Neutrino Masses and Interactions 
in a Model with Nambu-Goldstone Bosons

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Abstract. 
A natural scenario for the generation of neutrino masses is the see-saw mechanism, in which a large right-handed neutrino mass makes the left-handed neutrinos light. We review a special case when the Majorana masses originate from spontaneous breaking of a global U(1)XU(1) symmetry. The interactions of the right-handed with the left-handed neutrinos at the electroweak scale further break the global symmetry giving mass to one pseudo Nambu-Goldstone boson (pNGB). The pNGB can then generate a long-range force. Leptogenesis occurs through decays of heavy neutrinos into the light ones and Higgs particles. The pNGB can become the acceleron field and the neutrino masses vary with the value of the scalar field$^1$.

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INTRODUCTION

The standard model of electroweak theory is successful in explaining most of the phenomena we observed till now. The neutrino mass is the only indication we have that requires physics beyond the standard model. It has been established that the neutrino sector must be extended to accommodate the observed mass-squared difference for the neutrinos in the atmospheric, solar, as well as in the laboratory experiments.

The observed neutrino masses are too small to treat them at par with charged fermions. The most natural explanation for the smallness of the neutrino masses comes from the see-saw mechanism [1]. One introduces the right-handed neutrinos in the model that allows the usual Dirac masses for the neutrinos to be of the order of charged lepton masses. Since the right-handed neutrinos are singlets, it is now possible to allow very heavy Majorana masses $M_1, M_2, \ldots$, for right-handed neutrinos. This introduces the scale of lepton number violation in the model and in turn induces tiny Majorana masses for the left-handed neutrinos.

When Majorana masses of the right-handed neutrinos are introduced, they break lepton number explicitly. However, it is more natural to associate any new scale of the theory with a symmetry whose breaking produces neutrino masses. In a forthcoming article we propose [11], that the Majorana masses of the right-handed neutrinos originate from the spontaneous breaking of lepton number, which leads to a massless Goldstone boson, the Majoron. However, since the residual global symmetry is not exact, one Goldstone boson picks up small mass producing ultra low mass pseudo Nambu-Goldstone bosons (pNGB). An early proposal, along these lines, was the axion, which was introduced to solve the strong CP problem through the Peccei-Quinn symmetry [2, 3]. The axion is still being searched for as a candidate for dark matter [4]. In strong interactions, it is the breaking of symmetries within QCD that generates a large part of the proton’s mass, which is relatively large compared to the masses of current quarks. The result is the appearance of low mass pions – the pNGB’s of the spontaneously broken chiral symmetry. This concept of pNGB was applied [5] to the general formalism to a long-range force and phase transitions [6] occurring at a late epoch of the universe, thus developing quintessence models contemporaneously with other authors [7]. Neutrino physics has also found applications of quintessence model, in which the quintessence or the acceleron are formed as condensates and couple to neutrinos varying their masses [8]. This scenario has some interesting consequences [9, 10]. In this review we point out that the pNGB in models of Majorana neutrinos can play the role of the acceleron, introducing an exponential potential, and giving rise to a long-range force that may be of of phenomenological importance [11]. Finally, the model generates a lepton asymmetry through the decay of heavy neutrinos to light ones and Higgs particles.
TWO GENERATION PNGB MODEL

We demonstrate here the idea behind the pNGB model of Majorana neutrinos with a two generation example. The theory possesses a global $U(1)_A \times U(1)_B$ symmetry. The breaking of this symmetry spontaneously sets a large scale in the theory far above the scale of electroweak symmetry breaking. There are two right-handed neutrinos $N_1$ and $N_2$, which are singlets under the standard model gauge groups and transform under the global symmetry as $(1,0)$ and $(0,1)$ respectively. The global symmetry prevents any Majorana mass of the right-handed neutrinos. We also introduce two singlet scalar fields $\Phi_1(x)$ and $\Phi_2(x)$ transforming under the global symmetry as $(2,0)$ and $(0,2)$ respectively, whose interactions with the right-handed neutrinos are given by

$$\mathcal{L}_M = \frac{1}{2} \alpha_1 \bar{N}_1 N_1^c \Phi_1 + \frac{1}{2} \alpha_2 \bar{N}_2 N_2^c \Phi_2.$$  

(1)

The vacuum expectation values (vevs) of these scalars will give Majorana masses to the right-handed neutrinos and there will be two Goldstone bosons. As we shall demonstrate below, the Dirac masses of the neutrinos at the electroweak scale will break one combination of the global symmetry softly giving a small mass to one of the Goldstone bosons, which now becomes a pseudo Nambu-Goldstone boson (pNGB) of the model. The other Goldstone boson, the Majoron, remains massless. Since the Majoron couples only to singlet fields, the theory is consistent with phenomenology of known particles.

