Lepton flavor violating radion decays in the Randall-Sundrum scenario

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Abstract

We predict the branching ratios of lepton flavor violating radion decays $r \rightarrow e^\pm \mu^\pm$, $r \rightarrow e^\pm \tau^\pm$ and $r \rightarrow \mu^\pm \tau^\pm$ in the two Higgs doublet model, in the framework of the Randall Sundurum scenario. We observe that the branching ratios of the processes we study are at most at the order of $10^{-8}$ for the small values of radion mass and it decreases with the increasing values of $m_r$. Among the LFV decays we study, the $r \rightarrow \tau^\pm \mu^\pm$ decay would be the most suitable one to measure its branching ratio.

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1 Introduction

The hierarchy problem between weak and Planck scales could be explained by introducing the extra dimensions. One of the possibility is to bring the Planck scale down to TeV range by assuming the existence of compactified extra dimensions of large size \[^1\]. The assumption that the extra dimensions are at the order of submilimeter distance, for two extra dimensions, the hierarchy problem in the fundamental scales could be solved and the true scale of quantum gravity would be no more the Planck scale but in the order of electroweak (EW) scale. This is the case that the gravity is spreading over all the volume including the extra dimensions, however, the matter fields are restricted in four dimensions, called four dimensional (4D) brane. Another possibility was introduced by Randall and Sundrum \[^2\] (RS1 model) and it is related to the non-factorizable geometry where the gravity is localized in a 4D brane, so called Planck brane, which is away from another 4D brane, TeV brane where we live. In this model, the extra dimension is compactified to \( S^1/Z_2 \) orbifold with two 4D brane boundaries. The size of extra dimension is related to the vacuum expectation of a scalar field and its fluctuation over the expectation value is called the radion field (see section 2 for details). Radion in RS1 model has been studied in several works in the literature \[^3\]-\[^12\] (see \[^13\] for extensive discussion).

In the present work, we study the possible lepton flavor violating (LFV) decays of the radion field \( r \). The lepton flavor violating (LFV) interactions exist at least at one loop level in the extended standard model (SM), so called \( \nu \)SM, which is constructed by taking neutrinos massive and permitting the lepton mixing mechanism \[^14\]. Their negligibly small branching ratios (Brs) stimulates one to go beyond and they are worthwhile to examine since they open a window to test new models, to ensure considerable information about the restrictions of the free parameters, with the help of the possible accurate measurements. LFV interactions are carried by the flavor changing neutral currents (FCNCs) and in the SM with extended Higgs sector (the multi Higgs doublet model) they can exist at tree level. Among multi Higgs doublet models, the two Higgs doublet model (2HDM) is the candidate for LF violation. In this model, the LF violation is driven by the new scalar Higgs bosons \( S \), scalar \( h^0 \) and pseudo scalar \( A^0 \), and it is controlled by the Yukawa couplings appearing in \( lepton - lepton - S \) vertices.

Here, we predict the BRs of the LFV \( r \) decays in the 2HDM, in the framework of the RS1 scenario. We observe that the BRs of the processes we study are at most at the order of \( 10^{-8} \) for the small values of radion mass \( m_r \) and its sensitivity to \( m_r \) decreases with increasing values of \( m_r \). Among the LFV decays we study, the \( r \to \tau^\pm \mu^\pm \) decay would be the most suitable one to measure its BR.
The paper is organized as follows: In Section 2, we present the effective vertex and the BRs of LFV $r$ decays in the 2HDM, by respecting RS1 scenario. Section 3 is devoted to discussion and our conclusions. In appendix section, we present the interaction vertices appearing in the calculations.

2 The LFV RS1 model radion decay in the 2HDM

The RS1 model is an interesting candidate to explain the well-known hierarchy problem. It is formulated as two 4D surfaces (branes) in 5D world in which the extra dimension is compactified into $S^1/Z_2$ orbifold. In this model, the SM fields are assumed to live in one brane, the so called TeV brane, and the gravity is extended into the bulk with varying strength. Here the 5D cosmological constant is non vanishing and both branes have equal and opposite tension so that the low energy effective theory has flat 4D spacetime. The metric of such 5D world reads

$$ds^2 = e^{-2kL} \eta_{\mu\nu} dx^\mu dx^\nu - L^2 dy^2, \quad (1)$$

where $k$ is the bulk curvature constant, $L = R|\theta|$ is the size of the extra dimension and $e^{-kL}$ is the warp factor which causes that all the mass terms are rescaled in the TeV brane for $\theta = \pi$. With the rough estimate $L \sim 30/k$, all mass terms are bring down to the TeV scale. On the other hand the gravity peaks at so called Planck brane where it is located at $\theta = 0$.

