GRAIN ALIGNMENT AND POLARIZED EMISSION FROM MAGNETIZED T TAURI DISKS

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ABSTRACT

The structure of magnetic fields within protostellar disks may be studied via polarimetry provided that grains are aligned in respect to magnetic field within the disks. We explore alignment of dust grains by radiative torque in T Tauri disks and provide predictions for polarized emission for disks viewed at different wavelengths and viewing angles. We show that the alignment is especially efficient in outer part of the disks. In the presence of magnetic field, these aligned grains produce polarized emission in infrared wavelengths. We consider a simple model of an accretion disk and provide predictions for polarization that should be available to both instruments that do not resolve the disks and future instruments that will resolve the disks. As the surface magnetic field and the bulk magnetic field play different roles for the disk dynamics, we consider separately the contributions that arises from the surface areas of the disk and its interior. We find that the polarized emission drops for wavelengths shorter than $\sim 10\mu m$. Between $\sim 10\mu m$ and $\sim 100\mu m$, the polarized emission is dominated by the emission from the surface layer of the disks and the degree of polarization can be as large as $\sim 10\%$ for unresolved disks. We find that the degree of polarization at these wavelengths is very sensitive to the size distribution of dust grains in disk surface layer, which should allow testing the predicted grain size distributions. The degree of polarization in the far-infrared/sub-millimeter wavelengths is sensitive to the size distribution of dust grains in the disk interior. When we take a Mathis-Rumpl-Nordsieck-type distribution with maximum grain size of 500-1000 $\mu m$, the degree of polarization is around 2-3 $\%$ level at wavelengths larger than $\sim 100\mu m$. Our study indicates that multifrequency infrared polarimetric studies of protostellar disks can provide good insights into the details of their magnetic structure.

Subject headings: accretion, accretion disks — circumstellar matter — polarization — stars: pre-main-sequence — dust, extinction

1. INTRODUCTION

Magnetic field plays important roles in star formation, as well as formation and evolution of protostellar disks. Magnetic pressure can provide extra support to the disks and magnetic field can promote removal of angular momentum from disks (see Velikov 1959; Chandrasekhar 1961; Balbus & Hawley 1991). However, there are many uncertainties for the structure and effects of the magnetic field in protostellar disks.

Infrared (IR) polarimetry may be an important tool to investigate magnetic field structure in protostellar disks, provided that the grains are aligned in the disks in respect to magnetic field. Since the grain emissivity is larger for the long axis of a grain, emitted radiation has a polarization vector parallel to the grain’s long axis. If grains are aligned with their long axes perpendicular to magnetic field, the resulting grain emission has polarization directed perpendicular to the magnetic field. Therefore, by measuring the direction of polarization of IR emission from dust grains one can infer the direction of magnetic field. The key question thus is whether grain alignment is efficient in protostellar disks.

The notion that the grains can be aligned in respect to magnetic field can be traced back to the discovery of star-light polarization by Hall (1949) and Hiltner (1949), that arises from interstellar grains. Historically the theory of the grain alignment was developing mostly to explain the interstellar polarization, but grain alignment is a much wider spread phenomenon (see Lazarian 2003 for a review). Among the alignment mechanisms the one related to radiative torques (RTs) looks the most promising. We invoke it for our calculations below.

The RTs make use of interaction of radiation with a grain to spin the grain up. The RT alignment was first discussed by Dolginov (1972) and Dolginov & Mytrophanov (1976). However, quantitative studies were done only in 1990’s. In their pioneering work, Draine & Weingartner (1996, 1997, henceforth DW96 and DW97) demonstrated the efficiency of the RT alignment for a few arbitrary chosen irregular grains using numerical simulations. This work identified RTs as potentially the major agent for interstellar grain alignment. Cho & Lazarian (2005, henceforth CL05) demonstrated the rapid increase of radiative torque efficiency and showed that radiative alignment can naturally explain decrease of the degree of polarization near the centers of pre-stellar cores. Large grains are known to be present in protostellar disk environments and this makes the RT alignment promising.

The effect of RTs is two-fold. They can spin-up grains and drive the alignment. While the details of the second
process are a subject of intensive research (see Weingartner & Draine 2003; Lazarian & Hoang 2006; Hoang & Lazarian 2006), for our estimates we use the RTs spin-up efficiency to evaluate the efficiency of grain alignment. As grains of different temperatures are present in protostellar disks, the differential alignment of grains at different optical depths is expected to show itself through variations of polarization at different wavelengths.

Protostellar disks are often detected through far-infrared excess. Dust grains in the protostellar disks are the main cause of the infrared excess - dust grains absorb stellar radiation and re-emit at infrared wavelengths. The spectral energy distribution (SED) of the emitted light gives valuable information about the disk structure. Recently proposed hydrostatic, radiative equilibrium passive disk model (Chiang & Goldreich 1997; Chiang et al. 2001; hereafter CG97 and C01, respectively) fits observed SED from T Tauri stars very well and seems to be one of the most promising models. Here, passive disk means that active accretion effect, which might be very important in the immediate vicinity of the central star, is not included in the model. In this paper we use the model in C01.

Recently Aitken et al. (2002) studied polarization that can arise from magnetized accretion disks. They considered a single grain component consisting of the 0.6 $\mu$m silicate and assumed that grains were partially aligned with $R = 0.25$, where $R = < 3 \cos^2 \beta - 1 > /2$ is the Rayleigh reduction factor (Greenberg 1968). Here $\beta$ is the precession angle between the grain spin axis and the magnetic field and the angle brackets denote average (see Aitken et al. 2002 for details). In this paper we use both a theoretically motivated model of grain alignment and a more sophisticated model of accretion disk.