After the fields $\Phi_i, i = 1,2$ acquire vevs, we can express them in terms of their vevs $\sigma_i$ and decay constants $f$ (we assume their decay constants to be same, $f_1 \sim f_2 \sim f$) and the Nambu-Goldstone bosons $\phi_i$ as

$$\alpha_i \Phi_i \rightarrow \alpha_i \langle \Phi_i \rangle e^{2i\phi_i/f_i} = \alpha_i \sigma_i e^{2i\phi_i/f_i} = M_i e^{2i\phi_i/f_i}.$$  

(2)

At this stage it is possible to make phase transformations to the fields $N_i$ that eliminate the $\phi_i$, implying that both Goldstone bosons are massless. The self interactions of the Goldstone bosons, given by

$$\mathcal{L}_\phi = \frac{1}{2} M_\Phi \Phi^\dagger \Phi + \frac{1}{4} \lambda (\Phi^\dagger \Phi)^2 + \partial_\mu \Phi^\dagger \partial^\mu \Phi = \frac{\sigma^2}{f^2} \partial_\mu \Phi \partial^\mu \Phi + f (\Phi^\dagger \Phi)$$

also do not contain any term that can give masses to the Goldstone bosons.

We now write down all the Yukawa terms involving the neutrinos, including the Dirac mass terms

$$\mathcal{L}_\text{mass} = \frac{1}{2} M_1 \bar{N}_1 N_1^c e^{2i\phi_1/f} + \frac{1}{2} M_2 \bar{N}_2 N_2^c e^{2i\phi_2/f} + m e^{i\alpha} \bar{N}_1 \nu_1 + m e^{i\beta} \bar{N}_2 \nu_2$$

$$+ \lambda m e^{i\gamma} \bar{N}_2 \nu_1 + \lambda m e^{i\zeta} \bar{N}_2 \nu_2.$$  

(3)

We introduced the Dirac mass $m$ and some scaling parameters $\lambda, \epsilon, \epsilon'$ in order to write down the Dirac mass terms that break the $U(1)_{A-B}$ global symmetry softly. We included all the phases which
contribute to CP violation. Rephasing of the fields

\[ N_i \rightarrow e^{i\phi_2/f} N_i \quad \text{and} \quad \nu_i \rightarrow e^{i\phi_2/f} \nu_i \]  

and rephasing of the CP phases, leads to the full mass matrix

\[ L_\mu = \frac{1}{2} M_1 \bar{N}_1 \gamma \nu_1 + \frac{1}{2} M_2 \bar{N}_2 \gamma \nu_2 + m \bar{N}_1 \nu_1 + m \nu_1 + H.c., \]  

where \( 2\eta = \gamma - \alpha + \beta - \xi \). Thus we finally have one CP phase \( \eta \) and one combination of the fields \( \phi = \phi_1 - \phi_2 \), which becomes the pNGB. The other combination of fields \( \phi_1 + \phi_2 \) correspond to the invisible singlet Majoron and remains massless by decoupling from the theory. This conclusion is independent of the choice of phase transformation.

We shall now explicitly demonstrate how such \( U(1)_A \times U(1)_B \) global symmetry breaking soft Dirac mass term may appear in the theory after the electroweak symmetry breaking. If we assign the \( U(1)_A \times U(1)_B \) quantum numbers

\[ \ell_1 \equiv \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \equiv (1,0) \quad \text{and} \quad \ell_2 \equiv \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \equiv (0,1), \]  

then the usual Higgs doublet \( H \) with the assignment \( (0,0) \) (for this to give masses to the charged fermions) can give only the diagonal Dirac mass terms. We need two more Higgs doublets \( H_1 \equiv (-1,+1) \) and \( H_2 \equiv (+1,-1) \) in order to generate the complete Dirac mass we discussed above. The mass term of the Lagrangian now becomes

\[ -L_{mass} = \frac{1}{2} M_1 \bar{N}_1 \gamma \nu_1 + \frac{1}{2} M_2 \bar{N}_2 \gamma \nu_2 + m \bar{N}_1 \nu_1 + m \nu_1 + H.c., \]  

