. The

\(^2\)This is in keeping with the fact that fluxes are \( \alpha' \) effects, and so all flux induced effects should disappear for sufficiently large volume. This argument also applies to settings other than type IIB.
lowest eigenvalue corresponds to the flux induced mass of the four-dimensional dilaton. The
analysis confirmed the standard expectation for flux induced masses

\[ m_\tau \sim \frac{n_f \sqrt{\lambda}}{V^{2/3}} \]  \hspace{1cm} (2.18)

in the large-volume regime (2.17), with \( n_f \) measuring the number of flux quanta.

But [17] revealed a qualitatively distinct behavior in the regime with large warping. In
particular, for

\[ c \lesssim e^{-A_m} \]  \hspace{1cm} (2.19)

the lowest mass mode of the dilaton has

\[ m_\tau \sim e^{A_m} \sqrt{\lambda} / \sqrt{n'_f}, \]  \hspace{1cm} (2.20)

where \( \sqrt{n'_f} \) sets the length scale in the infrared end of the throat, and is determined by the
integer quanta of fluxes threading the cycles in this region.\(^3\) The wavefunction of the dilaton
is highly localized in the bottom of the throat. This mode is continuously connected to the
lowest mode in the regime (2.17); i.e. as the volume of the compactification is decreased the
wavefunction of the dilaton continuously varies from being uniformly spread on the internal
manifold to a highly localized function. Furthermore the KK mass gap in the strongly-warped
regime (2.19) is of the same order as the mass (2.20), and thus the dilaton-axion should be
integrated out of the effective field theory for the majority of applications. We see below that
the gravitino has a similar property.

The behavior of the flux-induced masses (2.18 – 2.20) has a simple interpretation. Linearizing
the equation of the dilaton for an excitation with four-dimensional mass \( m \) and
wavefunction in the internal direction \( \tau(y) \), one obtains [16]\(^4\)

\[ \lambda e^{4A} \nabla^2 \tau(y) + m^2 \tau(y) = \frac{g_s}{12} \lambda e^{8A} G_{mn} \tilde{G}^{\bar{m}n} \tau(y). \]  \hspace{1cm} (2.21)

The mass term generated by the flux,

\[ \frac{g_s}{12} \lambda e^{8A} G_{mn} \tilde{G}^{\bar{m}n}, \]  \hspace{1cm} (2.22)

is not a constant but has non-trivial variation over the internal manifold.

In the bulk region, the metric \( \tilde{g}_{mn} \) is of order unity, hence \( G_{mn} \tilde{G}^{\bar{m}n} \) behaves as \( n_f^2 \).
Also, \( e^{-4A} \sim c \), hence in the bulk

\[ g_s \lambda e^{8A} G_{mn} \tilde{G}^{\bar{m}n} \sim \frac{\lambda n_f^2}{V^{2/3}}. \]  \hspace{1cm} (2.23)

\(^3\)For instance in the infrared end of the KS throat \( n'_f \sim M \), the flux threading the \( S_3 \) of the conifold.
\(^4\)In obtaining (2.21) we have ignored mixing with the fluctuations of the three form and metric. We shall use
the equation only to give a qualitative explanation of the results of [17], a purpose for which these fluctuations
are not important.
in agreement with (2.18). In the throat region, the underlying Calabi-Yau metric $\tilde{g}_{mn}$ has a shrinking three cycle, but the presence of the fluxes makes the internal manifold (with metric $e^{-2A}\tilde{g}_{mn}$) have a characteristic length scale in this region of the order of $\sqrt{n_f'}$ in string units. Thus in the throat region $G_{mnp}\tilde{G}^{mnp} \sim 1/n_f'$ (there are approximately two powers of $n_f'$ from the three form flux and three inverse powers from the three inverse metrics). Hence at the bottom of the throat

$$g_s\lambda e^{8A}G_{mnp}\tilde{G}^{mnp} \sim \lambda e^{2A_m} \frac{1}{n_f'}.$$  

(2.24)

One expects the dilaton to localize in the infrared end of the throat region and acquire a mass of the order of (2.24) when it is energetically favorable to do so, i.e. when the volume is small enough so that (2.24) is less than (2.23). This yields the condition

$$\sqrt{2/3} \lesssim e^{-A_m},$$  

(2.25)

which is equivalent to (2.13).

The phenomenon of localization of massive modes should be fairly generic. As the volume of the compactification decreases, the redshift (2.13) becomes more and more prominent and one expects energetics to drive the wavefunction of excitations into the throat region. Our analysis of the gravitino (to follow in section 3) indicates that in the presence of SUSY breaking flux the gravitino wavefunction also localizes in the long throat region. We shall confine our discussion to this strongly-warped regime and examine implications for SUSY breaking. But first we comment on various important energy scales and possible effective descriptions in this regime.

2.1.2 Energy scales and effective descriptions in the strongly warped regime

Let us be more explicit about scales in the strongly-warped situation, with $c \lesssim e^{-A_m}$. The relevant energy scales are: the four-dimensional Planck mass $M_p$, which is the basic scale in the Einstein frame, and that we can set to $M_p = 1$ by measuring all mass scales in units of the Planck mass. The bulk string scale is then $M_s \sim 1/\sqrt{\alpha'} \sim g_s^{1/4}V^{-1/2}$, and the bulk Kaluza-Klein or compactification scale in the Einstein frame is $M_{KK} \sim V^{-2/3}$ in these units. Moreover, strong warping produces the warped string scale $M_s^w \sim g_s^{1/4}e^{A_m}V^{1/3}$, and the warped Kaluza-Klein scale, $M_{KK}^w \sim g_s^{1/4}\frac{\rho\sqrt{1/3}}{\rho V^{1/3}} \sim M_{KK}^w$ where $\rho > 1$ is a characteristic length of the tip of the throat (in units of the string length). Notice that the volume dependence of $M_{KK}^w$ and $M_s^w$ is the same and they only differ somewhat by the factor $\rho$. This is an important difference with respect to the bulk quantities since this implies that the tower of string states and Kaluza-Klein states at the tip of the throat can be at similar energies.

Based on this structure of scales we can distinguish the following energy regimes (and effective theories which describe them).

1. $E < M_{KK}^w$: This energy range is below the mass of moduli that acquire flux induced mass and all KK and string modes. The low-energy effective theory describing these
energies is necessarily a 4D field theory. It contains light degrees of freedom (like the Kähler moduli, which are massless until \( \alpha' \) corrections and non-perturbative effects are accounted for [1, 2]).

2. \( M_{KK}^w < E < M_s^w \): This small energy range is below the mass of all massive string states, warped or not, but contains a (finite) tower of those KK modes which are localized deep within the warped region. Because all string modes are massive, they can be integrated out leaving a low-energy effective theory which in this case is an explicitly higher-dimensional field theory. This effective theory is not the higher-dimensional field theory describing the full compact space. (If it were it would include states having masses larger than \( M_{KK} \), which is larger than the assumed cutoff.) Instead it only probes the warped geometry, with cutoff at a point \( y = Y \) where the warp factor, \( e^{A(Y)} \), is no longer sufficiently small. This energy scale represents both an UV cutoff (since it excludes KK and string states having energy higher than \( e^{A(Y)}/Y^{1/3} \)) and an IR cutoff, since the condition \( y < Y \) gives the space a finite volume, providing an example of UV/IR mixing.

3. \( M_s^w < E < M_{KK}^w \): The effective theory for this energy range contains both massive KK and string modes, as well as non-perturbative excitations like branes and black holes, but again only those with strongly warped-suppressed spectra. As such, the low-energy effective theory must be a string theory — again not the full string theory with which one starts, but rather a string theory which lives only within the cut-off volume of the warped geometry.

4. \( M_{KK} < E < M_s \): This energy range includes only strongly warped string modes, but contains the KK modes of the higher-dimensional field theory. The low-energy effective theory in this case would appear to consist of the string theory localized to the warped region (as above), but coupled to the full set of supergravity modes which can propagate outside of the warped region. Such a theory is somewhat novel and it would be interesting to elucidate further its explicit description.

5. \( M_s < E \): In this energy range the appropriate description is the full string theory, defined in the entire higher-dimensional geometry. It is believed that this theory can apply to arbitrarily high energies.

Cases 3 and 4 above may naïvely seem unusual inasmuch as they involve effective cut-off string theories, with the strings propagating in nontrivial, but cut-off, higher-dimensional background fields. They are indeed bona fide string theories since their cutoffs are much higher than the masses of the lightest (strongly warped) string states. Of course, in the light of AdS/CFT duality they seem less novel. One expects an alternate description of both regimes 2 and 3 above in terms of a cutoff gauge theory with the appropriate relevant deformation.