The size $L$ of extra dimension is related to the vacuum expectation of the field $L(x)$ and its fluctuation over the expectation value is called the radion field $r$. In order to avoid violations of equivalence principle $L(x)$ should acquire a mass and, to stabilize $r$, a mechanism, introducing a potential for $L(x)$, was proposed by Goldberger and Wise [3]. Finally the metric in 5D is defined as [4]

$$ds^2 = e^{-2A(y)} e^{-2F(x)} \eta_{\mu\nu} dx^\mu dx^\nu - (1 + 2F(x)) dy^2, \quad (2)$$

where the radial fluctuations are carried by the scalar field $F(x)$

$$F(x) = \frac{1}{\sqrt{6} M_{Pl}} e^{-kL} r(x), \quad (3)$$

and $A(y) = ky$. Here the field $r(x)$ is the normalized radion field (see [5]). At the orbifold point $\theta = \pi$ (TeV brane) the induced metric reads,

$$g^{ind}_{\mu\nu} = e^{-2A(y)} e^{-2F(x)} \eta_{\mu\nu}, \quad (4)$$
and the parameters $\gamma$ and $v$ are $\gamma = \frac{v}{\sqrt{6} \Lambda}$ with $\Lambda = M_{Pl} e^{-kL}$ and the vacuum expectation value of the SM Higgs boson. The radion is the additional degree of freedom of the 4D effective theory and we study the possible LFV decays of this field.

The flavor changing neutral currents (FCNCs) at tree level can exist in the SM with extended Higgs sector and they induce the flavor violating (FV) interactions with large BRs. The 2HDM with the FCNC at tree level allows comparably large BRs for such decays. The flavor violating $r$ decays, $r \rightarrow l_1^- l_2^+$, can exist at least in the one loop level in the framework of the 2HDM. The part of action which carries the interaction responsible for the LFV processes reads

$$S_Y = \int d^4x \sqrt{-g^{ind}} \left( \eta_{ij}^{L} i_{iL} \phi_1 E_{jR} + \xi_{ij}^{E} i_{iL} \phi_2 E_{jR} + h.c. \right), \quad (5)$$

where $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_i$ for $i = 1, 2$, are two scalar doublets, $i_{iL}$ ($E_{jR}$) are lepton doublets (singlets), $\xi_{ij}^{E}$ and $\eta_{ij}^{E}$, with family indices $i, j$, are the Yukawa couplings and $\xi_{ij}^{E}$ induce the FV interactions in the lepton sector. Here $g^{ind}$ is the determinant of the induced metric at the TeV brane where the 2HDM particles live. Here we assume that the Higgs doublet $\phi_1$ has a non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions, however, the second doublet has no vacuum expectation value, namely, we choose the doublets $\phi_1$ and $\phi_2$ and their vacuum expectation values as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \left( \begin{pmatrix} \sqrt{2} \chi^+ \\ i \chi^0 \end{pmatrix} \right) \right] ; \phi_2 = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + i H_2 \end{pmatrix} \right), \quad (6)$$

and

$$< \phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; < \phi_2 > = 0. \quad (7)$$

This choice ensures that the mixing between neutral scalar Higgs bosons is switched off and it would be possible to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one. The action in eq. (5) is responsible for the tree level $S - l_1 - l_2$ ($l_1$ and $l_2$ are different flavors of charged leptons with identical electric charges, $S$ is the neutral new Higgs boson(s), $S = h^0, A^0$) interaction (see Fig. 1-d, e) and the four point $r - S - l_1 - l_2$ interaction (see Fig. 1-c) where $r$ is the radion field. The latter interaction is coming from the determinant factor $\sqrt{-g^{ind}} = e^{-4A(y) - \frac{1}{2} r(x)}$. Notice that the term $e^{-4A(y)}$ in $\sqrt{-g^{ind}}$ is embedded into the redefinitions of the fields in the TeV brane,

\footnote{In the following, we replace $\xi_j^E$ with $\xi_{jN}^E$ where "N" denotes the word "neutral".}

\footnote{Here $H^1$ ($H^2$) is the well known mass eigenstate $h^0$ ($A^0$).}
namely, they are warped as $S \to e^{A(y)} S_{\text{warp}}$, $l \to e^{3A(y)/2} l_{\text{warp}}$ and in the following we use warped fields without the \textit{warp} index.