These dimension-4 terms do not break the global symmetry. However, after the electroweak symmetry breaking, when all the doublets \( H \) and \( H_i \) acquire vevs, the global symmetry is broken. The part of the Lagrangian with all neutrino mass terms is:

\[ -L_{mass} = \frac{1}{2} M_1 \bar{N}_1 \gamma \nu_1 + \frac{1}{2} M_2 \bar{N}_2 \gamma \nu_2 + m \bar{N}_1 \nu_1 + m \nu_1 + H.c., \]  

After rephasing of the fields it reduces to equation (5).
structure of the theory makes the loop finite. The reason is that the $\phi$ field could be eliminated if any of the vertices is set to zero. This diagram is also invariant under any rephasing of the fields, which can be confirmed by observing that the most general phase transformation of the neutrinos,

$$N_i \rightarrow e^{ip_i} N_i \quad \text{and} \quad \nu_i \rightarrow e^{iq_i},$$

transforms

$$m \rightarrow e^{i(q_1 - p_1)} m; \quad m \epsilon \rightarrow e^{i(q_2 - p_1)} \epsilon; \quad m \lambda \epsilon' \rightarrow e^{i(q_1 - p_2)} m \lambda \epsilon'; \quad \lambda m \rightarrow e^{i(q_2 - p_2)} \lambda m; \quad M_1 \rightarrow e^{-i2p_1} M_1; \quad M_2 \rightarrow e^{-i2p_2} M_2;$$

so that

$$(m) \cdot (M_1^* / f^2) \cdot (m \epsilon e^{i\eta}) \cdot (\lambda m \epsilon' e^{i\eta})^*$$

and hence the diagram remains invariant.

The explicit calculation gives

$$V_{\text{eff}}(\phi^2) = -\frac{m^4 \lambda^2 \epsilon \epsilon' M_1 M_2 \log \left( \frac{M_2^2}{M_1^2} \right)}{4\pi^2 \left( M_1^2 - M_2^2 \right)} \cos \left( \frac{2\phi}{f} \right),$$

which has the minima at $\phi = 0, \pi f, 2\pi f, \cdots$. Expanding around one of the minima, we can write down a mass term

$$V_{\text{eff}}(\phi) = \frac{m^4 \lambda^2 \epsilon \epsilon' M_1 M_2 \log \left( \frac{M_2^2}{M_1^2} \right)}{2\pi^2 \left( M_1^2 - M_2^2 \right)} \frac{\phi^2}{f^2}.$$
Thus the mass of the $\phi$ field is now
\[
m_\phi = \frac{m^2 \lambda \sqrt{\epsilon \epsilon'}}{\pi f} \frac{M_1 M_2 \log \left( \frac{M_1^2}{M_2^2} \right)}{M_1^2 - M_2^2}.
\] (12)

The symmetry at the scale of $f \sim M_i$ protects the mass of the pseudo Nambu-Goldstone boson, so an explicit soft breaking of the symmetry at the scale $m$ can generate a mass of the order of $m^2 / f$. This light pNGB ($\phi' = i \phi$) can generate a long-range force and also become the acceleron field to explain the smallness of dark energy.

**NEUTRINO MASSES**

We consider next the structure of neutrino masses in this model. After the global symmetries are broken by the vevs of the scalars, the model reduces to the usual see-saw models [1], except for the pNGB which generates a new long-range force because of its small mass. The interactions required for the usual leptogenesis [12, 13, 14] through the decays of right-handed neutrino are also present and will be discussed in the next section.