We should mention that challenges of observing polarization in mid-IR to far-IR (FIR) wavelengths have been dealt with successfully recently. For instance, Tamura et al. (1999) first detected polarized emission from T Tauri stars, low mass protostars. As technology develops, the future of polarimetric IR and submillimeter emission studies looks very promising. In this paper we try to theoretically predict polarized emission from T Tauri disks at different wavelengths and for different inclinations of the disks.

We calculate grain alignment by radiative torque using a T Tauri disk model in C01 and we predict polarized mid-IR/FIR/sub-millimeter emission. In §2, we discuss grain alignment in T Tauri disks. In §3, we give theoretical estimates for degree of polarization. In §4, we calculate the spectral energy distribution of maximally polarized emission, which will be useful only when we spatially resolve disks. In §5, we discuss the effect of inclination angle. In §6, we discuss observational implications. We give summary in §7.

2. GRAIN ALIGNMENT IN PROTOSTELLAR DISKS

2.1. The disk model used for this study

We assume that magnetic field is regular and toroidal (i.e. azimuthal). We use a T Tauri disk model in C01. Figure 1 schematically shows the model. The disk is in hydrostatic and radiative equilibrium (see also CG97) and shows flaring. According to CG97, flaring of disk is essential for correct description of SED. They considered a two-layered disk model. Dust grains in the surface layer are heated directly by the radiation from the central star and emit their heat more or less isotropically. Half of the dust thermal emission immediately escapes and the other half enters into disk interior and heats dusts and gas there. Both CG97 and C01 assumed that the disk interior is isothermal.

In both CG97 and C01, the disk surface layer is hotter than the disk interior. Thus, roughly speaking, the surface layer dominates in mid-infrared wavelengths and disk interior dominates in far-infrared/sub-millimeter wavelengths. The disk surface layer is both optically and physically thin.

The major difference between CG97 and C01 is the treatment of dust grain size distribution. CG97 assumed that all grains have a fixed size of 0.1 $\mu$m, while C01 assumes an MRN distribution (Mathis, Rumpl, & Nordsieck 1977) with maximum grain size of $a_{\text{max,i}} = 1000 \mu$m in the disk interior and $a_{\text{max,s}} = 1 \mu$m in the disk surface layer.

In our calculations, we use a grain model similar to that in C01. We use an MRN-type power-law distribution of grain radii $a$ between $a_{\text{min}} (= 0.01 \mu$m for both disk interior and surface layer) and $a_{\text{max}} (= 1000 \mu$m for disk interior and = 1 $\mu$m for disk surface layer) with a power index of -3.5: $dN \propto a^{-3.5} da$. As in C01 we assume that grain composition varies with distance from the central star in both disk interior and surface layer. We assume that grains in the surface layer are made of silicate only when the distance $r$ is less than 6 AU, and silicate covered with water ice when $r > 6$ AU. We do not use iron grains for the immediate vicinity of the star. We assume that grains in the disk interior are made of silicate when $r < 0.8$ AU and ice-silicate for $r > 0.8$ AU. The fractional thickness of the water ice mantle, $\Delta a/a$, is set to 0.4 for both disk surface and disk interior. Unlike C01, we use the refractive index of astronomical silicate (Draie & Lee 1984; Draie 1985; Loar & Draie 1993; see also Weingartner & Draie 2001). We take optical constants of pure water ice from a NASA web site (ftp://climate.gsfc.nasa.gov/wiscombe).

The column density of the disk is $\Sigma_0 r_{AU}^{-3/2}$ with $\Sigma_0 = 1000 g/cm^2$. Here $r_{AU}$ is distance measured in AU. The disk is geometrically flared and the height of the disk surface is set to 4 times the disk scale height $h$. The disk inner radius is $2 R_\ast$ and the outer radius is 100AU. The central star has radius of $R_\ast = 2.5 R_{\odot}$ and temperature of $T_\ast = 4000 K$. Temperature profile, flaring of disk, and other details of the disk model are described in C01.

2.2. Radiative torque for large grains

For most of the ISM problems, dust grains are usually smaller than the wavelengths of interest. However, this is no longer true in T Tauri disks because we are dealing with grains as large as $\sim 1000 \mu$m. To understand grain alignment in T Tauri disks we need to know radiative torque for large grains.

In this study, we do not directly calculate radiative torque for large grains. Instead, we use a simple scaling relation to model radiative torque for large grains.
In CL05, we used the DDSCAT software package (astro-ph/0309069 Draine & Weingartner 1996) to calculate radiative torque on grain particles and showed the relation between $\lambda Q_T$ and $\lambda/a$ for grains with radii between 0.1$\mu$m and 3.2$\mu$m. Here $Q_T$ is the radiative torque efficiency. Figure 2 obtained by reprocessing the earlier relation, shows that the radiative torque

$$Q_T = \begin{cases} \sim \mathcal{O}(1) & \text{if } \lambda \sim a, \\ \sim (\lambda/a)^{-3} & \text{if } \lambda > a, \end{cases}$$

(1)

where $a$ is the grain size and $\lambda$ the wavelength of the incident radiation. Note that the radiative torque peaks near $\lambda \sim a$ and that its value is of order unity there. A more general study on this issue is provided in Lazarian & Hoang (2006). This allows us to assume that the relation holds true both for small and large grains.

