Exploring backward pion electroproduction in the scaling regime

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Abstract. We use general relations between the Transition Distribution Amplitudes (TDAs), entering the description of the $p \rightarrow \pi^0$ transition, and the proton Distribution Amplitudes (DAs) in the soft-pion limit to estimate the size of the amplitude for backward electroproduction of $\pi^0$ at large $Q^2$.

We have recently \cite{1} shown that factorisation theorems \cite{2} for exclusive processes apply to $\gamma^- \pi^+ \rightarrow \gamma^* \gamma$ in the kinematical regime where the virtual photon is highly virtual but at small $t$. We also advocated the extension of this approach to $P \bar{P} \rightarrow \gamma^* \gamma$, to backward VCS $\gamma^* P \rightarrow P' \gamma$ \cite{3}, to backward pion electroproduction $\gamma^* P \rightarrow P' \pi$ and to $P \bar{P} \rightarrow \gamma^* \pi$ in the near forward region and for large virtual $Q^2$, which may be studied in detail at GSI.

For the $\gamma^*$ to $\rho$ transition, a perturbative limit of the TDA may be obtained \cite{5}. For $\gamma \rightarrow \pi$ one, where there are only four leading-twist TDAs \cite{1} related to $\langle \pi | e^{ijk} q^i_\alpha(z_1 n) [z_1; z_0] q^j_\beta(z_2 n) [z_2; z_0] q^k_\gamma(z_3 n) [z_3; z_0] | P \rangle$,\footnote{Presented at Quark Confinement and the Hadron Spectrum VII, September 2-7 2006, Ponta Delgada, Portugal and at Soft-Pions in hard processes, August 3-5 2006, Regensburg, Germany.} where $[z_1; z_0]$ denotes the Wilson line we have recently shown \cite{6} that experimental analysis of e.g. $\gamma^* \gamma \rightarrow \rho \pi$ and $\gamma^* \gamma \rightarrow \pi \pi$ could be carried out since the Bremsstrahlung contribution is small and rates are sizable at present $e^+e^-$ facilities. Whereas in the pion case, models used for GPDs (see \cite{7} and references therein) could be applied to TDAs, this is not obvious for baryonic ones, for which the soft limit considered here is therefore very interesting.

In Ref. \cite{4}, we have defined the leading-twist proton to pion $P \rightarrow \pi$ transition distribution amplitudes from the Fourier transform of the matrix element

$$\langle \pi | e^{ijk} q^i_\alpha(z_1 n) [z_1; z_0] q^j_\beta(z_2 n) [z_2; z_0] q^k_\gamma(z_3 n) [z_3; z_0] | P \rangle,$$ (1)
We define here the leading-twist TDAs for the \( P \rightarrow \pi^0 \) transition at \( \Delta T = 0 \) as:

\[
4\mathcal{F}\left(\langle \pi^0(p_\pi) | \epsilon^{ijk} u_\alpha(z_1 n) [z_1 ; z_0] u_\beta(z_2 n) [z_2 ; z_0] d_\gamma(z_3 n) [z_3 ; z_0] | P(p_1, s_1) \rangle\right)
= i \frac{f_N}{f_\pi} \left[ V_{1}^{\pi p_\pi} (\gamma C)_{\alpha\beta}(N^+)_{\gamma} + A_{1}^{\pi p_\pi} (\gamma_5^\mu N^+)_{\gamma} + T_{1}^{\pi p_\pi} (\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma_\mu N^+)_{\gamma}, \right]
\]

where \( \sigma^{\mu\nu} = 1/2[\gamma^\mu, \gamma^\nu] \), \( C \) is the charge conjugation matrix and \( N^+ \) is the large component of the nucleon spinor \( (N = (\gamma_5^\mu p + \gamma_\mu) N = N^+ + N^- \text{ with } N^+ \sim \sqrt{p_1^0} \text{ and } N^- \sim 1/\sqrt{p_1^0} \) \). \( f_\pi \) is the pion decay constant \( (f_\pi = 133 \text{ MeV}) \) and \( f_N \) has been estimated through QCD sum rules to be of order \( 5.2 \cdot 10^{-3} \text{ GeV}^2 \) \[8\]. All the TDAs \( V, A, \) and \( T \) are dimensionless.

Now, we shall derive the general limit of these three contributing TDAs at \( \Delta T = 0 \) when \( \xi \) gets close to \( 1 \). In that limit, the soft-meson theorems \[8\] derived from current algebra apply \[10\], which allow us to express these 3 TDAs in terms of the 3 Distribution Amplitudes (DAs) of the corresponding baryon. In the case of the proton DA \[8\], \( V^p(x_i) \), \( A^p(x_i) \), \( T^p(x_i) \) are defined such as

\[
4\mathcal{F}\left(\langle 0 | \epsilon_{ijk} u_\alpha(z_1 n) u_\beta(z_2 n) d_\gamma(z_3 n) | p(s) \rangle\right) = f_N \times \left[ V^p(x_i)(\gamma_5^\mu N^+)_{\gamma} + A^p(x_i)(\gamma_5^\mu N^+)_{\gamma} + T^p(x_i)(\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma_\mu N^+)_{\gamma} \right].
\]

We use the general soft pion theorem \[8\] to write:

\[
\langle \pi^a(p_\pi) | \mathcal{O} | P(p_1, s_1) \rangle = -i \frac{f_\pi}{f_\pi} \langle 0 | [Q_5^a, \mathcal{O}] | P(p_1, s_1) \rangle + \text{pole term}
\]

The second term, which takes care of the nucleon pole term, does not contribute at threshold and will not be considered in the following.