We shall first consider the time evolution of the light neutrino states without including the effects of the pNGB, which is determined by the matrix
\[
-\mathcal{L}_{\text{eff}} = m_{ij}^T M_i^{-1} m_{ij} v_i v_j = \frac{m^2}{M} \left( \begin{array}{c} v_1 \\ v_2 \end{array} \right) \left( \begin{array}{cc} 1 + (\lambda \epsilon')^2 e^{2i\eta} & e^{i\eta} (\epsilon + \lambda \epsilon') \\ e^{i\eta} (\epsilon + \lambda \epsilon') & \lambda^2 + \epsilon^2 e^{2i\eta} \end{array} \right) \left( \begin{array}{c} v_1 \\ v_2 \end{array} \right).
\] (13)

This gives a mixing angle
\[
\tan 2\theta = \frac{\epsilon + \lambda \epsilon'}{(1 - \lambda^2) e^{-i\eta} + (\lambda^2 e^{i\eta} - \epsilon^2) e^{i\eta}}
\] (14)

which is large for $\epsilon, \epsilon' \ll 1$ and $\lambda \approx 1$. This mass matrix can be diagonalized to
\[
M_{\text{diag}} = \left( \begin{array}{cc} m_1 e^{i\theta_1} & 0 \\ 0 & m_2 e^{i\theta_2} \end{array} \right)
\] (15)

with
\[
m_{1,2} = \frac{m^2}{M} \left[ 1 \pm (\epsilon + K) + O(\epsilon^2) \right], \quad \text{and} \quad \tan \theta_{1,2} = \frac{1 \pm (\epsilon + K) \cos \eta}{\pm (\epsilon + K) \sin \eta},
\]

where $K = \lambda \epsilon'$. This part of the discussion is the same as in any other model with a see-saw mechanism. It can account for the large mixing between two generations.

We shall now consider the effect of the light scalar field, the pNGB in the model. We obtain the interactions of the light neutrinos with the scalar field by keeping only the terms linear in pNGB,
given by

\[ \mathcal{L} = \frac{m_2^2}{2M} \overline{\Psi} \left\{ \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} - \frac{\phi}{f} e^{i\eta} \begin{pmatrix} (\epsilon - K) e^{-i\theta_1} & 0 \\ 0 & (K - \epsilon) e^{-i\theta_2} \end{pmatrix} \right\} \Psi + H.C. \] (16)

The long range force introduced by the pNGB in this model will have direct consequences in neutrino oscillation experiments [15]. The extension of the model to three generations and the implications of the new force will be discussed elsewhere [11].

**LEPTON ASYMMETRY OF THE UNIVERSE**

The direct connection between the neutrino masses and the baryon asymmetry of the universe is now well established. In our case, lepton number violating couplings were introduced in order to give masses to the neutrinos. These couplings generate a lepton asymmetry [12, 13, 14], which will be converted to a baryon number asymmetry as the universe expands and approaches the electroweak phase transition.

In general, the generation of the lepton asymmetry of the universe requires three ingredients:

1. Lepton number violation, which is also the source of neutrino masses;
2. CP violation, which comes from the interference of tree level and one loop diagrams
3. Departure of the lepton number violating interactions from equilibrium

The decays of the right-handed neutrinos violate lepton number and the Yukawa couplings contain a phase \( \eta \) which can give CP-even and CP-odd amplitudes. If the right-handed neutrino mass can now satisfy the out-of-equilibrium condition, this model will be able to generate the required baryon asymmetry of the universe.

In our model lepton number is violated at tree-level in decays of the right-handed neutrinos

\[ N_i \rightarrow \ell_j + H_a^\dagger \]
\[ \rightarrow \ell_c^j + H_a \]

The one loop diagrams, like vertex corrections and self energies [12, 13] will contain CP-even and -odd amplitudes. The structure of the Lagrangian in eqs (5) and (6) has loops only when the Higgs particles mix with each other. Let us denote by \( V_{01} \) the mixing between \( H_0 \) and \( H_1 \) and similarly by \( V_{02} \) the mixing between \( H_0 \) and \( H_2 \). Such mixings are generated by the quartic couplings of the potential when the scalar acquire vevs. Here we do not restrict ourselves to a specific form of the potential and represent the generic mixings by \( V_{0i} \). The interference between tree and vertex diagrams gives the asymmetry

\[ \delta = \frac{\Gamma(N_1 \rightarrow \ell H_a^\dagger) - \Gamma(N_1 \rightarrow \ell^c H)}{\Gamma(N_1 \rightarrow \ell H_a^\dagger) + \Gamma(N_1 \rightarrow \ell^c H)} \]
\[ \frac{3}{16\pi M_2} \frac{m^2 \lambda^2}{v_1^2 + v_0^2} \left[ |V_{02}|^2 \frac{v_1^2}{v_2^2} |\epsilon^2| - |V_{01}|^2 |\epsilon^2| \right] \sin 2\eta \]  