Fig. 1.— A schematic view of the disk model (see C01). The surface layer is hotter and heated by the star light. The disk interior is heated by re-processed light from the surface layers. We assume that the disk height, $H$, is 4 times the disk scale height, $h$.

Fig. 2.— Behavior of Torque. Torque is $\sim \mathcal{O}(1)$ when $\lambda \sim a$, where $a$ is the grain size. Roughly speaking, torque $\propto (\lambda/a)^{-3}$.

### 2.3. Rotation rate of dust grains by radiative torque

After some modifications, equation (67) in Draine & Weingartner (1996) reads

$$\left(\frac{\omega_{\text{rad}}}{\omega_T}\right)^2 = 4.72 \times 10^9 \frac{\alpha_1}{\delta^2} \rho_3 a^{-5} \left(\frac{u_{\text{rad}}}{n_H k T}\right)^2 \left(\frac{\lambda}{\mu m}\right)^2 [Q_T]^2 \left(\frac{\tau_{\text{drag}}}{\tau_{\text{drag, gas}}}\right)^2,$$

(2)

where $Q_T = Q_T \cdot \hat{a}_1$ and $\hat{a}_1$ is the principal axis with largest moment of inertia, $n_H$ is the hydrogen number density, $u_{\text{rad}}$ is the energy density of the radiation field, $\delta \approx 2$, $\alpha_1 \approx 1.745$, $\rho_3 = \rho h/3 g cm^{-3}$, $a_0 = a/10^5 cm$, and $\omega_T$ is the thermal angular frequency, which is the rate at which the rotational kinetic energy of a grain is equal to $kT/2$. The timescales $\tau_{\text{drag, gas}}$ and $\tau_{\text{drag, em}}$ are the damping time for gas drag and for electromagnetic emission, respectively, and they satisfy the relation $\tau_{\text{drag}}^{-1} = \tau_{\text{drag, em}}^{-1} + \tau_{\text{drag, gas}}^{-1}$ (see Draine & Weingartner (1996) for details). As we discussed in the previous subsection, $Q_T$ is of order of unity when $\lambda \sim a$ and declines as $(\lambda/a)$ increases. From this observation, we can write

$$\left(\frac{\omega_{\text{rad}}}{\omega_T}\right)^2 \approx \left(\frac{\omega_{\text{rad}}}{\omega_T}\right)_{\lambda=a} \left(\frac{Q_T}{Q_T, \lambda=a}\right)^2 \approx \left(\frac{\omega_{\text{rad}}}{\omega_T}\right)_{\lambda=a} \left(\frac{\lambda}{a}\right)^{-6}$$

(3)

for $\lambda > a$, where

$$\left(\frac{\omega_{\text{rad}}}{\omega_T}\right)_{\lambda=a} \approx 4.72 \times 10^9 \frac{\alpha_1}{\delta^2} \rho_3 a^{-5} \left(\frac{u_{\text{rad}}}{n_H k T}\right)^2 \left(\frac{\lambda}{\mu m}\right)^2 \left(\frac{\tau_{\text{drag}}}{\tau_{\text{drag, gas}}}\right)^2,$$

(4)

### 2.4. Minimum and maximum aligned grain size

As in Draine & Weingartner (1996), we assume that grains are aligned when $(\omega_{\text{rad}}/\omega_T)^2 > 10$. Suppose that a monochromatic radiation field illuminates dust grains and that $(\omega_{\text{rad}}/\omega_T)^2 > 10$ for $\lambda \sim a$. According to Eq. (1), the ratio $(\omega_{\text{rad}}/\omega_T)^2$ decrease as $a$ decreases and the ratio will drop below $\sim 10$. Applying Eq. (3) we can easily find the minimum aligned grains size:

$$a_{\text{lower}} \sim \lambda \left(\frac{10}{(\omega_{\text{rad}}/\omega_T)^2}_{\lambda=a}\right)^{1/6}$$

(5)
Then, how can we find the maximum aligned grain size? In other words, are all grains with \( a > \lambda \) aligned? In order to answer this question, we need to consider the behavior of torque \( Q_T \) for the limit \( a \gg \lambda \). Note that, when \( \lambda \sim a, Q_T \sim O(1) \). Then what happens when \( a \gg \lambda \)? In principle, we can calculate \( Q_T \) using numerical simulations. However, this is still infeasible because enormous computational power is required. Therefore we can only conjecture what will happen for \( a \gg \lambda \).

If one adopts the reasoning for the origin of RT in Dolginov & Mytrophanov (1976) one might expect that \( Q_T \) drops due to incoherent contributions. Theoretical considerations in Lazarian & Hoang (2006) show that this may not be true. While to be conservative we adopt a rule of thumb is that \( Q_T \) may begin to decline when \( a \geq 10 \lambda \), we will see below that this assumption does not alter our results in any appreciable way\(^1\).

We also need to consider many different time-scales: precession time-scale, time-scale for the alignment of angular momentum and \( \mathbf{a}_1 \), etc. However, fortunately, the exact knowledge on the maximum aligned grains size is not so important for our current study (see next section).