On the other hand, the part of new scalar action

$$S_2 = \int d^4x \sqrt{-g} \text{ind} \left( g^{\text{ind} \mu \nu} (D_\mu \phi_2)^\dagger D_\nu \phi_2 - m_S^2 \phi_2^\dagger \phi_2 \right)$$

(8)

leads to the one, with warped $S$ fields and their masses,

$$S'_{2} = \frac{1}{2} \int d^4x \left\{ e^{-2\pi r} \eta^{\mu \nu} \left( \partial_\mu h^0 \partial_\nu h^0 + \partial_\mu A^0 \partial_\nu A^0 \right) - e^{-4\pi r} \left( m_{h^0}^2 h^0 h^0 + m_{A^0}^2 A^0 A^0 \right) \right\}$$

(9)

which carries the $S - S - r$ interaction (see Fig. 1-b).

Finally, the interaction of leptons with the radion field is carried by the action (see [6])

$$S_3 = \int d^4x \sqrt{-g} \text{ind} \left( g^{\text{ind} \mu \nu} \bar{l} \gamma_\mu i D_\nu l - m_l \bar{l} l \right) ,$$

(10)

where

$$D_\mu l = \partial_\mu l + \frac{1}{2} w_{\mu}^{ab} \Sigma_{ab} l,$$

(11)

with $\Sigma_{ab} = \frac{1}{4} [\gamma_a, \gamma_b]$. Here $w_{\mu}^{ab}$ is the spin connection and by using the vierbein fields $e_\mu^a$, it can be calculated (linear in $r$) as

$$w_{\mu}^{ab} = -\frac{\gamma}{v} \partial_\mu r \left( e^{vb} e_\mu^a - e^{va} e_\mu^b \right),$$

(12)

Notice that the vierbein fields are the square root of the metric and they satisfy the relation

$$e_a^\mu e^{\mu \nu} = g^{\text{ind} \mu \nu}.$$ 

(13)

Using eq. (10)-(13), one gets the the part of the action in eq. (10) which is linear in the radion field $r$ as

$$S'_3 = \int d^4x \left\{ -3 \frac{\gamma}{v} r \bar{l} i \not{l} - \frac{3 \gamma}{2v} \bar{l} i \not{r} l + 4 \frac{\gamma}{v} m_l r \bar{l} l \right\},$$

(14)

and obtains the tree level $l - l - r$ interaction (see Fig. 1-a).

Now, we are ready to calculate the matrix element for the LFV radion decay. The decay of the radion $r$ to leptons with different flavors exits at least in the one loop order, with the help of internal new Higgs bosons $S = h^0, A^0$. The possible vertex and self energy diagrams are presented in Fig. 2. After addition of all these diagrams, the divergences which occur in the loop integrals are eliminated and the matrix element square for this decay is obtained as

$$|M|^2 = 2 \left( m_r^2 - (m_{l^-}^2 + m_{l^+}^2) \right) |A|^2,$$

(15)

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where

\[
A = f_{h^0}^{\text{self}} + f_{A^0}^{\text{self}} + f_{h^0}^{\text{vert}} + f_{A^0}^{\text{vert}} + f_{h^0 h^0}^{\text{vert}} + f_{A^0 A^0}^{\text{vert}},
\]

and their explicit expressions are given as

\[
f_{h^0}^{\text{self}} = \frac{\gamma}{128 \, v \pi^2 (w'_h - w_h)} \int_0^1 dx \, m_{h^0} \left\{ \left( \eta_i^+ (x - 1) \, w_h - \eta_i^+ z_{ih} \right) \left( 3 \, w'_h - 5 \, w_h \right) \ln \frac{L_{h^0 h^0}^\text{self} m_{h^0}^2}{\mu^2} \right. \\
+ \left. \left( \eta_i^V (x - 1) \, w'_h - \eta_i^+ z_{ih} \right) \left( 5 \, w'_h - 3 \, w_h \right) \ln \frac{L_{2,h^0 m_{h^0}^2}}{\mu^2} \right\},
\]

