New physics from ultrahigh energy cosmic rays

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Cosmic rays from outer space enter the atmosphere with energies of up to $10^{11}$ GeV. The initial particle or a secondary hadron inside the shower may then interact with an air nucleon to produce nonstandard particles. In this article we study the production of new physics by high energy cosmic rays, focusing on the long-lived gluino of split-SUSY models and a WIMP working as dark matter. We first deduce the total flux of hadron events at any depth in the atmosphere, showing that secondary hadrons can not be neglected. Then we use these results to find the flux of gluinos and WIMPs that reach the ground after being produced inside air showers. We also evaluate the probability of producing these exotic particles in a single proton shower of ultrahigh energy. Finally we discuss the possible signal in current and projected experiments. While the tiny flux of WIMPs does not seem to have any phenomenological consequences, we show that the gluinos could modify substantially the profile of a small fraction of extensive air showers. In particular, they could produce a distinct signal observable at AUGER in showers of large zenith angle.

PACS numbers: 96.50.sd, 12.60.-i

I. INTRODUCTION

The standard model (SM) has been very succesful when confronted with the experimental data from particle colliders. Despite that, not much is known about the nature of the Higgs sector. Hopefully, the Large Hadron Collider (LHC) at CERN will provide the value of the Higgs mass and will unveil the mechanism that explains that value. A dynamical mechanism would imply new physics just above the electroweak scale: SUSY, little Higgs, technicolor, extra dimensions, or some other unexpected physics. It is also possible, however, that the LHC just completes and confirms the SM as it was formulated, pushing above the TeV scale the limits for this new physics. This possibility has been seriously considered after recent astrophysical and cosmological data suggested a non-zero vacuum energy density. The preferred value $\Lambda \approx (10^{-3}\ eV)^4$ does not seem to be explained by any dynamics at that scale, and could be just telling us that the universe is larger and/or older than we expected: a multiverse. From the experimental point of view, precision data (flavor physics, EDMs) suggest that the scale of new physics is well above the TeV, whereas neutrino oscillations point to an even larger scale, above $10^{11}$ GeV. In contrast, dark matter strongly suggests a stable WIMP below (or at) the TeV. Therefore, the question of what (if any) physics beyond the SM we should expect at accessible energies is wide open.

On the other hand, it is known that accelerators are not the only place to look for collisions of TeV energy. There is a well established spectrum of cosmic rays that extends up to $E = 10^{11}$ GeV. When a proton from outer space enters the atmosphere and hits a nucleon, the center of mass energy $\sqrt{s} = \sqrt{2m_N E}$ in the collision may go up to 500 TeV. Depending on the energy of the initial particle, after the first interaction there will be many secondary hadrons with still enough energy to produce TeV physics in a collision with an atmospheric nucleon. If created, the massive particle would be inside the air shower together with thousands of other standard particles. Therefore, to have any chance to detect it the new particle should be long lived (its decay products inside the shower would be undetectable). This is precisely the case for the gluino of split-SUSY and for a WIMP as dark matter.
the case of the gluino of split-SUSY models \cite{2}. Such a particle could change substantially the profile of the air shower where it was produced or it may survive (together with muons and neutrinos) the shower and be observed far from the initial interaction point, in quasi-horizontal events or in neutrino telescopes.

In this paper we study the production of new physics by cosmic rays. To evaluate that we determine the total flux of hadrons in the atmosphere: primary plus secondary nucleons, pions and kaons. The decay length of these particles at these energies is much larger than their interaction length in air, so the probability that they produce new physics is just

$$ P_X = \frac{\sigma_X}{\sigma_T}, $$

where $\sigma_X$ is the cross section to produce the exotic particle and $\sigma_T$ the total cross section of the hadron with the air. We deduce as well the spectrum of secondary hadrons created by a single proton of ultrahigh energy.

We then show how to apply these results to evaluate the production rate of long-lived gluinos and also of a WIMP working as dark matter of the universe, and discuss possible signals in air showers and neutrino telescopes.

Other studies of the production of new physics by cosmic rays refer to primary neutrinos \cite{11}. In particular, there are several recent analyses of the possibility to observe long-lived staus at neutrino telescopes \cite{12}. A neutrino of very high energy may reach a telescope and interact there with a nucleon. Since the neutrino is weakly interacting, the relative effect of new physics is here more important than for protons or electrons. If IceCube \cite{13} establishes the neutrino flux at these energies, the search for deviations to the standard neutrino-nucleon cross sections will put strong constraints on some types of TeV physics \cite{14}.

