Relativistic generalization of the inertial and gravitational masses equivalence principle

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Abstract

The Newtonian approximation for the gravitational field equation should not necessarily involve admission of non-relativistic properties of the source terms in Einstein’s equations: it is sufficient to merely consider the weak-field condition for gravitational field. When a source has electromagnetic nature, one simply cannot ignore its intrinsically relativistic properties, since there cannot be invented any non-relativistic approximation which would describe electromagnetic stress-energy tensor adequately, even at large distances where the fields become naturally weak. But the test particle on which gravitational field is acting, should be treated as non-relativistic (this premise is required for introduction of the Newtonian potential $\Phi_N$ from the geodesic equation).

We use here (in parentheses if in a tetrad basis) Greek indices as 4-dimensional and Latin as 3-dimensional, $\kappa = 8\pi G$ ($G$ is the Newtonian gravitational constant), $R_{\mu\nu} = R^\alpha_{\mu\nu\alpha}$, and spacetime signature as $+,-,-,-$. Einstein’s equations then read as $R^{(\mu)}_{(\nu)} - \frac{1}{2} R \delta^{\mu}_{\nu} = -\kappa T^{(\mu)}_{(\nu)}$, thus $R = \kappa T$, and $R^{(\mu)}_{(\nu)} = -\kappa \left( T^{(\mu)}_{(\nu)} - \frac{1}{2} T \delta^{(\mu)}_{(\nu)} \right)$. We shall need only 00-component of Einstein’s equations,

$$R^{(0)}_{(0)} = -\frac{\kappa}{2} \left( T^{(0)}_{(0)} - T^{(i)}_{(i)} \right).$$  \hspace{1cm} (1)

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We call a source with \( T^{(\nu)}_{(\nu)} = 0 \) intrinsically relativistic since the spatial part of its stress-energy tensor is of the same order of magnitude as the temporal component (cf. the concept of a zero rest mass particle). An example is the Maxwell electromagnetic field which has this property even of its static solutions when any kind of motion is excluded. Similarly, a perfect fluid with its energy-momentum tensor

\[
T^{pf} = (\mu + p)u \otimes u - pg
\]

possesses this property in the particular case of incoherent radiation \((\mu = 3p)\), and the tensor (2) is written in the rest reference frame of the fluid. There is also the case of stiff matter \((p = \mu)\) in which the sound propagates with the velocity of light; we say that such objects are hyper-relativistic. Thus in the non-relativistic case \(\left| T^{(i)}_{(i)} \right| \ll T^{(0)}_{(0)} \) the 00-component of Einstein’s equations reads

\[
R^{(0)}_{(0)} \approx -\frac{\kappa}{2} T^{(0)}_{\text{non-rel}}(0),
\]

then in the intrinsically relativistic case,

\[
R^{(0)}_{(0)} = -\kappa T^{(0)}_{\text{intr-rel}},
\]

and finally in the hyper-relativistic case,

\[
R^{(0)}_{(0)} = -2\kappa T^{(0)}_{\text{hyper-rel}}.
\]

The Newtonian approximation is found from the geodesic motion of a non-relativistic test particle. Thus let us consider a static spacetime with \( g^{00} = 1 + 2\Phi_N, |\Phi_N| \ll 1 \) and choose a 1-form basis as

\[
\theta^{(0)} = e^\alpha dt, \quad \theta^{(k)} = g^{(k)}_j dx^j.
\]

Taking the inverse triad, so that \( dx^j = g_{(k)}^j \theta^{(k)}, dt = e^{-\alpha} \theta^{(0)} \), we find the necessary components of 1-form connections \( \omega^{(0)}_{(l)} \equiv \omega^{(l)}_{(0)} = \alpha_{,j} g_{(l)}^j \theta^{(0)} \), and finally from Cartan’s second structural equations,

\[
R^{(0)}_{(0)} = g^{(l)(k)} R^{(0)}_{(l)(k)(0)} \approx e^{-\alpha} (e^\alpha)_{,ij} g^{ij}
\]

where \( g^{ij} = -\delta^j_i + \text{higher-order terms (to be neglected)} \). Since \( e^\alpha \approx 1 + \Phi_N, R^{(0)}_{(0)} \approx -\Delta \Phi_N \) (\( \Delta \) is the usual Laplacian). Thus the Newton–Poisson
equations corresponding to (3), (4), and (5), are

- non-relativistic: $\Delta \Phi_N = 4\pi G\mu$,  
- intrinsically relativistic: $\Delta \Phi_N = 8\pi G\mu$  
- hyper-relativistic: $\Delta \Phi_N = 16\pi G\mu$

respectively (we wrote here the inertial mass density $\mu$ of the source instead of $T_{(0)}^{(0)}$). For any perfect fluid the Newton–Poisson equation takes the form

$$\Delta \Phi_N = 4\pi G(\mu + 3p),$$

so that for incoherent dust the old traditional equation follows, but if the fluid represents an incoherent radiation ($p = \mu/3$), the source term doubles (as this is the case for electromagnetic source), and for the stiff matter ($p = \mu$), it quadruples.

Since the equations (4) and (5) are exact ones, they strictly express the equivalence principle already generalized (to use an expression similar to “already unified” of J.A. Wheeler) in standard general relativity. The conclusions we came upon in this talk automatically add on relativistic features to the principle traditionally formulated in standard textbooks on general relativity as a completely non-relativistic approximation (for both test particle and sources of Einstein’s equations) just as it was used by Einstein in his first attempts to generalize the special relativity. But the Newtonian-type potential is generated by a wide class of distributions of matter, including intrinsically relativistic and hyper-relativistic cases: the only restriction here consists of weakness of the field and not the “state of motion” of the sources in Einstein’s equations (especially such an intrinsic property as to be relativistic which is so often realized by static configurations when the very idea of motion is out of question). Clearly, here we haven’t used any hypotheses at all.

As to the applications of this generalized principle of equivalence, it is worth pointing out the (post-) post-Newtonian approximations. Since some conclusions about validity of the principle of equivalence come from observations of stellar systems, a mere presence in them of intrinsically relativistic distributed or localized objects (say, high density of any kind of radiation, strong or widely distributed magnetic fields, existence of stiff matter in cores of exotic stars, jets of ultrarelativistic particles) would radically change interpretation of the observational data if their proper understanding depends
on adequate description of the sources of gravitational field, without any disregard for the pressure and stresses. These conclusions should definitively lead to a revision of the old problem of stability of young globular star clusters via the virial theorem (when the electromagnetic radiation between the stars is very intense) which seems to be done through approximated methods only. This is also the central point of evolution of the gravitation theory from Soldner [8] and Einstein-1911 [1] to Einstein-1915 [2], resulted in doubling [cf. (8) and (9)] of the light beams bending in the final self-consistent version of the theory. This doubling has two sides: one is mentioned just above, and another pertains to light beams and jets of ultra-relativistic particles via the 3rd Newtonian law, see comments on both in Refs. 4, 5 and 7. Another problem is connected to the interesting and stimulating question by D. Brill, the Chairman of the parallel Session GT4 at which this talk was delivered: How to relate Einstein’s first tentative considerations of photons’ absorption by a material sample, leading to its temperature rise, and the corresponding increase of its masses, both inertial and gravitating ones? My answer was that the gravitational mass does not satisfy a conservation law, at least that which follows from the Noether theorem [3, 6] under the general relativistic invariance of the action integral, in a contrast to the inertial mass, and it is clear that both masses cannot simultaneously be conserved, e.g. in the process of light absorption.

Finally, it should be emphasized once more that in this talk we made a revision of a too long persistent old viewpoint, but not of the same and mature theory.

References


