Studying the infrared behaviour of gluon and ghost propagators using large asymmetric lattices

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Abstract. We report on the infrared limit of the quenched lattice Landau gauge gluon propagator computed from large asymmetric lattices. In particular, the compatibility of the pure power law infrared solution \((q^2)^{2\kappa}\) of the Dyson-Schwinger equations is investigated and the exponent \(\kappa\) is measured. Some results for the ghost propagator and for the running coupling constant will also be shown.

Despite the success of Quantum Chromodynamics (QCD) as the theory of the strong interaction, a full understanding of the confinement mechanism is still missing. One line of research, very active in the last years, consists in the study of the QCD propagators for low momenta. Indeed, some works (for details, see [1] and references therein) relate the infrared behaviour of the gluon and ghost propagators in the Landau gauge with gluon confinement. In particular, the Zwanziger horizon condition implies a null zero momentum gluon propagator \(D(q^2)\), and the Kugo-Ojima confinement mechanism requires an infinite zero momentum ghost propagator \(G(q^2)\).

An investigation of the infrared behaviour of the gluon and ghost propagators should be done in a non-perturbative framework. At the moment, two first principles approaches are available for such a task, namely Dyson-Schwinger equations (DSE) and lattice QCD methods. Given the different nature of such approaches, a comparison between the results of the two methods is necessary.

A solution of the DSE [2] predicting pure power laws for gluon and ghost dressing functions,

\[
Z_{\text{gluon}}(q^2) \sim (q^2)^{2\kappa}, \quad Z_{\text{ghost}}(q^2) \sim (q^2)^{\kappa},
\]

with \(\kappa \sim 0.595\), has been extensively used in subsequent works (see [3] for a recent review). As shown in figure 1 of [11], these power laws are only valid for very low momenta, \(q < 200\,\text{MeV}\) (see also [4]). To test this solution of the DSE with lattice QCD, using a symmetric lattice, it would require a lattice volume much larger than a typical present day simulation (see, for example, [5]).

Large asymmetric lattices, in the form \(L_s^3 \times L_t\), with \(L_t \gg L_s\), give us a possibility to test these power laws on the lattice. In this paper we briefly report on the results [6, 7, 8, 9, 10, 11, 12] obtained by us, considering large asymmetric lattices with \(L_s = 8, 10, \ldots, 18\) and \(L_t = 256\), about the infrared behaviour of the gluon and ghost propagators and the strong running coupling defined from these propagators. Despite
the finite volume effects caused by the small spatial extension, the large temporal size of these lattices allow to access to momenta as low as 48 MeV.

In what concerns the gluon propagator, our results [9] show that the propagator dependence on the spatial volume is smooth. Indeed, for the smallest momenta, the bare gluon propagator decreases with the lattice volume, and increases for higher momenta. The values of the infrared exponent extracted from our lattices increase with the lattice volume. Although almost all values of $\kappa$ are below 0.5 (see table 1 in [9]), we obtain, by extrapolating the $\kappa$ values to the infinite volume, $\kappa$ values above 0.5, with a weighted mean of the various estimations giving $\bar{\kappa}_\infty = 0.5246(46)$.

Considering the gluon propagator as a function of the spatial volume, we can also extrapolate it, and fit the obtained propagator to a pure power law. Going this way, we get values for $\kappa \in [0.49, 0.53]$. Note that the lattice data favours the values in the right hand side of this interval.

The reader should be also aware that fits to our data considering higher momenta and other model functions give higher values for $\kappa$ [11, 13].

Similarly to other studies, it is possible to use our gluon data to verify the positivity violation for the gluon propagator [12, 13].

We have also computed the ghost propagator and the strong coupling constant $\alpha_S(q^2)$ defined from these propagators, for our smallest lattices [10]. Our lattice data for these quantities also show sizeable dependence on the spatial volume of the lattices involved in our calculations. We also have found visible Gribov copy effects in the ghost propagator as well as in the strong coupling constant.

Concerning the infrared behaviour of the ghost propagator, we were unable to extract an infrared exponent from our results. Possible reasons for this negative result can be either the finite volume effects associated to the small spatial volume of the lattices involved in the computation, or the lack of lattice data in the infrared region — remember that the DSE ghost power law lacks validity well below 200 MeV.

In the infrared region, $\alpha_S(q^2)$ shows a decreasing behaviour for the smallest momenta, in apparent contradiction with the continuum DSE prediction — an infrared fixed point, but in agreement with other lattice studies [5] and the solution of DSE on a torus [13].
FIGURE 2. On the left, the bare ghost dressing function in the infrared region computed from a plane wave source. On the right, the strong coupling constant. Here, we only consider pure temporal momenta.

However, the reader should be aware that $\alpha_s(q^2)$, for the smallest momenta, seems to increase with the volume.

In a near future, we will improve the statistics for our larger lattices and the extrapolations to the infinite volume limit. We also plan to perform simulations with larger lattices.

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