II. TOTAL FLUX OF HADRRONS

The flux of nucleons reaching the atmosphere in the range of energy from several GeV to $\approx 10^6$ GeV can be approximated by \cite{8}

$$ \frac{d\Phi_N}{dE} \approx 1.8 \times 10^{-2.7} \frac{\text{nucleons}}{\text{cm}^2 \text{sr} \text{GeV}}, $$

with $E$ given in GeV. Around 90% of these primary nucleons are protons (free or bound in nuclei) and 10% neutrons. At energies $\approx 10^6$ GeV (the knee) the differential spectral index changes from 2.7 to 3, and at $10^{10}$ GeV (the ankle) it goes back to 2.7 (see Fig. 1).

As this flux of nucleons enters the atmosphere, its interactions with the air will induce a flux of secondary and tertiary hadrons (all of them referred as secondary) at different vertical depths. Although approximate analytic expressions can be obtained in limited regions of energy \cite{9,10}, it is easy as well to perform a numerical simulation.

We use CORSIKA \cite{15} to obtain the number and energy distribution of secondary nucleons, (charged) pions and kaons (we neglect the baryons containing an s quark) produced inside the air shower started by a nucleon of energy $E$. In Fig. 1 we plot the total (primary plus secondary) flux of hadrons with energies from $10^4$ to $10^{11}$ GeV. We obtain this flux generating 1500 showers per decade of energy. We observe that at energies around $10^7$ GeV ($\sqrt{s} \approx 5$ TeV) the secondary nucleons increase in a 50% the number of primaries. Note also that the number of mesons (all of them secondary) is approximately a 15% the number of primary nucleons of the same energy.

The region of energies beyond $10^8$ GeV (the one relevant at AUGER \cite{16}) deserves a more detailed analysis. We notice an important factor that distinguishes two types of showers: the presence or not of a leading hadron carrying a significant fraction of energy after each hadronic interaction. In Table I we give two examples of
TABLE I: Spectrum of hadrons in a $10^{10}$ GeV shower with a first interaction of elasticity $x_F \approx 0.3$ (left, shower (a) in the text) and $x_F \approx 0.03$ (right, shower (b)).

III. FLUX OF MASSIVE LONG-LIVED PARTICLES

Let us now estimate the production rate of new physics in the collision of these cosmic hadrons with atmospheric nucleons. The probability that a hadron $h$ of energy $E$ produces the exotic particle(s) $X$ is determined by the cross section $\sigma_{Xh}^N(E)$ and the total cross section with the air, $\sigma_T^{ha}(E)$:

$$P_X^h(E) \approx \frac{A \sigma_{Xh}^N(E)}{\sigma_T^{ha}(E)}, \quad (3)$$

where we assume $A = 14.6$ nucleons in a nucleus of air and neglect nuclear effects in the interaction to produce showers, both of energy $E = 10^{10}$ GeV, with a very different spectrum of secondary hadrons. Clearly, shower (b) does not include a leading hadron after the first interaction. A relevant parameter to characterize this feature is the elasticity $x_F$ of the first hadronic interaction: in example (a) we have $x_F \approx 0.3$ (i.e., a secondary hadron keeps 30% of the initial energy) whereas (b) corresponds to $x_F \approx 0.03$. The analysis of 500 showers of $10^{10}$ GeV shows that around 15% of them have $x_F < 0.06$ (i.e., no leading hadron after the first interaction with the air).

$\lambda_{\tilde{G}}$ [g/cm$^2$]}

FIG. 2: Interaction length in air $\lambda_{\tilde{G}}$ of the gluino hadron. The interaction length in water is a 3.2% shorter.

$X$. Taking the total fluxes $d\Phi_h/dE$ of the three species of hadrons in Fig. 3 we obtain that the flux of exotic particles (of any energy) is just

$$\Phi_X = \sum_{h=N,\pi,K} \int_{E_{\text{min}}}^\infty dE \frac{d\Phi_h}{dE} P_X^h(E). \quad (4)$$

If we are interested in the energy distribution of $X$ we must evaluate

$$\frac{d\Phi_X}{dE_X} = \sum_h \int_{E_X}^{\infty} dE \frac{d\Phi_h}{dE} A \frac{d\sigma_X^N}{d\sigma_{T}^{ha}} \frac{d\sigma_{Xh}^N}{dE_X}, \quad (5)$$

where $E_X$ is the energy of $X$ and the cross sections inside the integral are evaluated at $E$. The (default) cross sections with the air used by CORSIKA above $10^4$ GeV can be approximated by $\sigma_{T}^{ha} \approx C_0^h + C_1^h \log(E/\text{GeV}) + C_2^h \log^2(E/\text{GeV})$, with the constants given in Table II.

