Constraining the density dependence of the symmetry energy in the nuclear equation of state using heavy ion beams

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Abstract

The density dependence of the symmetry energy in the equation of state of asymmetric nuclear matter \((N/Z > 1)\) is important for understanding the structure of systems as diverse as the atomic nuclei and neutron stars. Due to a proper lack of understanding of the basic nucleon-nucleon interaction for matters that are highly asymmetric and at non-normal nuclear density, this very important quantity has remained largely unconstrained. Recent studies using beams from the Cyclotron Institute of Texas A&M University, constraining the density dependence of the symmetry energy, is presented. A dependence of the form \(E_{\text{sym}}(\rho) = C(\rho/\rho_o)^\gamma\), where \(C = 31.6\) MeV and \(\gamma = 0.69\), is obtained from the dynamical and statistical model analysis. Their implications to both astrophysical and nuclear physics studies are discussed.

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I. INTRODUCTION

The fundamental goal of nuclear physics is to understand the basic building blocks of nature - neutrons and protons - and the force of interaction that binds them together into nuclear matter. Studying the nature of matter and the strength of nuclear interaction is key to understanding some of the fundamental problems such as, how elements are formed? How stars explode into supernova? What kind of matter exists inside a neutron star? How neutrons are compressed inside the neutron star to density trillions of times greater than on earth? What determines their density-pressure relation, the so-called equation of state?

Ordinary matter and their equation of state (the real gas equation) are well understood in terms of Van der Waal’s inter-molecular force of attraction. The nuclear matter equation of state on the other hand, is not so well understood due to a proper lack of understanding of the basic nucleon-nucleon interaction over a wide range of density, temperature and isospin (neutron-to-proton ratio). Until now our understanding of the nucleon-nucleon interaction has come from studying nuclear matter that are symmetric in isospin (i.e., neutron-to-proton ratio equal one) and those found near normal nuclear matter density ($\rho_o = 0.16 \; fm^{-3}$). It is not known how far these understanding remains valid as one go away from the normal nuclear density and symmetric nuclear matter. Various interactions used in “ab initio” microscopic calculations predict different forms of the nuclear equation of state above and below the normal nuclear matter density, and away from the symmetric nuclear matter [1]. As a result, the symmetry energy, which is the difference in energy between the pure neutron matter and the symmetric nuclear matter, show very different behavior above and below normal nuclear density. Fig. 1 shows the the density dependence of the symmetry energy as predicted by various microscopic calculations. One observes from the figure that in general two different forms are predicted. One, where the symmetry energy increases monotonically with increasing density (“stiff” dependence) and the other, where the symmetry energy increases initially up to normal nuclear density and then decreases at higher densities (“soft” dependence). Constraining the form of the density dependence of the symmetry energy is important not only for a better understanding of the nucleon-nucleon interaction, and hence their extrapolation to the structure of neutron-rich nuclei, but also to determine the structure of compact stellar objects such as the neutron stars. For example, a “stiff”
The density dependence of the symmetry energy is predicted to lead to a large neutron skin thickness compared to a “soft” dependence \[1\]. Similarly, a “stiff” dependence of the symmetry energy can result in rapid cooling of a neutron star, and a larger neutron star radius, compared to a soft density dependence \[3\].

Experimentally, one possible means of studying the nuclear equation of state at sub-normal nuclear density and high excitation energy is through the intermediate energy heavy ion collision reactions. In these reactions, an excited nucleus (the composite of the projectile and the target nucleus) expands to a sub-nuclear density and disintegrates into various light and heavy fragments in a process called multifragmentation. By studying the isotopic yield distribution of these fragments one can extract important information about the symmetry energy and their density dependence.

**II. SYMMETRY ENERGY AND THE ISOTOPIC YIELD DISTRIBUTION**

It has been shown from experimental observations that the ratio of the fragment isotopic yields in two different reactions, 1 and 2, \( R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z) \), obey an expo-
ential dependence on the neutron number \(N\) and the proton number \(Z\) of the isotopes; an observation known as isoscaling \[4, 5\]. The dependence is characterized by the relation,

\[
R_{21}(N, Z) = Ce^{(\alpha N + \beta Z)}
\]

Where, \(Y_2\) and \(Y_1\) are the yields from the neutron-rich and neutron-deficient systems, respectively. \(C\) is an overall normalization factor, and \(\alpha\) and \(\beta\) are the parameters characterizing the isoscaling behavior. The observation is also theoretically predicted by the statistical and the dynamical multifragmentation models \[4, 5, 6\]. In these models, the difference in the chemical potential of systems with different \(N/Z\) is directly related to the scaling parameter \(\alpha\), which is further related to the symmetry energy through a relation,

\[
\alpha = \frac{4C_{sym}}{T} \left( \frac{Z_2^2}{A_2^2} - \frac{Z_1^2}{A_1^2} \right)
\]

where, \(Z_1, A_1\) and \(Z_2, A_2\) are the charge and the mass numbers of the fragmenting systems, \(T\) is the temperature of the system and \(C_{sym}\), is the symmetry energy.