(17)

where \( v_a = \langle H_a \rangle, a = 0, 1, 2 \) and we assumed \( M_2 >> M_1 \), so that the asymmetry is generated by the decays of \( N_1 \). In the context of leptogenesis the lightest \( N_1 \) is naturally out of thermal equilibrium when it decays. The dilution factor depends on the relative magnitude of

\[ \Gamma_{N_1} = \frac{|f_{11}|^2}{16\pi M_1} \]  

(18)

relative to the Hubble constant

\[ H(M_1) = 1.7 \sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T = M_1. \]  

(19)

When the ratio \( \kappa = \Gamma / H(M_1) \) is small the lepton excess is large. On the other hand, for large \( \kappa \) the time of expansion is large relative to the lifetime of \( N_1 \) so that many decays and recombinations can take place in the cosmological scale \( 1/H \) giving a smaller excess of leptons. Finally, the sphaleron interactions convert a fraction of the produced \( (B-L) \) asymmetry to a baryon asymmetry

\[ \frac{n_B}{s} = \frac{24 + 4n_H}{66 + 13n_H} \frac{n_{B-L}}{s} \]  

(20)

This conversion takes place from the time of leptogenesis down to the time of the electroweak phase transition. Here \( n_H \) is the number of Higgs doublets in our model. In summary, it is evident that the model can generate the required amount of baryon asymmetry of the universe.

**ORIGIN OF DARK ENERGY**

One of the most challenging questions at present is why the cosmological constant is very small but nonvanishing. If the dark energy were due to the presence of a cosmological constant, then one expects

\[ \rho_{DE} = \rho_{vac} = E_0^4 \]

where \( E_0 \) is the energy associated with a particle or a field. The observed cosmological constant corresponds to an \( E_0 \sim 2 \times 10^{-3} \) eV, which is very small for the scales of most elementary particles. It is of the order of neutrino masses and it has recently been proposed that the dark energy tracks the neutrino density [8]. If the neutrino masses vary as functions of a light scalar field (called the acceleron) the dark energy density may track the neutrino masses. This would then explain why the scale of cosmological constant is the same as the scale of neutrino masses. In the present model, the pNGB introduces a spatial dependence into the Majorana mass \( M_1 \)

\[
\mathcal{L}_\mu = \frac{1}{2} M_1 \langle \phi \rangle \bar{N}_1 N_1 + \frac{1}{2} M_2 \bar{N}_2 N_2 + m \bar{N}_1 \nu_1 + m_{\epsilon} \bar{N}_1 \nu_2 \\
+ \lambda m \epsilon \bar{N}_2 \nu_1 + \lambda m \bar{N}_2 \nu_2 + H.c.
\]  

(21)
The first term $M_1(\mathcal{A})$ has an exponential functional dependence on the scalar field. The effective neutrino mass then varies explicitly with the acceleron field $m_\nu(\mathcal{A}) = m^T M^{-1}(\mathcal{A}) m$, as required by models of mass varying neutrinos [8].

**SUMMARY**

A large Majorana scale is a necessary ingredient of the see-saw mechanism. At this very high energy there may exist global symmetries which are broken in order to create the Majorana masses. Remnants of the global symmetry survive in the low energy interactions. In a two generation model developed with my collaborators Hill, Mocioiu and Sarkar [11], the breaking of the symmetry produces two Nambu-Goldstone bosons. The introduction of Dirac masses at the electroweak scale further breaks the symmetry softly giving a small and finite mass to one of the Nambu-Goldstone bosons and leaves the other scalar massless. The exchange of the scalar particle introduces a new long-range force between neutrinos.

Finally, the model has additional attractive properties. It describes correctly the masses and mixings of the light neutrinos. The decays of the heavy neutrinos generate a lepton asymmetry consistent with leptogenesis. The neutrino masses have an explicit dependence on the scalar field, which may bring density effects with the pNGB playing the role of the "acceleron".

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