\(^5\)Since we are assuming large warping we are taking \( M_{w}^w < M_{KK} \). For small warping we can have \( M_{KK} < M_{w}^w \) and the regime \( M_{KK} < E < M_{w}^w \) is the standard 10D supergravity, as in the usual unwarped case.
Moreover, one expects to be able to describe regime 4 in terms of the supergravity of the bulk manifold coupled to a cutoff gauge theory description of the throat dynamics. This is a variant of the description of the effective field theory of the single brane RS scenario \[\text{[8]}\] as that of a cutoff conformal field theory coupled to four-dimensional gravity and other ultraviolet brane degrees of freedom \[\text{[14, 20]}\]. It would be interesting, but goes beyond the scope of this article, to further investigate properties of such theories, and to moreover understand the space of such possible theories and their possible deformations through excitation of stringy modes above the threshold \(M_{\text{w}}\).

We now put aside such discussion and instead ask how supersymmetry breaking manifests itself in case 1.

### 2.2 Supersymmetry breaking

We next turn to the relevance of these various scales for supersymmetry breaking. Our goal is to identify the effective supersymmetry-breaking scale for any low-energy observers (like us) who might reside deep within the bottom of a warped throat.

#### 2.2.1 Supersymmetry breaking scales and warping

Consider, then, a higher-dimensional compactification of a supersymmetric field (or string) theory. Although a generic compactification will break all of the supersymmetries of the higher-dimensional theory, we choose to focus here on compactifications — like those considered for Type IIB vacua in ref. \[\text{[2]}\], say — chosen to preserve, or to approximately preserve, one of these supersymmetries (from the 4D perspective). ‘Approximate’ conservation here means that the scale associated with splittings within the various 4D supersymmetry multiplets is sufficiently small, in a way made more precise below.

Any higher dimensional supersymmetry parameter, \(\varepsilon(x, y)\), consists of many independent supersymmetries when viewed from the 4D perspective. Specifically, let \(a\) and \(\alpha\) be four- and six-dimensional spinor indices, respectively, so that the combination \((a\alpha)\) serves as a ten-dimensional spinor index. Then \(\varepsilon\) can be expanded

\[
\varepsilon^{a\alpha}(x, y) = \sum_k \varepsilon_k^{a}(x) \eta_k^{\alpha}(y)
\]  

(2.26)

in terms of an appropriate basis of 6D spinors, \(\eta_k(y)\), and each of the 4D spinors, \(\varepsilon_k(x)\), defines a separate local 4D supersymmetry transformation, gauged by the appropriate 4D component of the gravitino field,

\[
\Psi^{a\alpha}_{\mu}(x, y) = \sum_k \psi^{a}_{\mu k}(x) \eta_k^{\alpha}(y),
\]

(2.27)

with \(\delta \psi_{\mu k} = D_{\mu} \varepsilon_k + \ldots\) under a supersymmetry transformation. We suppress spinor indices in the sequel.

If precisely one 4D supersymmetry is unbroken then by assumption one of the spinor modes, \(\eta_0\), is an appropriate “Killing spinor,” for which the corresponding gravitino mode
ψ_{μ0} is precisely massless. Above this massless state will be a KK tower of massive 4D spin-3/2 states, ψ_{μk}, whose lightest elements we expect to have mass ∼ M_{KK} in an unwarped environment, or ∼ M_{KK}^w in a strongly warped environment.

Imagine now turning on some sort of supersymmetry-breaking bulk field (a flux or fluxes, non-perturbative effects, or perhaps an appropriate configuration of branes). We imagine that the supersymmetry breaking background field is not itself confined to the strongly warped region, and so has an amplitude set by a scale, ̂Λ, which is not warped. In what follows we take ̂Λ to be bounded above by M_{KK}, and so write ̂Λ ∼ f M_{KK}, where f is a small continuous or discrete parameter, f ≪ 1. In the limit that f → 0 one 4D supersymmetry is unbroken, and all particles residing within the same multiplet of this 4D symmetry have precisely the same mass. In particular, one of the 4D KK gravitino modes is precisely massless in this limit, making it degenerate with the massless 4D graviton. But for f ≠ 0 all 4D supersymmetries are broken and all KK mode masses are perturbed by an f-dependent amount which in general splits the masses of different particles residing within the same 4D supersymmetry multiplet.

We now ask: How large are the resulting supersymmetry-breaking splittings, ∆m_k, within multiplets? (In particular, how massive is the lightest 4D gravitino?) In the unwarped case, the answer for both questions would be m_{3/2} ∼ ∆m_k ∼ f M_{KK}, and it is tempting to suppose that this will also be the result in the strongly warped case. Indeed, explicit truncations of the 10D gravitino on the mode which is massless in the limit f → 0,

$$\Psi_μ(x, y) = ψ_{μ0}(x) η_0(y),$$

appear to give this result \[14\].

We argue here that when there is a strongly warped regime (2.19) this is not the correct answer, and that in general different multiplets of KK modes split by amounts which differ depending on whether or not they are localized in the strongly warped region. Although we expect ∆m_k ∼ f M_{KK} for modes not strongly warped, much smaller splittings occur within the warped region. In particular, we argue that the mass of the least massive gravitino is warped to smaller values, being at most of order the warped KK scale if ̂Λ is smaller than M_{KK}:

$$m_{3/2} ∼ f' M_{KK}^w.$$  

(again, f' ≪ 1 and one unbroken 4D supersymmetry must be recovered in the limit f' → 0).

This can be motivated as follows: (i) the generic masses of modes localized in the warped region are of order M_{KK}^w, and for sufficiently strong warping one expects energetics to drive the gravitino wavefunction into the throat; (ii) the lightest gravitino state must become massless as f' → 0. Since the least broken supersymmetry is by definition the one which corresponds to the lightest gravitino KK mode, this is at most the smallest of the warped KK masses: ∼ M_{KK}^w. The above result then only incorporates the additional information that the result must vanish as f' → 0, with the assumption that this vanishing is linear. (Of course, in particular cases there may also be additional suppressions by other small model-dependent factors.) We shall argue that the expectation for localization of the wavefunction of the
gravitino is correct in the presence of strong warping by analysis of the gravitino equations of motion in section 2.2.2.

How could the direct truncation to (2.28) give a much larger result? After all, the assumption underlying using the unbroken mode functions to estimate the gravitino mass is a familiar one from perturbation theory: it suffices to use unperturbed eigenfunctions in order to find the leading-order correction to the corresponding eigenvalues. The reason for failure of first order perturbation theory is that in the strongly warped regime, the wavefunction of the lowest mode in the presence of supersymmetry breaking fluxes is localized in the highly warped region and differs significantly from the Killing spinor on the Calabi Yau. This localization is responsible for the lowering of the mass to the warped KK scale, and is not captured by first order perturbation theory since that does not correct the wavefunction. To understand the result from the point of view of perturbation theory one has to use degenerate perturbation theory. It is the assumption of non-degeneracy which breaks down in strongly warped geometries, because there are many unperturbed states having masses $M_{KK}^w$ which are much smaller than the matrix element of the perturbation, $f M_{KK}$. Further discussion of the nature of the KK wavefunctions in highly warped geometries and the use of degenerate perturbation theory in this context is given in the Appendix, where a toy example is analyzed in detail.

To summarize, for the purposes of understanding 4D supersymmetry transformations it suffices to regard the higher-dimensional supersymmetric models as complicated 4D theories involving towers of 4D supermultiplets consisting of massive bosonic and fermionic KK modes. We have argued above that not all of these multiplets are split by equal amounts when supersymmetry is broken in a strongly warped geometry. In particular we expect KK modes which are not localized in the warped areas to generically have supersymmetric KK mass scales of order $M_{KK}$ and to have supersymmetry-breaking splittings within these multiplets which are of order $\Delta M \equiv f M_{KK}$. By contrast, multiplets of modes which are localized within strongly warped areas should instead have supersymmetric KK multiplet masses which are of order $M_{KK}^w$, and splittings within these multiplets should be of order $\Delta m \equiv f' M_{KK}^w$. We next illustrate these comments through explicit study of the gravitino.