2.5. Grain alignment in disks

We use eq. (1), instead of the DDSCAT software package, to obtain radiative torque on grain particles in the T Tauri disk described in \( 2 \). We take a conservative value of \( Q_T \) at \( \lambda \sim a \): \( Q_T \sim O(1) \). Apart from \( Q_T \), we also need to know \( u_{rad} \) and \( n_H \) to get the \( (\omega_{rad}/\omega_T)^{3/2} \) ratio (see Eq. (2)). We assume that \( \tau_{drag} \sim \tau_{drag, gas} \).

In the disk interior, there are two kinds of radiation fields: one from the surface layer and the other from the disk interior itself. We assume that both radiation fluxes direct only along the disk vertical axis (i.e. “\( z \)” axis). As in CG97, we assume that half of the stellar radiation flux that reaches the disk surface enters the disk interior. The radiation flux from the star is \( \sim (\alpha/2)(R_*/r)^2\sigma_B T_*^4 \), where \( \alpha \) is the grazing angle at which the starlight strikes the disk and \( R_* \) is the stellar radius, \( T_* \) is the stellar temperature, and \( \sigma_B \) is the Stephan-Boltzmann constant (CG97). We assume that the radiation flux from the surface layer has a narrow spectrum around the wavelength of \( \sim 3000/T_{ds} \) \( \mu m \), where \( T_{ds} \) is the grain temperature in the surface layer (see C01 for a graph for \( T_{ds} \)). The magnitude of the flux from the surface at a given height is less than the half of the incident stellar flux because the flux from the surface is attenuated by dust absorption in disk interior. We also need to consider that there are two surface layers - one above the mid-plane and the other below it. The flux from disk interior is also treated as a monochromatic wave with \( \lambda \sim 3000/T_i \) \( \mu m \).

Note that flux from the interior has longer wavelengths because the disk interior is cooler (see C01 for a graph for the disk interior \( T_i \)). From the two fluxes, we can calculate anisotropic component of the radiation energy density. We use the fact that the disk surface density is given by \( \Sigma = 1000 \mu m/\text{cm}^2 \) and that the height of the disk \( H(r) \) can be calculated from

\[
H/r \sim 4\sqrt{T_i/T_e}\sqrt{r}/R_*
\]

where \( T_e = 3 \times 10^{-24}GM_*/kR_* \) in cgs units. Here \( k \) is the Boltzmann constant.

We assume that grains are aligned when the ratio \( (\omega_{rad}/\omega_T)^{3/2} \) exceeds 10, which is overly conservative according to a recent study in Hoang & Lazarian (2006). Figure 3 shows that, at large \( r \), most grains are aligned even deep inside the interior. On the other hand, at small \( r \), only grains near the disk surface are aligned. This is because gas density is low and, therefore, the drag is smaller near the surface layer. The lower panel of Figure 3 shows that almost all grains are aligned when \( r \gg 10 \text{AU} \). Therefore, we expect strong polarized emission from outer part of the disk.

In the surface layer, grains are aligned by the star-light. Note that the radiation field scales as \( r^{-2} \). Since the column density scales as \( r^{-3/2} \) and the disk height is an increasing function of \( r \), the density in disk surface will drop faster than \( r^{-3/2} \). The gas temperature in the surface layer roughly scales as \( r^{1/2} \) (see CG97 and C01). Therefore we expect that the ratio \( \omega_{rad}/\omega_T \) is an increasing function of \( r \) (see eq. (4)). This implies that grains at large \( r \) are aligned. Grains near the central star cannot be aligned due to high gas density near the star. Indeed Figure 4 shows the ratio \( (\omega_{rad}/\omega_T)^{3/2} \) exceeds 10 when \( r \geq 1 \text{AU} \). This means that grains in the surface layer are aligned when \( r \geq 1 \text{AU} \). Note that the radiation from the central star has a \( \lambda_{max} \) at \( \sim 750 \text{nm} \). We expect that polarized emission from the surface layer is also originated from outer part of the disk.

3. THEORETICAL ESTIMATES OF DEGREE OF POLARIZATION

In this section, we estimate maximum degree of polarization of emitted radiation in IR wavelengths. We need only the grain size distribution for the calculation in this section. We do not use a detailed disk model. The discussion in this section is applicable for any system with large grains.

3.1. Maximum aligned size

3.2. Simple estimates for maximum degree of polarization

Suppose that all grains are perfectly aligned. Then, what will be the observed degree of polarization? Of course, we do not observe 100% polarization. The parameter \( 2\pi a/\lambda \) plays an important role here. There are three important points that determine the degree of polarization of emitted radiation.

First, when \( 2\pi a/\lambda < 1 \) (i.e. in the Rayleigh limit), grain’s intrinsic shape gives a limit. Suppose that grains are oblate spheroid, that the symmetry axis of the grain is parallel to \( y \)-axis, and that radiation is propagating along

\(^1\) Note that here \( \lambda \) is the wavelength at which radiation is strongest. That is, for most cases, \( \lambda = \lambda_{max} \propto 1/T \) when the radiation field is a black body radiation.
Grain alignment in disk interior. Upper panel: Contours show the ratio \( \left( \frac{\omega_{\text{rad}}}{\omega_{\text{T}}} \right)^2 \sim a \). Note that the disk vertical height is shown in units of the disk scale height \( h \). We assume that grains are aligned when \( \left( \frac{\omega_{\text{rad}}}{\omega_{\text{T}}} \right)^2 > 10 \). Lower panel: Fraction of aligned grains as a function of disk radius. After \( r > 10\text{AU} \), almost 100% of grains are aligned.