\[
f_{A^0}^{\text{self}} = \frac{\gamma}{128 \, v \pi^2 (w'_A - w_A)} \int_0^1 dx \, m_{A^0} \left\{ \left( \eta_i^+ (x - 1) \, w_A + \eta_i^+ z_{iA} \right) \left( 3 \, w'_A - 5 \, w_A \right) \ln \frac{L_{1,A^0 m_{A^0}^2}}{\mu^2} \right. \\
+ \left. \left( \eta_i^V (x - 1) \, w'_A + \eta_i^+ z_{iA} \right) \left( 5 \, w'_A - 3 \, w_A \right) \ln \frac{L_{2,A^0 m_{A^0}^2}}{\mu^2} \right\},
\]

\[
f_{h^0}^{\text{vert}} = \frac{\gamma}{128 \, v \pi^2} \int_0^1 dx \int_0^{1-x} dy \, \frac{m_{h^0}}{L_{h^0}^\text{vert}} \left[ \eta_i^V \left( 3 \, z_{rh} \left( y (1 - y) \, w'_h + x^2 (4 \, y - 1) \, w_h \right) \\
+ x \left( (1 - 3 \, y) \, w_h + y (4 \, y - 3) \, w'_h \right) \right) \\
+ 3 \, (x + y - 1) \left( x (4 \, x - 3) \, w_h^3 + y (4 \, y - 3) \, w'_h^3 \right) \\
+ 3 \, w_h \, w'_h (x + y - 1) \left( (1 - y + x (4 \, y - 2)) \, w_h + (1 - 2 \, y + x (4 \, y - 1)) \, w'_h \right) \\
+ 3 \, (x + y - 1) \left( (2 \, x - 1) \, w_h + (2 \, y - 1) \, w'_h \right) \\
+ \eta_i^+ z_{ih} \left( (x + y - 1) \left( -4 + 2 \, w'_h \, w_h + w_h^2 (8 \, y - 3) + w_h^2 (8 \, x - 3) \right) \\
- \left( 8 \, z_{ih}^2 + z_{rh} ((8 \, y - 3) \, x - 3 \, y) \right) \right) \\
- m_{h^0} \ln \frac{L_{h^0}^\text{vert} m_{h^0}^2}{\mu^2} \left( 9 \, \eta_i^V (w'_h (2 \, y - 1) + w_h (2 \, x - 1)) - 8 \, \eta_i^+ z_{ih} \right) \right].
\]

\[
f_{A^0}^{\text{vert}} = \frac{\gamma}{128 \, v \pi^2} \int_0^1 dx \int_0^{1-x} dy \, \frac{m_{A^0}}{L_{A^0}^\text{vert}} \left[ \eta_i^V \left( 3 \, z_{rA} \left( y (1 - y) \, w'_A + x^2 (4 \, y - 1) \, w_A \right) \\
+ x \left( (1 - 3 \, y) \, w_A + y (4 \, y - 3) \, w'_A \right) \right) \\
+ 3 \, (x + y - 1) \left( x (4 \, x - 3) \, w_A^3 + y (4 \, y - 3) \, w'_A^3 \right) \\
+ 3 \, w_A \, w'_A (x + y - 1) \left( (1 - y + x (4 \, y - 2)) \, w_A + (1 - 2 \, y + x (4 \, y - 1)) \, w'_A \right) \\
+ 3 \, (x + y - 1) \left( (2 \, x - 1) \, w_A + (2 \, y - 1) \, w'_A \right) \\
+ \eta_i^+ z_{iA} \left( (x + y - 1) \left( -4 + 2 \, w'_A \, w_A + w_A^2 (8 \, y - 3) + w_A^2 (8 \, x - 3) \right) \\
- \left( 8 \, z_{iA}^2 + z_{rA} ((8 \, y - 3) \, x - 3 \, y) \right) \right] \\
- \left( 8 \, z_{iA}^2 + z_{rA} ((8 \, y - 3) \, x - 3 \, y) \right) \right].
\]
In eq. (19) the flavor changing couplings $w_{\bar{l}_i^L l_j^R}^{(1)}$ and outgoing $j = 1, 2$ lepton (anti lepton). Here we choose the couplings $\xi_{N,l_i j_i}^E$ real.