2.2.2 The gravitino mass

In this subsection we examine the gravitino equations of motion to obtain the criterion for the localization of the gravitino wavefunction and its mass in this regime. We shall follow the conventions in Appendix B of [14] for the gravitino equations of motion, with $\kappa = 1$. First, we set our gamma matrix conventions. The 10 dimensional gamma matrices $\Gamma^M$ satisfy the algebra

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN}. \quad (2.30)$$

For the metric (2.3), we take

$$\Gamma^\mu = \frac{e^{-A}}{\sqrt{\lambda}} \gamma^\mu \otimes 1 \quad \Gamma^m = e^A \gamma_c \otimes \tilde{\gamma}^m, \quad (2.31)$$
where
\[ \{ \gamma^\mu, \gamma^\nu \} = 2 \eta^{\mu\nu}, \{ \tilde{\gamma}^m, \tilde{\gamma}^n \} = 2 \tilde{g}^{mn}, \] (2.32)
and \( \gamma_c = i \gamma_0 \gamma_1 \gamma_2 \gamma_3 \) is the four dimensional chirality matrix. The gravitino equation of motion is
\[ \Gamma^{MNP} \hat{D}_N \Psi_P = -i \frac{1}{2} \Gamma^P \Gamma^{\lambda} \Delta^* P_P - \frac{i}{48} \Gamma^{NPQ} \Gamma^M \hat{\lambda} G^*_N P_Q + \mathcal{O}(\Psi^3) \] (2.33)
where \( \hat{\lambda} \) is the dilatino, and \( P_M \) is the field strength of the dilaton
\[ P_M = \frac{1}{1 - B B^*} \partial_M B, \] (2.34)
with
\[ B = \frac{1 + i \tau}{1 - i \tau}. \] (2.35)
The supercovariant derivative acting on the gravitino is given by
\[ \hat{D}_N \Psi_P = D_N \Psi_P - R_P \Psi_N - S_P \Psi^*_N \] (2.36)
with
\[ R_M = \frac{i}{480} (\Gamma^{M_1 \ldots M_5} F_{M_1 \ldots M_5}) \Gamma_M \] (2.37)
and
\[ S_M = \frac{1}{96} (\Gamma^M N P Q G_{N P Q} - 9 \Gamma^N P G_{M N P}) \] (2.38)
The complicated nature of the equations makes it difficult to find an explicit solution corresponding to the massive 4D gravitino. In fact, it is not consistent to excite just the fields \( \Psi^\mu \); as in the case of dilaton and complex structure moduli \[16\], one finds mixing between the various 10D supergravity modes while carrying out the KK reduction. Given the intractable nature of the equations of motion, we shall use energetic arguments to determine the condition for localization of the gravitino motivated by the observations (2.22 – 2.25) for the dilaton in section 2.1.1.

We take the four dimensional gravitino \( \psi^\mu(x) \), to be embedded in the ten dimensional gravitino as
\[ \Psi^\mu(x, y) = \psi^\mu(x) \otimes \eta(y) \] (2.39)
where \( \eta(y) \) is the wavefunction of the gravitino in the extra dimensions. Thus, components of the gravitino equation of motion in the non-compact direction have the structure
\[ \frac{e^{-3A}}{\lambda^{3/2}} \gamma^{\mu\nu} \partial_{\nu} \psi_{\rho} \otimes \eta(y) + \frac{1}{24 \lambda} \gamma^{\mu\nu} \gamma_c \psi^*_\nu \otimes e^A G_{\eta\eta} \gamma^m \eta^* + \ldots = 0, \] (2.40)
where we have not explicitly written the contributions of the terms containing derivatives of \( \eta(y) \) and other contributions neglected due to the specific form of our Ansatz. The term
\[ \gamma^{\mu\nu} \partial_{\nu} \psi_{\rho} \] (2.41)
is the standard kinetic term for a spin 3/2 field in four dimensions. One expects the mass to be of the order of the relative strength of the kinetic term and the term involving fluxes in (2.40). We assume that the terms not written explicitly in (2.40) scale in a similar way as a function of the warping and Calabi-Yau volume as do the terms that are shown, justifying their neglect in obtaining an estimate for the mass. This should be reasonable since all such terms must adjust so that the equations can be satisfied, and appears confirmed by a crude analysis of their effects in the equations of motion.

Comparing the kinetic and potential terms in (2.40) we find that the flux-induced mass term for the gravitino scales like

$$\sqrt{\lambda} e^{A} G_{\mu \nu} \gamma^{\tilde{m} \tilde{n}} p,$$

(2.42)

and so varies over the internal manifold. Keeping in mind that the gamma matrices scale like the “square root of the inverse metric,” and comparing with (2.22), we see that the mass term has the same dependence on the warp factor and the fluxes as the mass term for the dilation defined in (2.22). Thus, as in the case of the dilaton, the minimum of the mass term lies in the infrared end of the throat and for

$$c \lesssim e^{-A m},$$

(2.43)

the wavefunction localizes. The mass acquired is of order

$$m_{3/2} \sim f' \sqrt{\lambda} e^{A m}.$$

(2.44)

We have explicitly included the factor $f'$ in (2.44) which for example represents the flux dependence. For $f'$ of order one, the mass of the lightest gravitino is not hierarchically separated from the other excitations. But for $f' \ll 1$ we can guarantee a small enough gravitino mass in order to include this field in an effective field theory description and use $N = 1$ supergravity formalism to describe the low energy physics. This is clear given the existence of a limit in which the fluxes are tuned to values such that supersymmetry is unbroken, and corresponds to tuning of the moduli-fixed value $W_0$ of the perturbative Gukov-Vafa-Witten superpotential.

One can also imagine a situation where the throat by itself is supersymmetric, so that the mass term (2.42) vanishes in the throat, but supersymmetry is broken purely in the bulk of the Calabi-Yau. In such a situation the model discussed in the appendix of [17] becomes relevant. Again, in the strongly-warped regime (2.43) one has a mass given by (2.44).

We now ask what these scales imply for the corresponding low-energy effective theories.

### 2.3 Criteria for supersymmetric 4D actions

From the 4D point of view, higher dimensional supergravity theories broken by bulk fields are special cases of ‘hidden sector’ models (for a review see [21]). In these models there is a collection of low-energy fields, $\ell^a$, of physical interest (describing, say, Standard Model particles). These are assumed to be coupled to a more generic set of fields, $h^m$, whose dynamics somehow breaks supersymmetry. Although supersymmetry is badly broken in the
‘hidden’ sector described by $h^m$, the weak $h - \ell$ couplings ensure it is only weakly broken in the ‘light’ sector described by $\ell^0$. In the best-case scenario these couplings are gravitational in strength. Notice that there is no requirement that the hidden-sector fields be light, although there is typically one state in this sector, the goldstone fermion,$^6$ which is much lighter than the others.

In the extra-dimensional case with strong warping, we take the light sector to consist of those modes which are localized within the strongly warped region, whose masses are typically of order $M^w_{KK}$ or smaller. The hidden sector consists of those fields which are not so localized. We have seen that supersymmetry-breaking splittings in the light sector are of order $\Delta m = f' M^w_{KK}$ while those in the hidden sector are of order $\Delta M = f M_{KK}$. More explicitly, we can write $f \sim W_0 \sqrt{\lambda}$ and $f' \sim W_0$, where as above $W_0$ denotes the value of the GVW superpotential.

What is the form of the low-energy 4D effective field theory which describes this kind of theory below a UV cutoff of order the lightest KK mass scale, $\Lambda \sim M^w_{KK}$? In hidden sector models there are generically three kinds of cases for low-energy supersymmetric actions, depending on the size of the cutoff relative to the two supersymmetry-breaking scales, $\Delta M \gg \Delta m$.

1. $\Delta M \ll \Lambda$: If the cutoff is larger than all supersymmetry-breaking mass splittings then the field content of the model can be grouped into supermultiplets. In this case the low-energy theory is itself described by a supergravity, even though it may include some of the hidden-sector fields. Any spontaneous supersymmetry breaking within the full theory can be understood within the effective theory as spontaneous supersymmetry breaking due to the appearance of a SUSY-breaking v.e.v. purely within the effective theory.

2. $\Lambda \ll \Delta m$: If the cutoff is well below the smallest SUSY-breaking scale, then typically most supermultiplets have some elements which are heavier than $\Lambda$ and so are integrated out, while others are lighter than $\Lambda$ and so remain in the low-energy theory. In this case the field content of the low energy theory cannot be organized into supermultiplets (it might contain just fermions with no bosons, for example), and so supersymmetry must be nonlinearly realized $[^2]$. (Notice that for a gauge theory a nonlinearly-realized spontaneous breaking is operationally indistinguishable from explicit breaking within the low-energy theory below the breaking scale $[^3]$, and so in this case the effective theory can be an arbitrary non-supersymmetric field theory.)

3. $\Delta m \ll \Lambda \ll \Delta M$: If the cutoff lies between the two splitting scales, then typically there are supermultiplets in the hidden sector (split by $\Delta M$) for which some elements of the multiplet are heavier than $\Lambda$ and so are integrated out, while others are lighter than $\Lambda$ and so remain in the low-energy theory. This is true in particular for the multiplet...

