Spin light of electron in matter

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Abstract

We further generalize the powerful method, which we have recently developed for description of the background matter influence on neutrinos, for the case of an electron moving in matter. On the basis of the modified Dirac equation for the electron, accounting for the standard model interaction with particles of the background, we predict and investigate in some detail a new mechanism of the electromagnetic radiation that is emitted by moving in matter electron due to its magnetic moment. We have termed this radiation the “spin light of electron” in matter and predicted that this radiation can have consequences accessible for experimental observations in astrophysical and cosmological settings.

1 Introduction

In a series of our papers [1–4] we have developed a rather powerful method of investigation of different phenomena that can appear when neutrinos and electrons move in the background matter. The method discussed is based on the use of the modified Dirac equations for the particles wave functions, in which the correspondent effective potentials, that account for the matter influence on particles, are included. It is similar to the Furry representation [5] in quantum electrodynamics, widely used for description of particles interactions in the presence of external electromagnetic fields. In [1–4] we apply the discussed method for elaboration of the quantum theory of the “spin light of neutrino” in matter. The spin light of neutrino in matter

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one of the four new phenomena studied in our recent papers [6], is a new type of electromagnetic radiation that can be emitted by a massive neutrino (due to its magnetic moment) when the particle moves in the background matter. Within quasi-classical treatment the existence of this radiation was proposed and studied in [7], while the quantum theory of this phenomenon was developed in [1–4, 8].

It has been shown [1] how the approach, developed at first for description of a neutrino motion in the background matter, can be spread for the case of an electron propagating in matter. The modified Dirac equation for an electron in matter has been derived [1] and on this basis we have considered the electromagnetic radiation that can be emitted by the electron (due to its magnetic moment) in the background matter. We have termed this radiation as the “spin light of electron” in matter. It should be noted here that the term “spin light” was introduced in [9] for designation of the particular spin-dependent contribution to the electron synchrotron radiation power.

2 Modified Dirac equation for electron in matter

Let us consider an electron having the standard model interactions with particles of electrically neutral matter composed of neutrons, electrons and protons. Indeed, we account below only for the neutron component that can be used as an abrupt model for modelling a real situation existed when electrons move in nuclear matter of a neutron star (see, for instance, [10]). We suppose that there is a macroscopic amount of the background particles in the scale of an electron de Broglie wave length. Then the addition to the electron effective interaction Lagrangian is

\[
\Delta L^{(e)}_{\text{eff}} = \tilde{f}^\mu \left( \bar{e} \gamma_\mu \left( 1 - 4 \sin^2 \theta_W + \gamma_5 \right) e \right),
\]

where the explicit form of \( \tilde{f}^\mu \) depends on the background particles densities, speeds and polarizations. The modified Dirac equation for the electron wave function in matter is [1]

\[
\left\{ i \gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 - 4 \sin^2 \theta_W + \gamma_5) \tilde{f}^\mu - m_e \right\} \Psi_e(x) = 0,
\]

where for the case of electron moving in the background of neutrons

\[
\tilde{f}^\mu = \frac{G_F}{\sqrt{2}} (n_n, n_n \nu).
\]

Here \( n_n \) is the neutrons number density and \( \nu \) is the speed of the reference frame in which the mean momentum of the neutrons is zero.

The solution of equation (2) can be found in analogy with the case of neutrino (for details see [2, 3]). The complete set of the electron wave functions in the matter
is:

\[ \Psi_{\varepsilon,p,s}(r,t) = \frac{e^{-i(E_\varepsilon t - pr)}}{2L^{3/2}} \begin{pmatrix} \sqrt{1 + \frac{m_e}{E_\varepsilon - \alpha_n m_e}} & \sqrt{1 + s \frac{m_e}{p}} \\ s \sqrt{1 + \frac{m_e}{E_\varepsilon - \alpha_n m_e}} & 1 - s \frac{m_e}{p} e^{i\delta} \\ \varepsilon \sqrt{1 - \frac{m_e}{E_\varepsilon - \alpha_n m_e}} & \sqrt{1 - s \frac{m_e}{p}} e^{i\delta} \end{pmatrix}, \quad (4) \]

where \( L \) is the normalization length, \( \delta = \arctan(p_2/p_1) \) and the so-called matter density parameter \( \alpha_n \) is defined as

\[ \alpha_n = \frac{G_F n_n}{2\sqrt{2} m_e}. \quad (5) \]