Finally, the BR for $r \to l_1^- l_2^+$ can be obtained by using the matrix element square as

$$BR(r \to l_1^- l_2^+) = \frac{1}{16 \pi m_r} \frac{|M|^2}{\Gamma_r}, \quad (20)$$
where $\Gamma_r$ is the total decay width of radion $r$. In our numerical analysis we consider the BR
due to the production of sum of charged states, namely

$$BR(r \to l_1^\pm l_2^\pm) = \frac{\Gamma(r \to (\bar{l}_1 l_2 + \bar{l}_2 l_1))}{\Gamma_r}. \quad (21)$$

**3 Discussion**

In four dimensions, higher dimensional gravity is observed as it has new states with spin 2,1
and 0, so called, graviton, gravivector, graviscalar. These states interact with the particles in
the underlying theory. In the RS1 model with one extra dimension, the spin 0 gravity particle
radion $r$ interacts with the particles of the theory (2HDM in our case) on the TeV brane and
this interaction occurs over the trace of the energy-momentum tensor $T^\mu_\mu$ with the strength
$1/\Lambda_r$,

$$\mathcal{L}_{\text{int}} = \frac{r}{\Lambda_r} T^\mu_\mu. \quad (22)$$

where $\Lambda_r$ is at the order of TeV. The radion interacts with gluon ($g$) pair or photon ($\gamma$) pair
in one loop order from the trace anomaly. For the radion mass $m_r \leq 150 \text{GeV}$, the decay
width is dominated by $r \to gg$. For the masses which are beyond the WW and ZZ thresholds
the main decay mode is $r \to WW$. In the present work, we study the possible LFV decays
of radion in the 2HDM and estimate the BRs of these decays for different values of radion
masses. We take the total decay width $\Gamma_r$ of the radion by considering the dominant decays
$r \to gg (\gamma\gamma, ff, W^+W^-, ZZ, SS)$ where $S$ are the neutral Higgs particles (see \[9\] for the explicit
expressions of these decay widths). Here, we include the possible processes in the $\Gamma_r$ according
to the mass of the radion.

The flavor violating $r$ decays $r \to l_1^- l_2^+$ can exist at least in the one loop level, in the
framework of the 2HDM, with the help of the internal new neutral Higgs bosons $S = h^0, A^0$
and the FV is carried by the the Yukawa couplings $\xi_{N,ij}^E$. In the version of 2HDM where
the FCNC are permitted, these couplings are free parameters which should be restricted by
using the present and forthcoming experiments. At first, we assume that these couplings are
symmetric with respect to the flavor indices $i$ and $j$. Furthermore, we take that the couplings
which contain at least one $\tau$ index are dominant and we choose a broad range for these couplings,
by respecting the upper limit predictions of $\xi_{N,\tau\mu}^E$ (see \[13\] and references therein), obtained by
using the experimental uncertainty, $10^{-9}$, in the measurement of the muon anomalous magnetic

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$^3$The dimensionfull Yukawa couplings $\xi_{N,ij}^E$ are defined as $\xi_{N,ij}^E = \sqrt{4G_F} \xi_{N,ij}^E$. 

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moment and by assuming that the new physics effects can not exceed this uncertainty. For the coupling \( \xi_{N,\tau e}^E \), the restriction is obtained by using this upper limit and the experimental upper bound of BR of \( \mu \rightarrow e\gamma \) decay, \( BR \leq 1.2 \times 10^{-11} \) [16] and, finally, the coupling \( \xi_{N,\tau e}^E \) is taken in the range \( 10^{-3} - 10^{-1} \) GeV (see [17]). For the Yukawa coupling \( \xi_{N,\tau \mu}^E \), we have no explicit restriction region and we use the numerical values which are greater than \( \xi_{N,\tau \mu}^E \).