$^6$Or, more precisely, the massive gravitino which ‘eats’ it.
which contains the goldstone fermion in the hidden sector. In this case supersymmetry is generically badly broken in the effective theory. What distinguishes this case from case 2 above, is that supersymmetry breaking is much smaller than $\Lambda$ within the light sector, which therefore has the field content to fill out complete supermultiplets. As a result, provided we restrict our attention only to light-sector observables the breaking of supersymmetry in this sector can be described as a supergravity coupled to a collection of soft-breaking terms which encode the couplings to supersymmetry breaking in the hidden sector.

The above categories show how the supersymmetry-breaking character of the low-energy effective theory differs between unwarped and strongly warped compactifications. In the unwarped case the effective theory is 4D once $\Lambda < M_{KK}$, and this is well-described by a 4D supergravity provided the supersymmetry-breaking scale satisfies $\Delta M \ll M_{KK}$.

In the strongly-warped case, however, the situation is different. In this case a 4D description is not valid at all until $\Lambda \ll M_{KK}^w$ (unless one works with the dual field theory), in which case some of the unwarped KK modes of the hidden sector typically already satisfy $\Lambda \ll \Delta M$. As a result we do not expect to obtain a low-energy description which consists purely of a 4D supergravity unless we are extremely close to the exactly supersymmetric limit: $W_0 \ll \mathcal{V}^{2/3}e^{A_m}$, where for the rest of section 2 we use the Einstein-frame relation, (2.12). However we can expect to capture the supersymmetry breaking in terms of an appropriate choice of soft-breaking terms provided we choose to focus only on observables involving fields which are localized in the strongly warped regime. We have illustrated in figure [f] the relevant scales for the light modes in both cases, $W_0 \ll \mathcal{V}^{2/3}e^{A_m}$ and $W_0 \gg \mathcal{V}^{2/3}e^{A_m}$.

### 2.3.1 4D action with unbroken supersymmetry

Suppose first we consider case 1, for which $W_0 \ll \mathcal{V}^{2/3}e^{A_m}$. In this case we expect the low energy limit to be given by an effective field theory consisting of a standard 4D supergravity, described, as usual, by a Kähler potential, $K(\ell, \ell^*)$, a superpotential, $W(\ell)$, and a gauge kinetic function, $F_{AB}(\ell)$. We expect this effective supergravity to ‘know’ about the higher-dimensional warping, inasmuch as all nonzero mass scales in the effective supergravity must themselves be warped. We now ask how this effective supergravity encodes this warping.

The appropriate overall scaling down of all energy scales (with fixed 4D Planck mass) for the states which are localized in the throat can be summarized within 4D supergravity, with no other unwanted effects, if the Kähler potential acquires a constant piece

\[ K(\ell, \ell^*) = 2A_m + \bar{K}(\ell, \ell^*) , \]  

(2.45)

with the holomorphic functions $W$ and $F_{AB}$ unchanged. Such a constant piece does not contribute at all to the Kähler metric, $\partial_a \partial^{\hat{a}} K$, or to the covariant derivative, $D_a W = \partial_a W +$
Figure 1: Relevant scales for the localized modes at the infrared bottom of a throat. In the case $W_0 \ll V^{2/3} e^{A_m \min}$, the effects of the fluxes are almost negligible and the system is nearly supersymmetric. For $V^{2/3} e^{A_m \min} \gg W_0$, the effects of the fluxes however become important. The masses of the unperturbed states $M_{KK}^w$ are much smaller than the flux scale and non-degenerate perturbation theory no longer applies to this case. The shaded region shows the range of energies at which an ultraviolet cutoff gives rise to a 4d supergravity description for the light modes.

$W \partial_a K$, but does have the effect of scaling the scalar potential by an overall factor: $V = e^{2A_m} \tilde{V}$, where $\tilde{V}$ is computed using the Kähler potential $\tilde{K}$. In particular, the entire scalar mass matrix gets scaled by a corresponding amount, $m^2 = e^{2A_m} \tilde{m}^2$. Fermion masses get scaled down in an identical way, due to the ubiquitous factor of $e^{G/2}$ to which they are proportional, with

$$G = K + \ln W + \ln W^*.$$  \hspace{1cm} (2.46)

Notice that this overall warping does not properly describe the suppression of KK modes, but this is not important because these cannot appear within the low-energy 4D theory in any case. The approximate description (2.45) is good only for modes localized in a region with definite value of the warp factor.

2.3.2 4D action with broken supersymmetry

We next turn to the case of more direct phenomenological interest, case 3, wherein the SUSY-breaking scale satisfies $\Delta m \ll M_{KK}^w \ll \Delta M$ and so the cutoff of the effective 4D theory lies between $\Delta m$ and $\Delta M$. In this case we expect supersymmetry to be badly broken for the
multiplets whose masses are split by $\Delta M$, but that the supersymmetry-breaking effects which
these induce for the multiplets split by $\Delta m$ to be encoded in the low-energy 4D theory by a
suitable class of weakly-coupled soft-breaking interactions. We argue here that the warping
of these masses may also be accomplished by the same shift as for the supersymmetric case,
$K = 2A_{m} + \tilde{K}$.

To show this we require a statement of what the resulting soft-breaking interactions
might be. These have been enumerated in the literature [24, 25], under the assumption that
there is a regime for which the full theory — both $\ell^a$’s and $h^m$’s — is given by a 4D $N = 1$
supergravity, described by an appropriate Kähler potential,

$$K = \tilde{K}(h, h^*) + \tilde{K}_{ab}(h, h^*)\ell^a \ell^b + \frac{1}{2} Z_{ab}(h, h^*)\ell^a \ell^b + \cdots ,$$

(2.47)

as well as a superpotential, $W$, and gauge-kinetic function, $F_{AB}$. (Here $\ell^a$ denotes $(\ell^a)^\dagger$.)
In this case the soft-breaking quantities can be computed in terms of the assumed coupling
functions and the SUSY-breaking hidden-sector auxiliary fields,

$$\mathcal{F}^{m} = e^{G/2}K^{m\pi}\partial_{h}G ,$$

(2.48)

using eq. (2.46). For instance, the resulting expressions for the scalar and gaugino masses are

$$m_{ab}^{2} = (m_{3/2}^{2} + V_{0})\tilde{K}_{ab} - \mathcal{F}^{m}\mathcal{F}^{\pi}\left(\partial_{m}\partial_{n}\tilde{K}_{ab} - \tilde{K}_{cd}\partial_{m}\tilde{K}_{ab}\partial_{n}\tilde{K}_{cd}\right)$$

$$M_{AB} = \frac{1}{2} \left[ (\text{Re } F)^{-1} \right]_{AC} \mathcal{F}^{m}\partial_{m}F_{CB} ,$$

(2.49)

where the gravitino mass is

$$m_{3/2} = e^{G/2} = e^{K/2}|W|$$

(2.50)

and $V_{0}$ is the value of the potential at its minimum.

We again expect the low-energy theory to know about the higher-dimensional warping,
since it must ensure that all of the nonzero masses associated with localized states within the
warped region are suppressed by a common factor $e^{A_{m}}$. As may be seen from the above, we
find that this can be done if the Kähler potential contains an additive constant, which can
be taken to be in the part describing the hidden (bulk) sector

$$\tilde{K}(h, h^*) = 2A_{m} + \mathcal{K}(h, h^*) ,$$

(2.51)

for essentially the same reason as for the supersymmetric case considered above. Such a
constant has the effect of scaling all of the supersymmetry breaking v.e.v.’s, $\mathcal{F}^{m}$, by a common
factor of $e^{A_{m}}$, thereby ensuring that all masses are properly suppressed by the warp factor.

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8Although in the previous section the hidden fields $h$ included moduli and matter fields living far from the
throat, here they are understood to include only moduli which may survive at low energies (such as Kähler
moduli).
Our knowledge of the gravitino mass can be used to develop some understanding of the Kähler potential of the low energy effective action. Let us first consider the compactification at very large volume such that there are no throats, (2.17). In this regime, the gravitino mass is \( m_{3/2} \sim 1/V \). From eq. (2.50) one concludes
\[
K \sim -2 \log V \tag{2.52}
\]
in agreement with the leading order behavior of the Kähler potential in the \( \alpha' \) expansion.

Now consider decreasing the parameter \( c \) to the regime \( c \ll e^{-A_m} \). Our analysis suggests the wavefunction of the gravitino continuously localizes in the throat\(^9\) and \( m_{3/2} \sim f' e^{A_m}/V^{1/3} \). Since the superpotential is expected to receive no corrections, (2.50) indicates that in this regime the Kähler potential behaves as
\[
K \sim 2A_m - \frac{2}{3} \log V \tag{2.53}
\]
Notice the volume dependence of \( K \) is apparently different from the leading order Kähler potential (2.52). This is understandable since in the highly warped regime \( (c \ll e^{-A_m}) \) one expects warping to significantly correct the Kahler potential.