As we mentioned above, to be of interest the exotic particle should be penetrating and survive the air shower where it was produced. We will then consider two different cases. The first one is the long-lived gluino $\tilde{g}$ of split-

TABLE II: Constants defining the total cross section with the air.

$M_{\tilde{g}} = 200$ GeV

$M_{\tilde{g}} = 300$ GeV
SUSY models \[2\]. In the most evasive scenario the gluino fragments always into an electrically neutral gluino-gluon hadronic state $\tilde{G}$ (a particular type of $R$-hadron). The experimental bounds on its mass are in this case around 170 GeV \[17\]. Notice that general bounds \[18\] or bounds based on the observed delay in the time of flight versus a muon \[19\] do not apply to a neutral long-lived hadron. The gluino hadron will interact often with the air, its interaction length (in Fig. 2) can be estimated \[20\] as the gluino mass \[17, 20, 21\]. Therefore, the gluino hadron is suggested by a NLO calculation \[23\].

The parton-parton cross sections to produce gluino pairs can be easily obtained taking the limit of large squark mass in the general expressions given in Ref. \[22\], and we include them in the Appendix. In Fig. 3 we plot the total $hN$ ($h = N, \pi, K$) cross sections $\sigma_{hN}^{\chi}$ to produce gluino pairs for values of the hadron energy between $10^4$ and $10^{11}$ GeV and $M = 200, 300$ GeV. We have assumed isospin symmetry to deduce the kaon PDFs and have taken a renormalization scale $\mu = 0.2M$, as suggested by a NLO calculation \[22\].

FIG. 3: Total cross sections $\sigma_{hN}^{\chi}$ with $X = \tilde{g}\tilde{g}$ and $\chi\chi$ for $h = N, \pi, K$.

We can now calculate the flux of gluino pairs produced in the atmosphere. In Fig. 4 we plot the flux $\Phi_{\tilde{g}\tilde{g}}$ for values of $M$ between 150 and 450 GeV. We express this flux in gluino pairs per year, squared kilometer and sterad. Multiplying by $2\pi$ sterad, for example, we can read that the $\approx km^2 IceCube$ detector would be exposed to one downgoing gluino pair per year if $M \approx 160$ GeV. Around a 64% of the gluino flux would be produced by the primary nucleons, whereas the rest corresponds to secondary nucleons (16%), pions (16%) and kaons (4%). In Fig. 5 we give the energy distribution of the gluinos for $M = 200, 300$ GeV. The energy $E$ is in this case the total energy of the two gluinos in the lab frame. In the next section we comment on the phenomenological relevance of these results.

The second nonstandard particle that we would like to consider is a stable WIMP $\chi$ constituting the dark matter of the universe (see \[24\] for a recent review on dark matter). To define a simple scenario, we assume that $\chi$ is a Majorana fermion with only weak interactions and a relatively light mass, $M_\chi \approx 70$ GeV, and that all other possible particles (neutral or charged in the same $SU(2)_L$ multiplet) are at least 30 GeV heavier. The later requirement is necessary to avoid collider bounds, and will imply that coannihilations at temperatures below $M_\chi$ are irrelevant. We also need a discrete symmetry similar to the R-parity to make $\chi$ stable. Under these hypotheses, the
only relevant parameter to determine the relic abundance of this particle is its coupling to the $Z$ boson. We will assume a coupling $(g/2c_W)\epsilon$. The parameter $\epsilon$ fixes the annihilation rate of $\chi$ pairs into quarks and leptons (with a $Z$ in the $s$ channel) and then the contribution of $\chi$ to the energy density of the universe. It is a simple exercise to obtain that for $M_\chi = 70$ GeV we must have $\epsilon \approx 0.25$. Ref. [25] provides an explicit realization of this model in a partly SUSY scenario.

We can now estimate the flux of $\chi$ pairs produced by cosmic rays. Again, the $\sigma^{hN}_{\chi\chi}$ cross section [26] (see the Appendix) depends only on the coupling with the $Z$ boson (we neglect virtual effects of heavier particles as well as the production of heavier fields that may decay into $\chi$ plus standard particles). In Fig. 5 we have included $\sigma^{hN}_{\chi\chi}$. The flux of downgoing dark-matter particles would be $4.6 \times 10^{-4}$ pairs per year, squared kilometer and sterad. Their energy distribution is shown in Fig. 5.