Grain alignment in surface layer. The ratio \( \left( \frac{\omega_{\text{rad}}}{\omega_{\text{T}}} \right)^2 \sim a \) exceeds 10 when \( r \geq 1\text{AU} \), which means that grains in the surface layer are aligned when \( r \geq 1\text{AU} \).

Relation between the absorption cross-section (= \( Q_{\text{abs}} \) times the geometric cross-section) and grain size. The wavelength at which we observe is 850 \( \mu \text{m} \). Grains are oblate spheroid made of silicate with the symmetry axis \( \hat{a}_1 \) parallel to y-axis. Axis ratio is 1:1.3. Radiation is propagating along x-axis. Left panel: When grains are smaller than \( \sim \frac{850}{2\pi} \), then we can observe polarization because two cross-sections are different. When grains are larger than \( \sim \frac{850}{2\pi} \), then we cannot observe polarization because two cross-sections are similar. Right panel: When the axis ratio is 1:1.3, the ratio of \( Q_{\text{abs}} \) is around \( \sim 1.6 \) for \( \lambda > 2\pi a \) (i.e. the Rayleigh limit) and \( \sim 1 \) for \( \lambda < 2\pi a \).

When we send two linearly polarized radiation fields with electric field parallel to and perpendicular to the grain’s symmetry axis, respectively, the radiation fields experience different cross-sections: the radiation with \( E \parallel \hat{a}_1 \) sees smaller cross-section. Here \( E \) is the electric field of the radiation field and \( \hat{a}_1 \) is the grain’s symmetry axis. As a result, we observe polarization because two cross-sections are different. So far we have dealt with polarization by absorption. Polarization by emission is caused by the exactly same fact that two cross-sections are different. Since \( Q_{\text{abs}} = Q_{\text{em}} \), where \( Q_{\text{em}} \) is the grain emissivity, grain emits more radiation with \( E \perp \hat{a}_1 \) than one with \( E \parallel \hat{a}_1 \). The degree of polarization of emitted light is \( (Q_{\text{abs},\perp} - Q_{\text{abs},\parallel})/(Q_{\text{abs},\perp} + Q_{\text{abs},\parallel}) \), where \( \parallel \) and \( \perp \) refer to directions parallel and perpendicular to the grain’s symmetry axis \( \hat{a}_1 \).

Figure 5 obtained from the DDSCAT package, shows this effect clearly. The radiation fields have \( \lambda = 850\mu\text{m} \) and the grains’ long to short axis ratio is 1:3:1. The ratio of two cross-sections is around 1.6 for \( 2\pi a/\lambda < 1 \). When we observe emission from those grains, the degree of polarization can be as large as \( (1.6-1)/(1.6+1)=22\% \). If we assume that grains’ long to short axis ratio is 1.5:1, the ratio of the cross-sections is 2:1 and the resulting degree of polarization for emission is as large as \( (2.1-1)/(2.1+1)=35\% \). The ratio of the cross-sections varies as wavelength of radiation varies. The ratio seems to be fairly constant for \( \lambda > 100\mu\text{m} \). However, for shorter wavelengths, the ratio decreases (Figure 6). The shape of grains in protostellar disk is uncertain (see, for example, Hildebrand & Dragovan 1995 for the general ISM cloud cases).

Second, when \( 2\pi a/\lambda > 1 \) (i.e. outside the Rayleigh limit), we do not observe polarization. Figure 5 clearly shows that the ratio of cross-sections becomes very close to 1 when \( 2\pi a/\lambda > 1 \). This means that the usual argument about polarization by absorption or emission works only for the Rayleigh limit: \( \lambda > 2\pi a \). That is, in the Rayleigh limit,
we are concerned only with the absolute magnitude of the polarization.  

be zero for a face-on disk when magnetic field is perfectly azimuthal and the disk is cylindrically symmetric. In this section, we assume that the disk is face-on. The degree of polarization will smaller values because not all grains are aligned and some part of the disk is optically thick.

when radiation meets an elongated grain, it recognizes the elongated shape and interacts differently depending on the direction of the electric field of the radiation. However, when \( \lambda < 2\pi a \), radiation do not recognize that grains are elongated. We can easily understand this fact when we consider an elongated macroscopic object: Cross-sections are same regardless of the electric field directions for visible light. This second point is somewhat tricky: Even in the case grains are actually “aligned”, we do not “observe” polarization when the Rayleigh limit is violated.

This observation has an important consequence. When we calculate polarization, large grains (i.e. grains with \( a > \lambda/2\pi \)) do not contribute to polarization even when they are aligned. This fact reduces the degree of polarization significantly in protostellar disks. For example, suppose that we has grains as large as 1000\( \mu \)m in a disk. If we observe emission from the disk at \( \lambda = 850\mu \)m, grains with \( a > 850/2\pi \sim 100\mu \)m do not contribute polarization although they dominate extinction when the grain size distribution is a power-law (\( dN \propto a^{-q} da \)) with \( q < 3 \).

Thirds, if the medium is optically thick, the degree of polarization reduces. The intensity of radiation from a uniform slab is \( S_\nu(1 - \exp(-\tau)) \), where \( S_\nu \) is the source function and \( \tau \) is the optical depth. We can observe polarization when \( \tau \), is different for parallel and perpendicular (to grain symmetry axis or generally any spatial direction) directions. However, in the optically thick limit, the intensity becomes equal to \( S_\nu \). Therefore, difference in optical depth does not produce observable level of polarization when the slab is opaque.