An important feature of the form of the Kähler potential for the compactifications of [2] is the no-scale structure associated with the vanishing of the cosmological constant,
\[
K^{a\bar{b}} \partial_a K \partial_{\bar{b}} K = 3, \tag{2.54}
\]
where \( K^{a\bar{b}} \) is the inverse of the Kähler metric \( \partial_a \partial_{\bar{b}} K \) and the derivatives are taken with respect to holomorphic coordinates for Kahler moduli. This is satisfied by \( K = -2 \log V \).

While (2.53) has different volume dependence and thus appears to violate the no-scale condition, we expect important warping corrections to the definition of the holomorphic coordinates such that the no-scale structure is preserved in this regime. Related corrections have been described in [26, 16, 27].

3. SUSY breaking in the microscopic theory

In this section we give examples of how visible-sector supersymmetry-breaking masses might arise within a microscopic calculation within a warped environment. To this end we compute the warp-factor dependence of the masses which are induced for some brane moduli as a consequence of bulk fluxes.

3.1 Flux-induced masses on a D3-brane

We consider a D3-brane filling the non-compact dimensions of a generic GKP vacuum [2]. The matter content of the theory in the world-volume of the brane will contain six real scalars \( Y^m \) parameterizing the position of the D3-brane in the transverse space. The dynamics of these

\(^9\)In case of more than one throats the most strongly warped throat is the relevant one.
scalars is described in terms of the corresponding Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions

\[ S_3 = -|\mu_3| \int d^4x \ e^{-\phi} \sqrt{-\det(P[E])} + \mu_3 \int P[C_4] + \ldots , \]  

(3.1)

where \( \mu_3 \) is the D3-brane charge, and \( P[E] \) denotes the pullback to the brane of the tensor \( E_{MN} = g_{MN} + B_{MN} \). With brane coordinates \((x^\mu, Y^m(x^\mu))\), this can be written

\[ P[E]_{\mu\nu} = E_{\mu\nu} + E_{mn} \partial_\mu Y^m \partial_\nu Y^n + \partial_\mu Y^m E_{m\nu} + \partial_\nu Y^n E_{\mu n} . \]  

(3.2)

\( P[C_4] \) similarly denotes the pull-back of the RR 4-form potential. One expects additional terms in the CS piece due to the other RR fields, but these turn out to be irrelevant for the purpose of computing the scalar masses, as one may check. The metric in these expressions is the string-frame metric, which is related to (2.11) by an \( e^{\phi/2} \) scaling factor,

\[ ds_{\text{str}}^2 = e^{\phi/2} \left( \lambda e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn} dy^m dy^n \right) , \]  

(3.3)

with \( \lambda \) defined in (2.12).

Assuming a constant background for the ten dimensional dilaton, as generically happens in the absence of D7 branes, one easily shows that

\[ \sqrt{-\det(P[E])} = \lambda^2 e^{4A} e^{\phi} \left[ 1 + \frac{1}{2} \lambda^{-1} e^{-4A} g_{mn} \partial_\mu Y^m \partial^\mu Y^n \right] \]  

(3.4)

where we have kept only the terms relevant for the computation of the scalar masses and where the internal metric \( \tilde{g}_{mn} \) in (3.4) is evaluated at the position of the brane. From this we find

\[ S_3 \simeq - \int d^4x \left[ |\mu_3| \lambda \tilde{g}_{mn} \partial_\mu Y^m \partial^\mu Y^n + |\mu_3| \lambda^2 e^{4A} - \mu_3 C_{0123} \right] . \]  

(3.5)

Thus the potential for a D3 position \( Y^m \) is given by

\[ V = \lambda e^{4A} - \frac{1}{\lambda} C_{0123} . \]  

(3.6)

This can be expanded around the minimum value \( e^{4A_m} \) to give the mass matrix,

\[ V \simeq \lambda e^{4A_m} + \frac{\partial_m \partial_n V}{2} Y^m Y^n . \]  

(3.7)

The trace of the mass matrix can be computed (see e.g. \[ \text{[28, 29]} \)) using the supergravity equations \[ \text{[2]} \):

\[ \tilde{g}^{mn} \partial_m \partial_n V = \lambda \tilde{\nabla}^2 \left( e^{4A} - \frac{1}{\lambda^2} C_{0123} \right) \simeq \lambda \frac{e^{8A}}{24F m^7} (G_3 + i \tilde{\sigma}_6 G_3)_{mnp}(\tilde{G}_3 - i \tilde{\sigma}_6 \tilde{G}_3)^{\text{mpn}} . \]  

(3.8)

The fact that the scalar masses vanish for imaginary self dual fluxes can be traced back to the no-scale structure of these Type IIB vacua \[ \text{[14, 16]} \]. However, imaginary anti-self dual components for the 3-form flux can be thought of as being induced by the backreaction of
effects which break the no-scale structure. Comparing (3.8) to (2.24), we find that the flux-induced scalar masses are of the same size as both the bulk moduli masses discussed there and the gravitino mass (2.44),

\[ m_Y \sim e^{A_m \sqrt{\lambda}}. \]  

(3.9)

Notice that, with very little effort, the computation can be extended to the case of the scalar masses for a \( D3 \) brane, as its action differs only in the sign of the last term in (3.7). Thus, the expression (3.9) remains valid for an antibrane, but now the left hand side of (3.8) is proportional to the imaginary self dual components of the fluxes 29.

\subsection*{3.2 Flux-induced masses on a D7 brane}

For D7 branes one can proceed as in the previous section. However, now the D7 brane wraps a 4-cycle \( \Sigma \) in the extra dimensions. We will consider a trivial normal bundle for the embedding of \( \Sigma \) in the compact manifold, so there will be just two geometric moduli \( Y^i \) in the four dimensional theory parameterizing the position of the D7-brane.

We will use 4D indices \( \alpha, \beta, \ldots \) for the compact dimensions within the brane volume; and 2D indices \( i, j, \ldots \) for the compact dimensions transverse to the D7-brane. If we imagine the D7 to wrap directions 4,5,6,7 of the compact space so that 8,9 denote the transverse directions, then \( \alpha, \beta, \ldots = 4, \ldots, 7 \) and \( i, j, \ldots = 8, 9 \).

For simplicity, we will only consider the reduction of the DBI piece of the action, which for a D7-brane reads,

\[ S_7 = -|\mu_7| \int_{\mathbb{R}^4 \times \Sigma} d^8 \xi e^{-\phi} \sqrt{-\det(P[E])}. \]  

(3.10)

Notice that, as for D3-branes, one expects also contributions from the CS action, cancelling part of the mass terms in the DBI piece.\(^{10}\) However, for the purpose of computing the warp suppression of the scalar masses, it suffices to analyze the DBI contributions.

The pull-back (3.2) is then given by

\[
\begin{align*}
P[E]_{\mu\nu} &= e^{2A} \lambda e^{\phi/2} \eta_{\mu\nu} + e^{-2A} e^{\phi/2} \hat{g}_{ij} \partial_\mu Y^i \partial_\nu Y^j, \\
P[E]_{\alpha\beta} &= e^{-2A} \lambda e^{\phi/2} \hat{g}_{\alpha\beta} + B_{\alpha\beta},
\end{align*}
\]  

(3.11)

where we have restricted ourselves to the case \( B_{\mu\nu} = 0 \) and \( \partial_\nu Y^i = 0 \). The two fields \( Y^i(x) \) denote low-energy fluctuations in the transverse position, \( y^i = Y^i(x) \), of the D7 brane.