**IV. NEW PHYSICS IN A SINGLE EVENT OF ULTRAHIGH ENERGY**

We would also like to find the probability for the production of gluino or WIMP pairs in single events of the highest energy (the ones to be measured at AUGER). In Table I we give two simulations of showers of high ($x_F \approx 0.3$) and low ($x_F \approx 0.03$) elasticity, both with a proton primary of energy $10^{10}$ GeV. For $M = 200$ (300) GeV the probability to produce a gluino pair in each shower is $P^a_{g\bar{g}} \approx 5.4 \times 10^{-5}$ ($9.7 \times 10^{-6}$) and $P^b_{g\bar{g}} \approx 4.3 \times 10^{-5}$ ($6.6 \times 10^{-6}$), respectively. Around 20% of this probability corresponds to the primary proton, whereas the rest expresses the probability that the gluino pair is produced by secondary hadrons inside the shower. Notice that the shower with no leading hadron after the first interaction has more hadrons of intermediate and lower energies. For a light gluino these tend to compensate the effect of the leading hadron (i.e., $P^b_{g\bar{g}} \approx P^a_{g\bar{g}}$). However, as the gluino mass $M$ increases the less energetic hadrons are not able to produce gluinos and $P^b_{g\bar{g}}$ becomes significantly smaller than $P^a_{g\bar{g}}$.

The probability to produce the pair of WIMPs in these two showers is $P^a_{XX} \approx 2.5 \times 10^{-8}$ and $P^b_{XX} \approx 3.1 \times 10^{-8}$. In this case the exotic particles, of mass $M_\chi = 70$ GeV, can be effectively produced by hadrons of intermediate energy and $P^b_{XX} > P^a_{XX}$.

In Fig. 6 we plot the average probability to produce gluinos (and WIMPs) for a primary proton of energy above $10^8$ GeV. We separate the probability that the gluinos are produced by the primary or secondary hadrons inside the shower. We find that secondary pions are the main source of gluinos in showers above $10^8$ GeV.
V. DETECTABILITY OF THE GLUINO PAIRS

The small flux of WIMPs that we have obtained does not seem to imply any experimental consequences in neutrino telescopes or air shower experiments. The flux of gluino pairs, however, might be visible.

As mentioned above, IceCube (around 1 km² of area) would be exposed to one gluino pair per year if $M \approx 160$ GeV. Since this mass is already below the experimental limits from the Tevatron, such an event would be unexpected. For example, if $M = 200$ (300) GeV the probability to have the two gluinos crossing IceCube in one year is just around 0.22 (0.015). Let us assume that such unexpected event actually happens. The mean energy of the gluino pair is around $E_0 = 2.1 \times 10^6$ (5.1 $\times$ 10⁶) GeV, and we obtain an average angle between the two gluinos $\theta_{\tilde{g}} \approx 8.8 \times 10^{-4}$ (5.4 $\times$ 10⁻⁴) rad. We will approximate a constant interaction length in water $\lambda_G \approx 175$ g/cm² for the gluino hadron and a linear energy loss per interaction $\Delta E = k\gamma$ with $k \approx 0.2$ GeV. This means that a $\tilde{G}$ of $M = 200$ GeV deposits a 1 per mille of its energy every 1.8 meters of water. It is then easy to deduce that the energy of the gluino pair at a depth $x$ is just

$$E_G \approx E_0 \exp\{-x/x_0\},$$

(6)

where $x_0 = \lambda_G M/k \approx 1.8 \times 10^5$ (2.6 $\times$ 10⁵) g/cm² for $M = 200$ (300) GeV. Therefore, two vertical gluinos would reach a depth $x_1 = 1.4 \times 10^5$ g/cm² (the top of IceCube) with a total energy around $E_1 = 1.1 \times 10^6$ (2.1 $\times$ 10⁶) GeV. If they are produced at an altitude $H = 20$ km, the gluinos will enter IceCube separated by 18 (11) meters and will deposit $\Delta E = 4.7 \times 10^5$ (1.5 $\times$ 10⁵) GeV of energy through the kilometer long detector. Such a signal could be clearly distinguished from a typical muon bundle in an air shower core.