In the present work, we estimate the BRs of the LFV \( r \) decays \( r \rightarrow l_1^\pm l_2^\mp \) in the RS1 model. We assume that the flavor violation comes from the new neutral Higgs scalars at tree level. Throughout our calculations we use the input values given in Table (1).

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<td>( G_F )</td>
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Table 1: The values of the input parameters used in the numerical calculations.

Fig. 3 is devoted to \( m_r \) dependence of the BR \( (r \rightarrow \tau^\pm \mu^\pm) \). The solid-dashed lines represent the BR \( (r \rightarrow \tau^\pm \mu^\pm) \) for \( \xi_{N,\tau \tau}^E = 100 \) GeV, \( \xi_{N,\tau \mu}^E = 10 \) GeV, \( \xi_{N,\tau \tau}^E = 10 \) GeV, \( \xi_{N,\tau \mu}^E = 1 \) GeV. It is observed that the BR \( (r \rightarrow \tau^\pm \mu^\pm) \) is at the order of the magnitude of \( 10^{-8} \) for the large values of the couplings and the radion mass values \( \sim 200 \) GeV. For the heavy masses of the radion this BR stabilizes to the values \( 10^{-9} \).

In Fig. 4 we present \( m_r \) dependence of the BR \( (r \rightarrow \tau^\pm e^\pm) \) and BR \( (r \rightarrow \mu^\pm e^\pm) \). The solid-dashed lines represent the BR \( (r \rightarrow \tau^\pm e^\pm) \) for \( \xi_{N,\tau \tau}^E = 100 \) GeV, \( \xi_{N,\tau e}^E = 0.1 \) GeV, \( \xi_{N,\tau \tau}^E = 10 \) GeV, \( \xi_{N,\tau \mu}^E = 0.1 \) GeV. The small dashed line represents the BR \( (r \rightarrow \mu^\pm e^\pm) \) for \( \xi_{N,\tau \mu}^E = 1 \) GeV, \( \xi_{N,\tau e}^E = 0.1 \) GeV. This figure shows that the BR \( (r \rightarrow \tau^\pm \mu^\pm) \) is at the order of the magnitude of \( 10^{-12} \) for the large values of the couplings and the radion mass values \( \sim 200 \) GeV. For the heavy masses of the radion this BR reaches to the values less than \( 10^{-14} \). The BR \( (r \rightarrow \mu^\pm e^\pm) \) is at the order of the magnitude of \( 10^{-15} \) for \( m_r \sim 200 \) GeV and the intermediate values of Yukawa couplings. These BRs, especially BR \( (r \rightarrow \mu^\pm e^\pm) \), are negligibly small.

Now, we present the Yukawa coupling dependencies of the BRs of the decays under consid-
eration, for different radion masses.

Fig. 5 represents the $\xi_{E_N,\tau \tau}$ dependence of the BR($r \rightarrow \tau^\pm \mu^\pm$) for $\xi_{E_N,\tau \mu} = 10 \, GeV$. The solid-dashed-small dashed lines represent the BR for the radion masses $m_r = 200 - 500 - 1000 \, GeV$. This figure shows that the BR is sensitive to the radion mass and, obviously, increases two order in the range $10 \, GeV \leq \xi_{E_N,\tau \tau} \leq 100 \, GeV$.

In Fig. 6, we present the $\xi_{E_N,\tau \tau}$ dependence of the BR($r \rightarrow \tau^\pm e^\pm$) for $\xi_{E_N,\tau e} = 0.1 \, GeV$. The solid-dashed-small dashed lines represent the BR for the radion masses $m_r = 200 - 500 - 1000 \, GeV$. Similar to the $r \rightarrow \tau^\pm \mu^\pm$ decay, the BR is strongly sensitive to the radion mass.

As a summary, the LFV decays of radion in RS1 model strongly depends on the radion mass and the Yukawa couplings. The BR for $r \rightarrow \tau^\pm \mu^\pm$ decay is at the order of $10^{-8}$ for the small values of radion mass $m_r$ and it decreases with the increases values of $m_r$. On the other hand the BRs for $r \rightarrow \tau^\pm e^\pm$ ($r \rightarrow \mu^\pm e^\pm$) decays are at the order of $10^{-12}$ ($10^{-15}$) for the small values of $m_r$. These results show that the detection of the LFV radion decays is not easy. Among them the LFV $r \rightarrow \tau^\pm \mu^\pm$ decay would be the appropriate one to measure its BR. With the possible production of the radion (the most probable production is due to the gluon fusion, $gg \rightarrow r$ [9]), hopefully, the future experimental results of this decay would be useful to test the possible signals coming from the extra dimensions and new physics which results in flavor violation.