For the disk interior, let us consider only the first and the second points mentioned above. That is, let us assume that the grains are perfectly aligned and the disk is optically thin. We can estimate the degree of polarization \( p \) for the disk interior by integrating the following:

\[
 p(\lambda) = \frac{\int_{a_{\text{min}}}^{a_{\text{max}}} \left( -\frac{Q_{\text{abs},\perp}(a) - Q_{\text{abs},\parallel}(a)}{Q_{\text{abs},\perp}(a) + Q_{\text{abs},\parallel}(a)} \right) a^2 N(a) da}{\int_{a_{\text{min}}}^{a_{\text{max}}} \left( -\frac{Q_{\text{abs},\perp}(a) + Q_{\text{abs},\parallel}(a)}{Q_{\text{abs},\perp}(a) + Q_{\text{abs},\parallel}(a)} \right) a^2 N(a) da},
\]

where \( Q_{\text{abs}} \) is grain absorption efficiency, or grain emissivity, and \( a_{\text{min}} = 0.01\mu \)m. In left panel of Figure 6 we use \( a_{\text{max}} = 1000\mu \)m. We use \( dN \propto a^{-q} da \) with \( q = 3.5 \) and assume that grains are oblate sphere (to long short axis ratio of 1.5:1). It is not surprising that the degree of polarization rises when we use smaller \( a_{\text{max}} \): when \( a_{\text{max}} \) is smaller, more grains are outside the Rayleigh limit. Right panel of Figure 4 shows this effect.

Note that we do not use actual disk models here. Actual numerical simulations using actual disk models will give smaller values because not all grains are aligned and some part of the disk is optically thick.

Left panel of Figure 6 shows that the maximum degree of polarization for \( \lambda \leq 100\mu \)m is slightly larger than that for \( \lambda \geq 100\mu \)m. However, in reality, we do not expect a significant degree of polarization for \( \lambda < 100\mu \)m. The reason is that the whole disk becomes optically thick for \( \lambda < 100\mu \)m. The opacity per unit mass for \( \lambda = 100\mu \)m is around \( \sim O(0.1) \) (see Figure 3 in C01). The outer most disk has column density of \( 1\text{g/cm}^2 \). Therefore, even the outer-most part of the disk becomes optically thick when the wavelength drops below \( \sim 100\mu \)m.

For the disk surface layer, the second point mentioned above is irrelevant because grains are smaller in the surface layer: \( a_{\text{max}} = 1\mu \)m. The third point is also irrelevant because the surface layer is optically thin at far-infrared and submillimeter wavelengths. Therefore, if all grains are perfectly aligned, the degree of polarization of the emitted radiation is determined only by the grain shapes (see the first point above).

4. ESTIMATES FOR SPECTRAL ENERGY DISTRIBUTION

In this section, we calculate the degree of polarization of emitted infrared radiation from a disk with structure and parameters described in C01. In this section, we assume that the disk is face-on. The degree of polarization will be zero for a face-on disk when magnetic field is perfectly azimuthal and the disk is cylindrically symmetric. In this section, we are concerned only with the absolute magnitude of the polarization.

4.1. Spectral energy distribution
Fig. 7.— Expected maximum degree of polarization for disk interior. *Left panel*: $a_{\text{max}} = 1000\mu$m. We assume grains are oblate spheroids with a size ratio of 1.5:1. *Right panel*: The degree of polarization is very sensitive to the maximum grain size, $a_{\text{max}}$. Results are for ice-silicate. When $a_{\text{max}}$ is smaller, more grains are outside the Rayleigh limit and, hence, the degree of polarization rises for a given observing wavelength.

When $(\omega_{\text{rad}}/\omega_T)^2_{\lambda=\omega}$ is larger than 10, we find the minimum aligned size from Eq. (5):

$$a_{\text{lower}} \sim \left( \frac{10}{(\omega_{\text{rad}}/\omega_T)^2_{\lambda=\omega}} \right)^{1/6} \lambda_{\text{max}}, \quad (8)$$

where $\lambda_{\text{max}}$ is the peak wavelength of the aligning radiation. Grains smaller than $a_{\text{lower}}$ are not aligned.

When $(\omega_{\text{rad}}/\omega_T)^2_{\lambda=\omega}$ is larger than 10, we find the maximum size of grains that give rise to polarization from

$$a_{\text{upper}} = \min[10\lambda_{\text{max}}, \lambda_{\text{obs}}/2\pi], \quad (9)$$

where $\lambda_{\text{obs}}$ is the observing wavelength. In most cases, $a_{\text{upper}} = \lambda_{\text{obs}}/2\pi$ because $\lambda_{\text{max}}$ falls in far-infrared wavelengths. Note again that the actual maximum aligned size can be larger than $a_{\text{upper}}$.