Because of its block-diagonal structure, the determinant of \( P[E] \) becomes

\[
\det(P[E]) = (e^{2A} \lambda e^{\phi/2})^4 \det \left[ \eta_{\mu\nu} + e^{-4A} \lambda^{-1} \hat{g}_{ij} \partial_\mu Y^i \partial_\nu Y^j \right] \times (e^{-2A} \lambda e^{-\phi/2})^4 \det \left[ \hat{g}_{\alpha\beta} + e^{2A} e^{-\phi/2} B_{\alpha\beta} \right],
\]  

(3.12)

which, when expanded in powers of the fluctuations gives

\(^{10}\)See [31] for further details.
\[ \sqrt{-\det(P[E])} = \lambda^2 e^{2\phi} \sqrt{g_4} \left( 1 + \frac{1}{2} e^{-4A} \lambda^{-1} \bar{g}_{ij} \partial_{\mu} Y^i \partial^\mu Y^j + \cdots \right) \times \left( 1 + \frac{1}{4} e^{4A} e^{-\phi} B_{\alpha\beta} B^\alpha B^\beta + \cdots \right), \] (3.13)

where we denote \( \bar{g}_{\alpha\beta} \), the determinant of the pullback on the four cycle, by \( \bar{g}_4 \). We can use this in the DBI action, working to quadratic order in the fluctuations; we assume that the flux \( B_{\alpha\beta} \) is constant over the four cycle, and hence it does not induce Freed-Witten anomalies \[35\] in the worldvolume of the brane. This gives

\[ S_7 = -|\mu_7| \int_{\mathbb{R}^4 \times \Sigma} d^4x d^4y \sqrt{g_4} e^\phi \left( \lambda^2 + \frac{1}{2} e^{-4A} \lambda \bar{g}_{ij} \partial_{\mu} Y^i \partial^\mu Y^j + \frac{1}{4} e^{4A} \lambda^2 e^{-\phi} B_{\alpha\beta} B^\alpha B^\beta + \cdots \right) \]

\[ = -|\mu_7| \int d^4x \left( V + \frac{1}{2} G_{ij} \partial_{\mu} Y^i \partial^\mu Y^j \right) \] (3.14)

where the potential and kinetic terms are given by

\[ V = \lambda^2 \int_{\Sigma} d^4y \sqrt{g_4} e^\phi \left[ 1 + \frac{1}{4} e^{-\phi} e^{4A} B_{\alpha\beta} B^\alpha B^\beta \right], \quad G_{ij} = \lambda \int_{\Sigma} d^4y \sqrt{g_4} e^{-4A} e^\phi \bar{g}_{ij}. \] (3.15)

We are now in a position to see why the presence of a nonzero background flux, \( H_{mnp} \neq 0 \), can generate a mass for the brane modulus \( Y^i \). Notice that since \( dB = H_3 \), if the normal to \( \Sigma \) has components along the cycle supporting \( H_3 \), in the vicinity of \( \Sigma \) we may write \( B_{\alpha\beta} \sim H_{\alpha\beta i} Y^i \). This gives

\[ B_{\alpha\beta} B^\alpha B^\beta \sim H_{\alpha\beta i} H^\alpha_j Y^i Y^j, \] (3.16)

leading to a potential matrix \( V_{ij} \sim C \lambda^2 H_{\alpha\beta i} H^\alpha_j Y^i Y^j \). A mass is produced if the D7 brane wraps a cycle which overlaps the cycle which supports the nonzero flux.

Whether such masses break 4D supersymmetry or not depends on the details of the fluxes involved. Using a complex basis in the compact dimensions, this mass can preserve supersymmetry if the corresponding flux is purely of (2,1) or (1,2) type, and breaks supersymmetry \[32\] if it contains fluxes of (3,0) or (0,3) type.

Since our interest is in tracking how masses depend on warping for states localized in warped regions, it is instructive to specialize the above analysis to the special case where the D7 brane is localized within such a region,\(^{11}\) analogously to what we did with the D3 branes. Suppose then that the warp factor is \( e^A(y) = e^{A_m} e^{A(y)} \) throughout \( \Sigma \), where \( e^{A_m} \ll 1 \) and \( e^{A(y)} \) integrates over the cycle to give a result which is \( O(1) \) (notice that this implies that the flux relevant for generating masses for D7 moduli is also nonzero in the highly warped region, which is not the generic case). In this case, using the explicit formula for the warped throat metric we find fluctuation \( G_{ij} \sim \lambda e^{2A_m} \) and \( V_{ij} \sim \lambda^2 e^{4A_m} \). Thus the mass for the fields \( Y^i \) is

\[ m_Y \sim e^{A_m} \sqrt{\lambda}, \] (3.17)

which coincides with the ones obtained in the previous section for the geometric moduli of a D3 brane.

\(^{11}\)For an example of such a construction, see \[33\].
3.3 Effective 4D description

We now record what would be the corresponding description of such flux-induced masses for the brane geometric moduli from the point of view of the low-energy 4D theory. We consider in turn the cases where the flux preserves and breaks supersymmetry. Our main interest is in how the overall warp suppression factor, $e^{A_m}$, appears in the low-energy theory.

3.3.1 Unbroken supersymmetry

If the relevant flux does not break $N = 1$ 4D supersymmetry then the effective 4D theory is described by the standard supergravity lagrangian, within which the D-brane moduli are represented by complex scalars residing within chiral supermultiplets. More precisely, the D7 moduli are described by a single complex scalar $Y$, whereas the six geometric D3 moduli are arranged into three complex scalars, $\tilde{Y}^a$, with $a = 1, 2, 3$. We will discuss both kind of moduli under the same context.

The appropriate choice of Kähler potential is found by inspecting the kinetic terms for the D3 and D7 moduli, leading to the form $K = 2A_m + K$, with

$$K = K_c(\phi, \phi^*) + K_Y(\phi, \phi^*)Y^*Y + \tilde{K}_{\tilde{Y}^a}(\phi, \phi^*)\tilde{Y}^{a*}\tilde{Y}^a + \cdots,$$

(3.18)

where $\phi$ collectively denotes all the other moduli and the ellipses denote terms involving higher powers of $\tilde{Y}^a$, $Y$ and their complex conjugates. We include an overall constant $2A_m$, as was argued above to be required in order to generically warp all nonzero masses for localized states by a factor $e^{A_m}$. This additive factor does not affect the $\tilde{Y}^a$ and $Y$ kinetic terms, however, so agreement with the microscopic calculation requires we also choose

$$K_Y = |\mu_7| k(\phi, \phi^*) \quad K_{\tilde{Y}^a} = |\mu_3| k^a(\phi, \phi^*),$$

(3.19)

with no $A_m$ dependence. For the present purposes the functions $K_c$, $k$ and $k^a$ could be arbitrary, although they are known explicitly for specific types of compactifications [34, 36, 37].

In this case the superpotential is not given only by the GVW form, since it also acquires a dependence on the D7 moduli [31, 37], which we expand to lowest order in $Y$:

$$W = W_{GVW}(\phi) + \frac{\mu_7}{2} w(\phi) Y^2 + \cdots.$$  

(3.20)

As for D7 branes in the unwarped regions, we will assume that $w$ does not carry any factors of the small quantity $e^{A_m}$.

The corresponding Kähler derivatives become

$$D_{\tilde{Y}^a}W = \partial_{\tilde{Y}^a}W + W\partial_{\tilde{Y}^a}K = \mu_3 k^a \tilde{Y}^{a*}W_{GVW} + \cdots,$$

(3.21)

$$D_YW = \partial_Y W + W\partial_Y K = \mu_7 \left[ w Y + k Y^*W_{GVW} \right] + \cdots,$$

(3.22)

which show that $\tilde{Y}^a = Y = 0$ does not break supersymmetry (or perturb the vacuum away from vanishing potential $V$).
In this case, keeping in mind the no-scale nature of the low-energy theory which ensures that $W_{GV W} = V = 0$ at the minimum, the scalar mass term for the brane moduli is given by the contribution

$$V = e^K \sum_{\Phi = Y, \tilde{Y}^a} \frac{|D\Phi W|^2}{K} + \cdots = |\mu_7| e^{2A_m} e^K \frac{|w Y|^2}{k} + \cdots ,$$  \hspace{1cm} (3.23)

Notice that the $Y$ mass term now scales in the same way as it did in the microscopic computation, whereas on the other hand, the D3 moduli remain massless, in agreement with the no-scale structure of the potential.

### 3.3.2 Broken supersymmetry

Next consider the case where the mass-generating flux breaks supersymmetry. Since the supersymmetry-breaking field is not within the low-energy theory, and since the mass splitting generated is much smaller than the generic KK mass, in this instance we expect the effective 4D theory to be described by a 4D supergravity supplemented by soft-breaking terms.

In this case the $Y$ and $\tilde{Y}^a$ kinetic terms are unchanged from the supersymmetric case, and the additive term for $K$ is also required to ensure that all generic masses are warped by a factor of $e^{A_m}$, and so $K = 2A_m + \mathcal{K}$, with $\mathcal{K}$ given by eq. (3.18). The additional suppression of the D7 modulus mass is then described by the relevant soft-breaking terms in the scalar potential. Specializing the result given in eq. (2.49) to the case of a diagonal metric, and canonically normalizing the kinetic terms, gives the following result for the physical $Y$ mass

$$m_Y^2 = m_{3/2}^2 - \mathcal{F}^m \mathcal{F}^\pi \partial_m \partial_\pi \ln K_Y ,$$  \hspace{1cm} (3.24)

where we use $V_0 = 0$, and as before $m_{3/2} = e^{K/2} |W|$ and $\mathcal{F}^m$ is given by (2.48). Notice that both terms of (3.24) include a factor of $e^{2A_m}$, so that the soft-breaking mass of $Y$ indeed has a factor of $e^{A_m}$.