A The vertices appearing in the present work

In this section we present the vertices appearing in our calculations. Here $S$ denotes the new neutral Higgs bosons $h^0$ and $A^0$.
\[
\frac{-i\gamma}{v} \left[ \frac{3}{2} (p_{1} + p_{2}) - 4m_{i} \right]
\]

\[
\frac{-2i\gamma}{v} (p_{1} \cdot p_{2} - m_{S}^{2})
\]

\[
(S = h^{0}) \quad \frac{A_{ij}}{2\sqrt{v}} \left[ (\xi_{ij}^{E} + \xi_{ji}^{E*}) + (\xi_{ij}^{E} - \xi_{ji}^{E*})\gamma_{5} \right]
\]

\[
(S = A^{0}) \quad \frac{-4\gamma}{2\sqrt{v}} \left[ (\xi_{ij}^{E} - \xi_{ji}^{E*}) + (\xi_{ij}^{E} + \xi_{ji}^{E*})\gamma_{5} \right]
\]

\[
\frac{-i}{2\sqrt{2}} \left[ (\xi_{ij}^{E} + \xi_{ji}^{E*}) + (\xi_{ij}^{E} - \xi_{ji}^{E*})\gamma_{5} \right]
\]

\[
\frac{1}{2\sqrt{2}} \left[ (\xi_{ij}^{E} - \xi_{ji}^{E*}) + (\xi_{ij}^{E} + \xi_{ji}^{E*})\gamma_{5} \right]
\]

Figure 1: The vertices used in the present work.
References


Figure 2: One loop diagrams contribute to $r \rightarrow l_1^- l_2^+$ decay due to the neutral Higgs bosons $h_0$ and $A_0$ in the 2HDM. $i$ represents the internal lepton, $l_1^-$ ($l_2^+$) outgoing lepton (anti lepton), internal dashed line the $h_0$ and $A_0$ fields.
Figure 3: $m_r$ dependence of the BR ($r \rightarrow l_1^\pm l_2^\pm$). The solid-dashed lines represent the BR($r \rightarrow \tau^\pm \mu^\pm$) for $\xi_{N,\tau\tau}^E = 100 \text{ GeV}$, $s_{N,\tau\mu} = 10 \text{ GeV}$- $s_{N,\tau\tau}^E = 10 \text{ GeV}$, $\xi_{N,\tau\mu}^E = 1 \text{ GeV}$.

Figure 4: $m_r$ dependence of the BR ($r \rightarrow l_1^\pm l_2^\pm$). The solid-dashed lines represent the BR($r \rightarrow \tau^\pm e^\pm$) for $\xi_{N,\tau\tau}^E = 100 \text{ GeV}$, $\xi_{N,\tau e}^E = 0.1 \text{ GeV}$- $\xi_{N,\tau\tau}^E = 10 \text{ GeV}$, $\xi_{N,\tau e}^E = 0.1 \text{ GeV}$. The small dashed line represents the BR ($r \rightarrow \mu^\pm e^\pm$) for $\xi_{N,\tau\mu}^E = 1 \text{ GeV}$, $s_{N,\tau e}^E = 0.1 \text{ GeV}$. 
Figure 5: $\tilde{\xi}_{N,\tau\tau}$ dependence of the BR($r \rightarrow \tau^\pm \mu^\pm$) for $\tilde{\xi}_{N,\tau\mu} = 10$ GeV. The solid-dashed-small dashed lines represent the BR for the radion masses $m_r = 200 - 500 - 1000$ GeV.

Figure 6: $\tilde{\xi}_{N,\tau\tau}$ dependence of the BR($r \rightarrow \tau^\pm e^\pm$) for $\tilde{\xi}_{N,\tau e} = 0.1$ GeV. The solid-dashed-small dashed lines represent the BR for the radion masses $m_r = 200 - 500 - 1000$ GeV.