For the transition \( P \rightarrow \pi^0 \), \( Q_5^a = Q_5^a \) and the flavour content of \( \mathcal{O} \) is \( u_\alpha u_\beta d_\gamma \). Since the commutator of the chiral charge \( Q_5 \) with the quark field \( \psi \) (\( \tau^a \) being the isospin matrix) is \( [Q_5, \psi] = -\frac{a}{2} \gamma_5 \psi \), the first term in the rhs of Eq. \[8\] gives three terms from \( (\gamma^5 u)_{\alpha} u_\beta d_\gamma \), \( u_\alpha (\gamma^5 u)_{\beta} d_\gamma \) and \( u_\alpha u_\beta (\gamma^5 d)_{\gamma} \). The corresponding multiplication by \( \gamma_5 \) (or \( (\gamma_5)^T \) when it acts on the index \( \beta \)) on the vector and axial structures of the DA (Eq. \[8\]) gives two terms which cancel each other and the third one, which remains, is the same as the one for the TDA, up to the modification that in the DA decomposition \( p \) is the proton momentum, whereas for the TDA one, \( p \) is the light-cone projection of \( P \equiv (p_1 + p_\pi)/2 \), \( i.e. \) half the proton momentum if one neglects \( p_\pi \). This introduces a factor 2 in the relation between the DA \( V^p \) (\( V^p \)) and the TDA \( V_{1}^{\pi p_\pi} (A_{1}^{\pi p_\pi}) \), which cancels the factor 1/2 from \( [Q_5^a, \psi] \). To what concerns the tensorial structure multiplying \( T^p \), the three terms are identical at leading-twist accuracy and yield a factor 3 in \( T_1 \).

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1 In the following, we shall use the notation \( \mathcal{F} = \langle P,n \rangle^3 \int \frac{d^4z}{z} e^{i\Sigma x_i z_i p.n} \).
We eventually have the soft limit\(^2\) for our three TDAs at \(\Delta_T = 0\):

\[
V_1^\pi\pi^0(x_i, \xi, t) \rightarrow V^p(x_i), \quad A_1^\pi\pi^0(x_i, \xi, t) \rightarrow A^p(\frac{x_i}{2}), \quad T_1^\pi\pi^0(x_i, \xi, t) \rightarrow 3T^p(\frac{x_i}{2}).
\] (5)

At leading order in \(\alpha_s\), the amplitude for \(\gamma^*(q)P(p_1, s_1) \rightarrow P'(p_2, s_2)\pi^0(p_\pi)\) is

\[
M^\mu = -ieF^\pi\pi^0(Q^2, \xi, t)\bar{u}(p_2)\gamma^\mu\gamma^5u(p_1), \quad F^\pi\pi^0 = \frac{Cf_N^2}{f_\pi Q^4} \int_{-1+\xi}^{1+\xi} d^3x \int_0^1 d^3y \sum_{\alpha=1}^{14} T'_\alpha(x_i, y_j),
\] (6)

to be compared with the leading amplitude for the baryonic form factor \(\mathcal{M}^\mu = -ieF^p(Q^2)\bar{u}(p_2)\gamma^\mu u(p_1), \quad F^p = \frac{Cf_N^2}{Q^4} \int_0^1 d^3x \int_0^1 d^3y \sum_{\alpha=1}^{14} T_\alpha(x_i, y_j).
\] (7)

Considering, for now, only the contribution from the ERBL region \(x_i > 0\), the integration between \(-1 + \xi\) and \(1 + \xi\) can be converted into one between 0 and 1 by a change of variable. Since the expressions of \(T'_\alpha\) and \(T_\alpha\) are identical up to the 3 replacements the initial-state DAs by the \(P \rightarrow \pi^0\) TDAs, they would in fact differ only by the factor 3 in the last relation of Eq. (5) extrapolating the \(\xi \rightarrow 1\) limit to the ERBL region.

Due to this factor, whereas the asymptotic choice \(\mathcal{M}^\mu\), \(120 \times x_1 x_2 x_3\), for the DAs gives a vanishing result for \(F^p\) or \(G^p_M\), the result is nonzero for \(F^\pi\pi^0\). This lets us therefore hope that the onset of the dominance of the perturbative contribution to \(\gamma^*P \rightarrow P'\pi^0\) with TDAs may happen at much lower \(Q^2\) than for the proton form factor. Quantitative results to be compared with the measurement of [11] will be presented soon.

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\(^2\) The factor \(\frac{1}{3}\) in the argument of the DA in Eq. (5) comes from the fact that for the TDAs, the \(x_i\) are defined with respect to \(p\) (see e.g. \(F \equiv (p.n)^3 \int_{-\infty}^{\infty} dz e^{z_1 x_i z_2 p.n}\) and \(p \rightarrow \frac{\vec{p}}{4}\) when \(\xi \rightarrow 1\). Therefore, they vary within the interval \([0 : 2]\), whereas for the DAs, the momentum fraction are defined with respect to the proton momentum \(p_1\) and vary between 0 and 1.