The electron energy in matter

\[ E_{\varepsilon}^{(e)} = \varepsilon \sqrt{p^2 \left( 1 - s \alpha_n \frac{m_e}{p} \right)^2 + m_e^2 + c\alpha_n m_e} \quad (6) \]

depends on the electron helicity \( s = \pm 1 \) and the quantity \( \varepsilon = \pm 1 \) which splits the solutions into positive- and negative-frequency branches.

## 3 Spin light of electron in matter

To the lowest order of the perturbation theory, the corresponding quantum process is described by the one-photon emission diagram with the initial \( \psi_i \) and final \( \psi_f \) electron states and with the vertex corresponding to the standard electromagnetic interaction with the photon due to the electron charge \( e \). Thus, the transition amplitude is given by

\[ S_{fi} = -ie \sqrt{4\pi} \int d^4x \bar{\psi}_f (x)(\gamma^\mu e^\mu) \frac{e^{ikr}}{\sqrt{2\omega L^3}} \psi_i (x), \quad (7) \]

where \( k^\mu = (\omega, k) \) and \( e^\mu \) are the photon momentum and polarization vector, respectively. Choosing the three-dimensional transversal gauge and performing integration over the time variable, we get

\[ S_{fi} = ie \sqrt{\frac{2\pi}{\omega L^3}} 2\pi \delta(E - E' - \omega) \int d^3x \bar{\psi}_f (r)(\gamma e^r) e^{ikr} \psi_i (r), \quad (8) \]

The \( \delta \)-function in the last expression stands for the energy conservation law with \( E \) and \( E' \) being energies of the initial and final neutrino states. Performing further integration over the spatial coordinates we get also the momentum conservation law, which together with the energy conservation law,

\[ E = E' + \omega, \quad p = p' + k, \quad (9) \]

leads to the only possible transition of the electron when its chirality changes from \( s_i = -1 \) to \( s_f = 1 \). The corresponding photon energy then is given by the expression [1]:

\[ \omega = \frac{2\alpha_n m_e p[\bar{E} - (p + \alpha_n m_e) \cos \theta]}{(\bar{E} - p \cos \theta)^2 - (\alpha_n m_e)^2}, \quad (10) \]
where \( \theta \) is the angle between the directions of initial neutrino and emitted photon propagation. We also use the following notation \( \tilde{E} = E - \alpha_n m_e \). In the case of relativistic electrons and small values of the matter density parameter \( \alpha_n \) the photon energy is

\[
\omega_{SLe} = \frac{1}{1 - \beta_e \cos \theta} \omega_0, \quad \omega_0 = \frac{G_F}{\sqrt{2}} \alpha_n \beta_e,
\]

(11)

here \( \beta_e \) is the electron speed in vacuum. From this expressions we conclude that for the relativistic electron the energy range of the SLe may even extend up to energies peculiar to the spectrum of gamma-rays. We also predict the existence of the electron-spin polarization effect in this process.