III. EXPERIMENT

The isotopic yield distributions for the present study were obtained by carrying out measurements at the Cyclotron Institute of Texas A&M University (TAMU), using the K500 Superconducting Cyclotron, and the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University (MSU). Reactions such as, 
\(^{40}\text{Ar, }^{40}\text{Ca + }^{58}\text{Fe, }^{58}\text{Ni and }^{58}\text{Ni, }^{58}\text{Fe + }^{58}\text{Ni, }^{58}\text{Fe,}\) at beam energies from 25 MeV/nucleon to 53 MeV/nucleon, were studied to measure the fragment yield distribution. Details of the measurement can be found in \[7\], and references therein.

IV. RESULTS AND DISCUSSIONS

Fig. 2 shows the form of the density dependence of the symmetry energy obtained by comparing the experimentally measured isoscaling parameter with those obtained from the statistical and the dynamical multifragmentation models. Also shown in the figure are comparisons with several other recent independent studies. The solid curve corresponds to the one obtained from Gogny-AS interaction in dynamical AMD model that explains the
FIG. 2: Comparison of the density dependence of the symmetry energy obtained from various different studies. See text for details and table I.

The present results, assuming the sequential decay effect to be small [8, 9]. The triangle and the circle symbols also correspond to the present measurements obtained by comparing with the statistical multifragmentation model (see Ref. [10] for more details). The dashed curve correspond to those obtained recently from an accurately calibrated relativistic mean field interaction, used for describing the Giant Monopole Resonance (GMR) in $^{90}$Zr and $^{208}$Pb, and the IVGDR in $^{208}$Pb by Piekarewicz et al. [11]. The dot-dashed curve correspond to the one used for explaining the isospin diffusion data of NSCL-MSU by Chen et al. [12]. This dependence has now been modified to include the isospin dependence of the in-medium nucleon-nucleon cross-section, and in good agreement with the present study. The shaded region in the figure corresponds to those obtained by constraining the binding energy, neutron skin thickness and isospin analogue state in finite nuclei using the mass formula of Danielewicz [13]. The dotted curve correspond to the parameterization adopted by Heiselberg et al. [14] in their studies on neutron stars, and obtained by fitting the
predictions of the variational calculations of Akmal et al. \cite{15}. Finally, the solid square point in the figure correspond to the value of symmetry energy obtained by fitting the experimental differential cross-section data in a charge exchange reaction using the isospin dependent optical potential by Khoa et al. \cite{16}. The parameterized forms of the density dependence of the symmetry energy obtained from all these studies are as shown in table I. The close agreement between various independent studies show that a constraint on the density dependence of the symmetry energy, given as $E_{\text{sym}}(\rho) = C(\rho/\rho_o)^\gamma$, where $C = 31.6$ MeV and $\gamma = 0.69$ can be obtained.

V. CONCLUSIONS

A number of reactions were studied at TAMU to investigate the density dependence of the symmetry energy in the equation of state of asymmetric nuclear matter. The results were analyzed within the framework of dynamical and statistical model calculations. It is observed that a dependence of the form $E_{\text{sym}}(\rho) = 31.6 (\rho/\rho_o)^{0.69}$ MeV, agrees better with the experimental data, indicating that a “stiff” form of the nucleon-nucleon interaction provides a better understanding of the nuclear matter EOS. The observed constrain leads to nuclear matter compressibility $K \sim 230$ MeV, and a neutron skin thickness $R_n - R_p \sim 0.21$ fm, for $^{208}$Pb nuclei. It also predicts a neutron star mass of 1.72 solar mass and a radius, $R = 11 - 13$ km for the “canonical” neutron star. Furthermore, it predicts a direct URCA cooling for neutron stars above 1.4 times the solar mass. These results have important implications for nuclear astrophysics and future experiments probing the properties of nuclei using beams of neutron-rich nuclei.

VI. ACKNOWLEDGEMENTS

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TABLE I: Parametrized form of the density dependence of the symmetry energy obtained from various independent studies.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Parameterization</th>
<th>Studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heiselberg et al.</td>
<td>$32.0(\rho/\rho_0)^{0.60}$</td>
<td>Variational calculation</td>
</tr>
<tr>
<td>Danielewicz et al.</td>
<td>$31(33)(\rho/\rho_0)^{0.55(0.79)}$</td>
<td>BE, skin, isospin analog states</td>
</tr>
<tr>
<td>Chen et al.</td>
<td>$31.6(\rho/\rho_0)^{1.05}$</td>
<td>Isospin diffusion</td>
</tr>
<tr>
<td>Piekarewicz et al.</td>
<td>$32.7(\rho/\rho_0)^{0.64}$</td>
<td>Giant resonances</td>
</tr>
<tr>
<td>Shetty et al.</td>
<td>$31.6(\rho/\rho_0)^{0.69}$</td>
<td>Isotopic distribution</td>
</tr>
</tbody>
</table>