Now we know $a_{\text{upper}}$ and $a_{\text{lower}}$. Note that $a_{\text{upper}}$ and $a_{\text{lower}}$ are functions of the distance to the central star, $r$, and the distance to the disk mid-plane, $z$. We calculate the parallel (with respect to the local magnetic field) and perpendicular opacity respectively:

$$\tau_{//}(r,z) \propto \int_{a_{\text{min}}}^{a_{\text{max}}} Q_{\text{abs}}(\pi a^2)N(a)f_{//}da, \quad (10)$$

$$\tau_{\perp}(r,z) \propto \int_{a_{\text{min}}}^{a_{\text{max}}} Q_{\text{abs}}(\pi a^2)N(a)f_{\perp}da, \quad (11)$$

where $N(a)da \propto a^{-3.5}$ (i.e. an MRN-type distribution) and

$$f_{//} = \begin{cases} 0.77 \text{ (or } 0.65) & \text{if } a_{\text{lower}} < a < a_{\text{upper}} \\ 1 & \text{otherwise} \end{cases}, \quad (12)$$

and

$$f_{\perp} = \begin{cases} 1.23 \text{ (or } 1.35) & \text{if } a_{\text{lower}} < a < a_{\text{upper}} \\ 1 & \text{otherwise} \end{cases}, \quad (13)$$

where we assume that the long to short axis ratio of the oblate spheroid is 1.3:1 (or 1.5:1).

From this, we can calculate emission in parallel and perpendicular directions:

$$L_{\lambda,//} \propto \lambda \int_{r_{\text{min}}}^{r_{\text{max}}} dr \int_{-4h}^{4h} dz \frac{d\tau_{//}}{dz} e^{-\tau_{//}} B_{\lambda}(T), \quad (14)$$

$$L_{\lambda,\perp} \propto \lambda \int_{r_{\text{min}}}^{r_{\text{max}}} dr \int_{-4h}^{4h} dz \frac{d\tau_{\perp}}{dz} e^{-\tau_{\perp}} B_{\lambda}(T), \quad (15)$$

where $\tau_{//}$ and $\tau_{\perp}$ measure optical depths from $z$ to $4h$ along the axis perpendicular to the disk mid-plane (see CG97). We use BHCOAT.f and BHMIE.f codes in Bohren & Huffman (1983) to calculate grain emissivity $Q_{\text{abs}}$.

The degree of polarization is

$$p(\lambda) = (L_{\lambda,\perp} - L_{\lambda,//})/(L_{\lambda,\perp} + L_{\lambda,//}). \quad (16)$$

Figure 7 (left panel for 1.3:1 oblate spheroid and right panel for 1.5:1 oblate spheroid) shows the results. The degree of polarization can be as large as $\sim 5\%$ in FIR/sub-millimeter wavelengths and $\sim 10\%$ in mid-IR regimes. The polarized emission at FIR is dominated by the disk interior and that at mid-IR is dominated by the disk surface layer. Note again that, in these calculations, we ignored the direction of polarization and we only take the absolute value of it.
4.2. Radial energy distribution

Figure 9 shows radial distribution of emitted radiation. For $\lambda = 850\mu m$, both radiations from the disk interior and the surface layer are dominated by the outer part of the disk. But, for $\lambda = 10\mu m$, the inner part of the disk contributes significant portion of total emission and polarized emission.

![Figure 9: Radial distribution of emitted radiation.](image)

**Figure 8.**—Spectral energy distribution. The vertical axis (i.e. $\lambda F_{\lambda}$) is in arbitrary unit. Thick solid line: total (i.e. interior + surface) emission from disk. Thin dotted line: total emission from disk surface. Thick dotted line: polarized emission from disk surface. Thin dashed line: total emission from disk interior. Thick dashed line: polarized emission from disk interior. Note that, in these calculations of polarized emission, we ignored the direction of polarization vectors and we only take the absolute value of them. **Left panel:** Results for oblate spheroid grains with axis ratio of 1.3:1. **Right panel:** Results for oblate spheroid grains with axis ratio of 1.5:1. The degree of polarization is larger than that in the left panel.

5. EFFECTS OF DISK INCLINATION

In this section we calculate actual degree of polarization that we can observe. Chiang & Goldreich (1999) calculated spectral energy distribution (SED) from inclined disks. We follow a similar method to calculate the SED of polarized emission. The SED of disk interior is the integral of the source function:

$$L_{\lambda}^{\text{int}} \propto \lambda \int_{-r_{\max}}^{r_{\max}} dx \int_{-y(x)}^{y(x)} dy \int d\tau_{\lambda} B_{\lambda}(T_{i}) e^{-\tau_{\lambda}},$$

where

$$y(x) = \sqrt{r_{\max}^2 - x^2 \cos \theta} - H(r_{\max}) \sin \theta,$$

and $r_{\max}$ is the outer disk radius, $H(r_{\max})$ is the height of disk at the disk outer radius, $\theta$ is the angle between disk symmetry axis and the line of sight, $T_{i}$ is the temperature of disk interior, $B_{\lambda}$ is the Planck function, and $\tau$ is the optical depth. The integral $\int d\tau_{\lambda}$ is taken over the line of sight (see Chiang & Goldreich 1999 for details). The SED of the disk surface is obtained from the integral

$$L_{\lambda}^{\text{surf}} \propto \lambda \int_{-r_{\max}}^{r_{\max}} dx \int_{-y(x)}^{y(x)} dy \sum B_{\lambda}(T_{ds}) \left[ 1 - \exp \left( -\frac{\alpha \epsilon_{s}}{\hat{n} \cdot \hat{l}} \right) \right] \exp(-\tau_{\lambda}),$$

where $\hat{n}$ and $\hat{l}$ are unit vectors normal to the surface and parallel to the line of sight, respectively, $\epsilon_{s}$ is the Planck averaged dust emissivity at the surface. The summation is performed whenever the line of sight intersects the surface (see Chiang & Goldreich 1999 for details).