4 Rate and power of the radiation

With the expressions for the amplitude (8) and the emitted photon energy (10) we arrive to general formulas for the total rate and power of the radiation:

\[
\Gamma = \frac{e^2}{2} \int_0^\pi \frac{\omega}{1 + \beta_{te}^2} \left( 1 - \beta_e \bar{\beta}_e - \frac{m_e^2}{E'} \right) \left( 1 - y \cos \theta \right) \sin \theta d\theta,
\]

(12)

\[
I = \frac{e^2}{2} \int_0^\pi \frac{\omega^2}{1 + \beta_{te}^2} \left( 1 - \beta_e \bar{\beta}_e - \frac{m_e^2}{E'} \right) \left( 1 - y \cos \theta \right) \sin \theta d\theta,
\]

(13)

where

\[
\tilde{\beta}_e = \frac{p + \alpha_n m_e}{\tilde{E}}, \quad \tilde{\beta}_e' = \frac{p' - \alpha_n m_e}{E'},
\]

(14)

are the quantities, describing the initial and final electron group velocities, and

\[
E' = E - \omega, \quad p' = K_e \omega - p,
\]

\[
K_e = \frac{\tilde{E} - p \cos \theta}{\alpha_n m_e}, \quad y = \frac{\omega - p \cos \theta}{p'}.
\]

(15)

Taking integration in (12) and (13) we obtain:

\[
\Gamma = e^2 \frac{m_e^2 \left[ (1 + 2\alpha_n^2) m_e^2 + 2p^2 \right]}{4p^2 (4 \alpha_n p + m_e)^2 \sqrt{(p + m_e \alpha_n)^2 + m_e^2}} \times \left[ (4 \alpha_n p + m_e)^2 \ln \left( 1 + \frac{4 \alpha_n p}{m_e} \right) - 4 \alpha_n p (m_e + 6 \alpha_n p) \right],
\]

(16)

and

\[
I = e^2 \frac{m_e^2}{2p^2 (4 \alpha_n p + m_e)^2} \left\{ \left( 1 + \alpha_n^2 \right) m_e^2 + p^2 \right\} (4 \alpha_n p + m_e)^3 \ln \left( 1 + \frac{4 \alpha_n p}{m_e} \right) - \frac{4}{3} \alpha_n p \left[ 88 \alpha_n^2 p^4 + 3 (1 + \alpha_n^2) m_e^4 + 30 \alpha_n (1 + \alpha_n^2) m_e^3 p + p^2 m_e^2 (3 + 88 \alpha_n^2 (1 + \alpha_n^2)) + 2 \alpha_n (15 + 16 \alpha_n^2) m_e p^3 \right].
\]

(17)
Let us estimate the total rate $\Gamma$ of the radiation, and also the corresponding lifetime $T_{SLe}$ of the electron in respect to the considered process using expression (16). Consider the electron with momentum $p = 1\ MeV$ moving in matter characterized by the number density $n_n \sim 10^{37}\ cm^{-3}$; in this case the matter density parameter is $\alpha_n = 0.6 \times 10^{-6}$. Then, for the rate of the process we get $\Gamma \sim 3.2 \times 10^{-10} MeV$ which corresponds to the characteristic life-time of the electron $T_{SLe} \sim 2 \times 10^{-2}\ s$.

Finally, we have developed the approach to description of the matter influence on an electron which is based on the exact solutions of the correspondent modified Dirac equation for the particle wave function. The approach developed (we have previously used it for the case of neutrino) is similar to the Furry representation in quantum electrodynamics. Note that our focus has been on the standard model interactions of electrons with the background matter. A similar approach, which implies the use of the exact solutions of the correspondent modified Dirac equations, can be developed in the case when electrons interact with different external fields predicted within various extensions of the standard model (see, for instance, [11, 12] and the paper of V.Zhukovsky et al in this book). We have predicted and investigated in some detail a new type of electromagnetic radiation (the spin light of electron in matter, $SLe$) that can be emitted by the electron due its magnetic moment within the standard model of interaction with the background matter. The obtained $SLe$ energy spectrum shows that for ultra-relativistic electrons it can even extend up to energy range peculiar to the spectrum of gamma-rays. Comparing the rates of the spin light of neutrino and spin light of electron in matter, we have predicted (see also [1]) that the latter is more effective then the former. We have predicted also that the $SLe$ emitted by ultra-relativistic electrons moving in dense astrophysical and cosmological media can have consequences accessible for experimental observations.

References