More frequent events could be obtained in the larger air shower experiment AUGER. The number of gluino pairs of $M = 200$ (300) GeV hitting per year the 3000 km² AUGER area would be around

$$N_{\tilde{g}\tilde{g}} \approx \int_0^1 d\cos\theta \ 2\pi A \cos\theta \ T \ F_{\tilde{g}\tilde{g}} \approx 330 \ (22),$$

(7)

where $\theta$ is the zenith angle. However, this is not a very significant number, as these gluinos are inside the air shower where they were produced, and many of these showers have an energy below the 10⁶ GeV threshold in AUGER. In Fig. 7 we plot the distribution of the 330 (22) gluino pairs as a function of the primary energy. We obtain an estimate of 20 (2) gluino events per year inside showers above 10⁶ GeV. A 75% of these events come from a zenith angle below 60° whereas the remaining 25% are showers reaching AUGER quasi-horizontally, with a larger zenith angle.

Let us then analyze a typical gluino event inside a 10¹⁰ GeV shower for $M = 200$ GeV. Once created, the average angle between the gluinos is $4.7 \times 10^{-4}$ rad, whereas the total energy of the pair is around $5.2 \times 10^7$ GeV. The gluino-gluon R-hadrons will interact every 160 g/cm², depositing in each interaction an average one per mille of their energy (this corresponds to the value $k = 0.2$ given above). Notice that the signal produced by a gluino hadron is quite different from that produced by a light hadron of the same energy [27, 28]. A proton deposits most of its energy within a couple of vertical atmospheres (2000 g/cm²), whereas the gluino would interact in the same interval around 12 times, depositing a fraction $10^{-3}$ of hadronic energy every time. The gluino signal is more homogeneous, after 4 or 5 interaction lengths it is a trace of constant energy. Notice also that the number of muons from pion decays that the gluino produces grows proportional to the length of that trace. This will affect the curvature in the shower front hitting the ground. The series of mini-showers started by the gluinos will produce muon bundles at different slant-depths, and the front will exhibit a more pronounced curvature than what one would expect for a shower of the same energy interacting only
in the upper atmosphere. For isolated gluino showers (i.e., not inside a proton event) these features have been discussed by Anchordoqui et al. \cite{27} and confirmed with a Montecarlo simulation in \cite{28}. Previous works on air showers started by (lighter) long-lived gluinos were proposed as a possible explanation of the events beyond the GZK cutoff \cite{29}.

Therefore, one needs to distinguish two types of events, the vertical ones (with a zenith angle $\theta < 60^\circ$) and the quasi-horizontal ones (with $\theta > 60^\circ$). If the gluinos are produced in a $10^{10}$ GeV vertical shower they will take a 0.5% of the initial energy and deposit a 6% of that energy in two traces separated by $\approx 9$ m. Each trace is the superposition of hadronic showers of $2.6 \times 10^4$ GeV separated by 160 g/cm$^2$, i.e., around $1.6 \times 10^5$ GeV of energy in each gluino trace. It seems unlikely that such a distortion in the profile of a vertical shower can be detected in AUGER. Quasi-horizontal showers, however, could provide a more promising signal. At large zenith angles the depth of the atmosphere increases up to 36000 g/cm$^2$ for $\theta = \pi/2$, and this has several important effects on the gluino signal. First, the two gluino hadrons will cross the AUGER surface with a larger separation than in vertical showers. In particular, horizontal events come from a distance $D \approx \sqrt{2HR}$, where $H \approx 20$ km and $R_T$ is the radius of the Earth. This gives a separation $\approx D\theta_{gg}$. Second, after a certain depth most of the hadronic energy of the shower has been absorbed by the atmosphere, leaving only muons (and invisible neutrinos). In contrast, the hadron content in the gluino trace is basically constant, as each $\tilde{G}$ starts a $2.6 \times 10^4$ GeV shower every 160 g/cm$^2$. Therefore, the detectors would observe charged hadrons produced by the two gluinos in addition to the muons. The third effect on these inclined events is that the total number of muons produced in the gluino traces is larger than in vertical showers, as it grows almost linearly with the depth. In this way, the two gluinos inside the $10^{10}$ GeV event would deposit around $10^7$ GeV (a 20% of their total energy) if they come horizontally crossing a depth of 36000 g/cm$^2$. Notice that these muons produced deep in the atmosphere increase the curvature of the shower front.