Regarding the D3 moduli $\tilde{Y}^a$, due to the no-scale structure of the scalar potential for these vacua, the gravitino mass is generically related to $K_{\tilde{Y}^a}$ in such a way that

$$m_{3/2}^2 = \mathcal{F}^m \mathcal{F}^\pi \partial_m \partial_\pi \ln K_{\tilde{Y}^a} ,$$  \hspace{1cm} (3.25)

and the soft masses for the D3 brane moduli vanish, even though supersymmetry is being broken.

In general the no-scale structure of the scalar potential is however spoiled by both $\alpha'$ corrections and non-perturbative corrections to the superpotential, and the relation (3.25) will not hold. In that case, the soft masses (2.49) for the $\tilde{Y}^a$ scalars become

$$m_{\tilde{Y}^a}^2 = V_0 + m_{3/2}^2 - \mathcal{F}^m \mathcal{F}^\pi \partial_m \partial_\pi \ln K_{\tilde{Y}^a} ,$$  \hspace{1cm} (3.26)

where we have taken again a diagonal metric and canonically normalized kinetic terms. The value of the potential, $V_0$, now is also generically different from zero and given by the formula

$$V_0 = e^K (|DW|^2 - 3|W|^2) .$$  \hspace{1cm} (3.27)
Thus, from the scaling $e^K \propto e^{2A_m}$ we find that in scenarios with broken no-scale structure the soft masses for the D3 moduli are also suppressed by a $e^{A_m}$ factor, in agreement with the results obtained in section 3.1 from a microscopical point of view.

4. Towards Phenomenology

Clearly our results may have interesting phenomenological implications. The study of soft supersymmetry breaking in the KKL T scenario [38, 40] has not included the effects of warping. Warping has only been considered in the mechanism for lifting to de Sitter space, but in this scenario we expect the standard model to appear from D-branes wrapping non-trivial cycles of the Calabi-Yau manifold, and a natural possibility is that these D-branes lie in a strongly-warped region (for explicit constructions in this direction see [41, 33]). This was actually part of the original motivation in [2], since the redshift in a throat is a natural mechanism to solve the hierarchy problem if the standard model sector lives on the tip of the throat.

The substantial redshifting due to the warp factor dependence of $M_w$ implies that we can consider different scenarios depending on the corresponding warped string scale which can take any value between the electroweak scale (1 TeV) and the GUT scale ($\sim 10^{17}$ GeV). On the other hand, all the other relevant scales are also suppressed by the same warp factor (and bulk volume dependence). As mentioned before, we expect the warped Kaluza-Klein scale $M^w_{KK}$ to take values somewhat smaller than $M_w$ due to the characteristic curvature scale $1/\rho$ of the tip of the throat. Furthermore, the gravitino mass and all soft supersymmetry breaking terms are further reduced by the flux factor $m_{3/2} \sim M^w_{KK} W_0$. If the flux superpotential $W_0$ is very small the gravitino mass is hierarchically smaller than $M^w_{KK}$ and a low-energy supergravity action with standard soft supersymmetry breaking terms are obtained. Notice that a very small $W_0$ is required in the original KKL T scenario, but for a different reason. This was to have the tree-level action to be comparable to the non-perturbative contributions to the superpotential and justify neglecting of the perturbative corrections to the Kähler potential.

For a GUT string scale $W_0 \sim 10^{-13}$ gives rise to a TeV gravitino. This is similar to the scenario discussed in [38] without taking warping into account. A GUT scale does not seem very plausible if we have $e^{-4A_m} \gg \nu^{2/3} \gg 1$ since $M^w_{KK} / M^w_{KK} \sim e^{A_m} / \nu^{1/3} \sim 10^2$ requires a relatively small volume. Smaller warped string scales arise more naturally in our scenario, including two potentially attractive possibilities. Having the warped string scale at the intermediate scale, $M^w_{KK} \sim 10^{11}$ GeV, could permit warped realizations of intermediate-scale string scenarios [34], which are also attractive from the point of view of some string inflationary models [4]. Getting a TeV gravitino mass in such models requires a very small flux superpotential, $W_0 \sim 10^{-7}$, which is still in the range of validity of the KKL T approximations.

Alternatively, the warped string scale could be of order $M^w_{KK} \sim 10$ TeV and $W_0 \sim 1/10$, which still justifies the use of effective field theory. Since statistically speaking a very small value of $W_0$ is not preferred, one might argue that having such a low warped string scale is a more natural scenario to have. This would indicate a very interesting phenomenological scenario with a very small string scale and approximately supersymmetric effective action
with soft SUSY breaking terms:

$$M_{1/2} \sim m_0 \sim A \sim m_{3/2} \sim \frac{1}{10} M_s^w \sim 1 \text{ TeV}$$

(4.1)

This can be considered on the same footing as dynamical supersymmetry breaking in the approach towards the solution of the hierarchy problem, in the sense that the exponential hierarchy is obtained from the warped geometry rather than strongly coupled dynamics in a field theory. These two mechanisms are dual from the AdS/CFT correspondence.

For $W_0 \sim 1$ we know that the KKLT approximations fail. In a large class of models [42] an exponentially large volume stabilization is obtained by including perturbative corrections to the Kähler potential. But for very large volumes the effects of warping are less and less relevant since the condition $e^{-4A_m} \gg V^{2/3}$ would be more difficult to satisfy. Interestingly enough, in our set-up, values of $W_0 \gtrsim 1$ would imply a collapse of the supersymmetric field theory approximation since the gravitino mass would be as heavy as the KK and string modes. In this case we do not expect an effective supersymmetric 4D action to play any role and we may have to consider directly an effective string theory phenomenology with distinctive signatures as compared with standard effective field theories, such as the presence of towers of KK and string states, etc. Alternatively, one might describe this situation in terms of the AdS/CFT dual.

We might foresee further scenarios depending on the location and the source of supersymmetry breaking as compared to the standard model.

5. Conclusions

We have given a first step towards understanding the effective description of broken supersymmetric theories in strongly-warped throats. Warped compactifications are very natural in IIB string theory [43, 44] and provide a very rich, local and stringy scenario to discuss supersymmetry breaking with potentially different properties from standard scenarios of supersymmetry breaking in terms of gravity, gauge and anomaly mediation.

A typical compactification may have many throats and depending on the structure of the fluxes on each throat, the physics in each of the throats can be very different. Therefore within one single compactification we may have local models that feel differently the scale of supersymmetry breaking and therefore different structure of soft-breaking terms. In a large class of models this may not be describable in terms of effective 4D supergravities since the gravitino mass will be degenerate with the string and KK scales.

If the flux superpotential is small enough, a natural hierarchy is generated between the KK scale and the gravitino mass, justifying a supersymmetric effective field theory treatment. The exponentially large warp factor is a natural source of hierarchy as in the Randall-Sundrum model. In our case it provides the exponentially small scale of SUSY breaking instead of dynamical SUSY breaking.

Our investigation allowed us to solve a puzzle regarding the gravitino mass in warped throats. The original supersymmetric partner of the 4D graviton, the gravitino zero mode,
acquires a very large, unwarped mass if supersymmetry is broken in the bulk, however in the throat the scalar masses are warped and cannot then be proportional to the gravitino mass as the general 4D expressions for soft terms require. The puzzle is solved by realizing that the effective partner of the 4D graviton is the lightest KK mode of the gravitino which is warped down since the gravitino is localized in the throat, and therefore this gravitino’s mass is the basic scale of all supersymmetry breaking terms.

This analysis also leads us to suggest that the effective Kähler potential is shifted by the warp function $A$ at the tip of the throat, but a general interpolation between the regimes of weak and strong warping is unknown.

We identify several potentially interesting phenomenological scenarios depending on the amount of warping and the tuning of the flux superpotential $W_0$. Further investigation of the detailed phenomenology of these new scenarios is certainly desirable. We also expect potential applications for cosmology, and in particular for inflationary scenarios depending on the existence of warping.

Acknowledgements

We thank L. Ibáñez for collaboration in the early stages of this work. We acknowledge useful conversations on the subject of this paper with S. Abdussalam, J. Conlon, D. Cremades, O. DeWolfe, S. Hartnoll, S. Kachru, P. Kumar, S. Thomas, E. Silverstein, A. Sinha, and H. Verlinde. One of us (AM) would like to especially thank A. Frey for many useful discussions. CB’s research is supported in part by funds from Natural Sciences and Engineering Research Council of Canada, the Killam Foundation and McMaster University. SBG and AM acknowledge the support of the Department of Energy under Contract DE-FG02-91ER40618. The work of PGC was supported by the EU under the contracts MEXT-CT-2003-509661, MRTN-CT-2004-005104 and MRTN-CT-2004-503369. KS wishes to thank Trinity College, Cambridge for financial support. C.B. thanks the kind hospitality of the Galileo Galilei Institute, where some of this work was done. SdA, SBG, FQ thank KITP Santa Barbara and the organizers of the workshop on ‘String Phenomenology’ for the same reasons, and acknowledge the partial support of the National Science Foundation under Grant No. PHY99-07949. FQ is partially funded by PPARC and a Royal Society Wolfson award. SdA wishes to thank the Perimeter Institute and DOE grant No. DE-FG02-91-ER-40672 for partial support.

A. Warping and degenerate perturbation theory

In this appendix we use the example of a bulk scalar in the Randall-Sundrum scenario as a toy model to discuss some features of KK reduction in highly warped regions. The KK reduction can be carried out explicitly in this model, thus the model is helpful for building intuition. We use it to examine the common practice of identifying supersymmetry-breaking mass shifts by truncating the KK reduction using supersymmetric configurations, and show
when this can be justified in terms of a perturbative analysis. We show in particular why this analysis breaks down in the presence of strong warping.

Explicit diagonalization

Reference [45] considered a massive 5-d scalar

\[ S = \int d^5x \sqrt{g} \left( - \partial_M Y \partial^M Y - M^2 Y^2 \right) \]

in a finite domain of AdS (of radius \(R\))

\[ ds^2 = \frac{r^2}{R^2} \eta_{\mu \nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2, \quad r_0 < r < R \]

with Neumann boundary conditions at both ends.

Before discussing the KK reduction, we note that the above model has the essential features to capture the dynamics of the gravitino in IIB constructions in the highly warped regime \((c \ll e^{-A_m})\). In the throat regions of IIB constructions, the metric typically factorizes to an AdS5 \(\times X_5\) structure, for some manifold \(X_5\), and so the wavefunction has a product structure. Thus, to understand the effects of warping it suffices to truncate to a 5D model. Also the ten dimensional mass term for the gravitino, \(e^{3A} G_{mnp} \gamma^m \gamma^np\) is approximately a constant in the throat region\(^{12}\) and typically scales as the inverse of the local AdS radius. Thus the simplest model to consider is that of a minimal massive scalar \((A.1)\) with \(MR \sim 1\).

We give a brief outline of the results of explicit Kaluza-Klein reduction (for details see [45]). For dimensional reduction, consider the 5D equations of motion for the ansatz

\[ Y(x, r) = \sum_n u_n(r) \phi_n(x) \]

with \(\partial_\mu \partial^\mu \phi_n = m_n^2 \phi_n \ (m_n\ \text{are the four dimensional masses})\). This yields an equation for \(u_n\)

\[-\frac{1}{R^4 r^5} \partial_r (r^5 \partial_r u_n) + \frac{r^2}{R^2} M^2 u_n = m_n^2 u_n. \]

A general solution to the differential equation \((A.4)\) can be written in terms of Bessel functions of order \(\nu = \sqrt{4 + (MR)^2}\),

\[ u_n(r) = \frac{N_n}{r^2} \left[ J_\nu \left( \frac{m_n R^2}{r} \right) + b_n Y_\nu \left( \frac{m_n R^2}{r} \right) \right] \]

where \(N_n\) and \(b_n\) are constants. The masses \(m_n\) and the constants \(b_n\) are fixed by the boundary conditions at \(r_0, R\). For large warping \((r_0/R \ll 1)\) one finds that the masses are determined by the equation

\[ 2J_\nu (x_n) + x_n J'_\nu (x_n) = 0 \]

where \(x_n = \frac{R^2}{r_0} m_n\). In the regime \(RM \sim 1\), the lowest root of \((A.6)\) is of the order of unity and

\[ m_0 \sim \frac{r_0}{R} M. \]

The wavefunction \((A.5)\) is highly localized in the region close to \(r_0\).

\(^{12}\)For a discussion see section 2.2.2 and related discussion of the dilaton mass term in section 2.1.1.
Perturbative analysis

It is illustrative to examine this result in the context of perturbation theory, since it provides a toy example which shares many of the features which commonly arise when supersymmetry is broken in an extra-dimensional model. In supersymmetric models it is often useful to imagine turning on supersymmetry breaking in a parametrically small way, such as by turning on a small flux. In this case our interest is often in the size of the SUSY-breaking mass splittings which arise, as computed perturbatively in the SUSY-breaking parameter. This kind of problem has an analogue in the present example in the limit of small $M$, since the model acquires a new symmetry in this limit corresponding to shifts of the form $Y \rightarrow Y + \epsilon$. We therefore now consider computing the scalar mass in the small-$M$ limit, in order to compare the results obtained perturbatively with the exact results found above. The perturbative methods that we discuss can also have applications to situations where an explicit diagonalization is not possible.

To proceed we treat the term involving the five dimensional mass $M$ in (A.4) as a perturbation. We take the masses and wavefunctions of the unperturbed “hamiltonian” to be $\mu_n$ and $v_n$, i.e.

$$-\frac{1}{R^4 r} \partial_r (r^5 \partial_r v_n) \equiv H_0 v_n = \mu_n^2 v_n. \quad (A.8)$$

In the absence of the five dimensional mass term, the lowest mode is massless ($\mu_0 = 0$) and has a constant wavefunction. For the higher modes, the masses and wavefunctions ($\mu_i, v_i i > 0$) are given by (A.5) and (A.6) with $\nu = 2$. We note that the first roots of (A.6) are of the order of unity, hence the scale of the unperturbed KK tower is

$$m_{KK} \sim \frac{r_0}{R^2}. \quad (A.9)$$

To set up the perturbative computation, we begin by introducing an inner product

$$\langle a|b \rangle = \int_{r_0}^{R} \, dr \, ra(r)b(r) \quad (A.10)$$

under which the unperturbed Hamiltonian, $H_0$, for the KK states is hermitian. We normalize $v_n$ as

$$\langle v_n|v_m \rangle = \delta_{nm}. \quad (A.11)$$

Given the perturbation Hamiltonian

$$H' \equiv \frac{r^2}{R^2} M^2, \quad (A.12)$$

the mass of the lowest mode in first order non-degenerate perturbation theory is

$$m_0^2 = \langle v_0|H'|v_0 \rangle. \quad (A.13)$$

With the normalization (A.11),

$$v_0 \sim \frac{1}{R}. \quad (A.14)$$
Then \( (A.13) \) gives

\[
m_0 \sim M.
\]  

(A.15)

This is the analogue of supergravity calculation which estimates the lowest gravitino KK mass by evaluating the supersymmetry-breaking action using the Killing spinor which defines the wavefunction of the mode which is massless in the supersymmetric limit.

Note that the perturbative result, eq. \( (A.13) \), is much larger than the correct value of the mass of the lowest mode \( (A.7) \). First order non-degenerate perturbation theory fails because the strength of the perturbation is large compared to the KK scale mass \( (A.9) \). Under such a circumstance one expects the wavefunction of the lowest excitation \( (u_0) \) to be significantly different from the lowest mode in the absence of the perturbation due to mixings between the zero mode \( (v_0) \) and the KK modes \( (v_i) \) introduced by the perturbation. These mixings are not captured by first order non-degenerate perturbation theory which does not incorporate the corrections to the wavefunction.

In situations where the KK scale is small compared to the strength of the perturbation, a better perturbative tool is degenerate perturbation theory, since effectively the KK modes \( (v_i) \) and the zero mode \( (v_0) \) are degenerate compared to the scale of the perturbation. Typically, one has to include a large number of KK modes to obtain quantitatively reliable results. Since our purpose here is to be illustrative we will include just one mode. We shall see that this is sufficient to reproduce correct estimates.

For the purposes of estimate, we approximate the first KK mode by its power law behavior; then with the normalization condition \( (A.11) \)

\[
v_1 \sim \frac{r_0^3}{r^4}.
\]  

(A.16)

In the subspace spanned by \( v_0 \) and \( v_1 \), the perturbation \( H' \) has approximate matrix elements

\[
\begin{pmatrix}
M^2 & \frac{r_0^3}{r^4} \ln(R/r_0) M^2 \\
\frac{r_0^3}{r^4} \ln(R/r_0) M^2 & \frac{r_0^2}{r^3} M^2
\end{pmatrix}.
\]  

(A.17)

The lowest eigenvalue of the perturbation matrix \( (A.17) \) is of the order of \( \frac{r_0^2}{r^3} M^2 \), which is in agreement with \( (A.7) \).

References


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