**VI. SUMMARY AND DISCUSSION**

Cosmic ray experiments could provide a window to explore physics beyond the standard model, complementing accelerator experiments. The primary nucleons or secondary hadrons inside an air shower may produce new massive particles when they interact with atmospheric nucleons. We first have determined the total flux of hadrons within cosmic rays. We think Fig. II is a relevant result, as it is this (and not just the flux of primaries) the relevant flux to evaluate the production rate of new physics by cosmic rays. Then we have discussed how to calculate the creation of a massive particle, focusing on the long-lived gluino of split-SUSY models. We have shown that the flux of gluino hadrons produced by very energetic cosmic rays allows the possibility of isolated gluino events crossing a km$^2$ neutrino telescope like IceCube. In the larger detector AUGER, for a gluino mass $M = 200$ GeV we obtain an estimate of 20 gluino events per year inside air showers of energy above $10^8$ GeV. We have argued that the gluinos could modify substantially the profile of the showers with a large zenith angle. The gluino hadrons propagate losing a 1 per mille of their energy in every interaction length, defining an air shower with an approximately constant number of hadrons and with a number of muons that grows linearly with the depth. Therefore, the signal is stronger if they are created inside showers that reach AUGER from a large zenith angle. We think that the possibility of detecting in AUGER such a signal deserves a more detailed study.

We thank Carlos García-Canal and Paolo Lipari for useful discussions. This work has been supported by MEC of Spain (FPA2003-09298-C02-01 and FPA2006-05294) and Junta de Andalucía (FQM-101 and FQM 437). We also acknowledge financial support from a grant CICYT-INFN (06-13).

**APPENDIX A: PARTONIC CROSS SECTIONS**

Making use of the Mandelstam variables, $s + t + u = 2M^2$, and the polar angle $\theta$ in the center of mass frame, $t = -(1 + \beta^2 - 2\beta \cos \theta)s/4$, with $\beta = \sqrt{1 - 4M^2/s}$ and $\gamma = (1 - \beta^2)^{-1/2}$, the cross-sections read:

$$\frac{d\sigma}{d\cos \theta} = \frac{s}{2} \frac{d\sigma}{dt}, \quad \sigma = \frac{1}{2} \int_{-1}^{1} d\cos \theta \frac{d\sigma}{d\cos \theta}. \quad (A1)$$
\[ q\bar{q} \rightarrow q\bar{q}: \]
\[
\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{4s^2} \left\{ \frac{2(t - M^2)(u - M^2)}{s^2} + \frac{M^2(s - 4M^2)}{(t - M^2)(u - M^2)} \right. \\
+ \frac{(t - M^2)(u - M^2) - 2M^2(t + M^2)}{(t - M^2)^2} \\
+ \frac{(t - M^2)(u - M^2) + M^2(u - t)}{s(t - M^2)} \\
\left. + \frac{(u - M^2)(t - M^2) - 2M^2(u + M^2)}{s(u - M^2)} \\
+ \frac{(u - M^2)(t - M^2) + M^2(t - u)}{s(u - M^2)} \right\}, \quad (A2)
\]
\[
\sigma = \frac{3\pi\alpha_s^2}{4s} \left\{ 3 \left( 1 + \frac{1}{\gamma} - \frac{1}{4\gamma^2} \right) \log \frac{1 + \beta}{1 - \beta} \\
- \left( 4 + \frac{17}{2\gamma^2} \right) \beta \right\}. \quad (A3)
\]

\[ q\bar{q} \rightarrow gg: \]
\[
\frac{d\sigma}{dt} = \frac{8\pi\alpha_s^2}{3s^2} \left( t - M^2 \right)^2 + \frac{(u - M^2)^2 + 2M^2s}{s^2}, \quad (A4)
\]
\[
\sigma = \frac{8\pi\alpha_s^2}{9s} \left( 1 + \gamma \right) \beta. \quad (A5)
\]

\[ q\bar{q} \rightarrow XX: \]
\[
\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha_s^2\epsilon^2}{32s^4\epsilon^4_W} \frac{s\beta^3}{(s - M_Z^2)^2} \\
\times (1 - 4\eta + 8\eta^2)(1 + \cos^2\theta), \quad (A6)
\]
\[
\sigma = \frac{\pi\alpha_s^2\epsilon^2}{24s^4\epsilon^4_W} \frac{s\beta^3}{(s - M_Z^2)^2} (1 - 4\eta + 8\eta^2), \quad (A7)
\]

with \( \eta = |Q_q|s_W^2 \) and \( \epsilon = 0.25 \) (see discussion in Sect. III).