In our calculations, we explicitly take care of the fact that grain symmetric axis is changing along a given line of sight. We follow the description in Lee & Draine (1985) to calculate this effect and we obtain optical depths with respect to the x and y directions in the above integrals (Eqs. (17) and (19)). After calculating the optical depths, we calculate $L_{\lambda,x}$ and $L_{\lambda,y}$, and we calculate the degree of polarization based on the luminosity: $p(\lambda) = (L_{\lambda,y} - L_{\lambda,x})/(L_{\lambda,y} + L_{\lambda,x})$. 

![Equations 17 and 19](image)
Fig. 9.— Radial energy distribution. (a) $\lambda = 10\,\mu m$. Inner part of the disk emits substantial amount of radiation. But it emits negligible amount of polarized radiation. Note that, when $r < 1$ AU, grains in the surface layer are not aligned and only negligible fraction of grains are aligned in the interior (see Figures 3 and 4). (b) $\lambda = 50\,\mu m$. (c) $\lambda = 100\,\mu m$. (d) $\lambda = 850\,\mu m$. The result for $\lambda = 450\,\mu m$ (not shown) is very similar to that for $\lambda = 850\,\mu m$.

Figure 10 shows the effects of the disk inclination. We calculate the polarized emissions from the disk interior. The viewing angle $\theta$ (the angle of disk inclination) is the angle between the disk symmetry axis and the line of sight. We plot the direction of polarization for 3 different wavelengths and 2 different viewing angles. The lines represent the direction of polarization. Since we assume that magnetic field is azimuthal, the direction of polarization is predominantly radial (see lower panels). In Figure 11 we show similar plots for radiation from the disk interior only. For $\lambda > 100\,\mu m$, the polarization patterns in Figure 11 is very similar to those in Figure 10. But near the disk edges, Figure 10 shows larger degree of polarization than Figure 11. This is because the emission from the disk interior is very weak there compared with that from the disk surface layer. For $\lambda < 100\,\mu m$, the polarization patterns in Figure 11 are very different from those in Figure 10 because polarized emission from the disk surface layer dominates that from the disk interior. Note that, since the degree of polarization of emission from the disk surface layer is very sensitive to the maximum grain size in the surface layer, the results for $\lambda < 100\,\mu m$ should be very sensitive to the maximum grain size in the surface layer.

While the polarimetry of the spatially resolved accretion disks is promising with new generation of instruments (see §6.2), at present one can study disk magnetic fields with unresolved accretion disks. Below we provide predictions for this case. Figure 12 shows spectral energy distribution for such a disk for four different viewing angles. When $\theta = 90$ (i.e. for edge-on disk), inner part of the disk (i.e. region close to the star) is invisible due to high opacity. Therefore the spectral energy distribution truncates for $\lambda < 10\,\mu m$. When $\theta = 0$ (i.e. for face-on disk), the polarized emission is zero as expected.

Finally, Figure 13 shows the change of the degree of polarization for selected wavelengths. Left panel shows the degree of polarization for total emission, while the right panel shows that for radiation from the interior only. The degree of polarization is large when the angle $\theta$ is small (see left panel). The sudden drop for $\lambda = 10\,\mu m$ is due to the following reason. As the viewing angle drops, the inner part of the disk suddenly becomes visible, which causes a sudden increase of the the total intensity (or flux). But, the polarized intensity (or flux) does not change much because the inner part of disk does not emit polarized emission. Note that grains are not aligned in the inner part of the disk. For $\lambda = 50\,\mu m$, polarization is dominated by the disk surface layers.

6. DISCUSSION

6.1. Other Alignment Processes

That is, we do not see thick dotted or thick dashed lines in Figure 12(d).
In the paper above we concentrated on only one alignment process, namely, the RT mechanism. This mechanism has become so promising in explaining of interstellar polarization partially because the main competitor, namely, paramagnetic alignment mechanism (Davis-Greenstein 1951) and its later modifications were shown to have problems with aligning interstellar grains. In fact, the fast spin-up mostly due to $\text{H}_2$ formation on the catalytic sites over grain surface as suggested in Purcell (1979) is a textbook process that is invoked to explain the efficient paramagnetic alignment. Indeed, fast rotating grains should be immune to randomization by atomic bombardment and thus get aligned well. However, the Purcell’s spin-up was shown to be inefficient for most of grains in diffuse interstellar gas due to thermal flipping of grains that was reported in Lazarian & Draine (1999ab). The thermal flips arise from the coupling of vibrational and rotational degrees of freedom stemming from the processes of internal relaxation within interstellar grains, in particular, Barnett (Purcell 1979) and nuclear relaxation (Lazarian & Draine 1999b). As a result of flipping the direction of the Purcell’s torques acting on a grain alters and the grain gets “thermally trapped”. It rotates at a thermal velocity and therefore is subjected the randomization due to random gaseous and ionic bombardment.

The difference of the interstellar and disk grains is their size. The larger grains in the disk are not thermally trapped. Therefore, potentially, the processes of Purcell’s spin-up are applicable. If temperatures of grains and ambient gas are different, this may result in a spin-up that arises from variations of the accommodation coefficient over grain surface (Purcell 1979), provided that the temperatures of the gas and the grains in the disk differ. The problem of such a scenario is that the paramagnetic alignment is slow, unless grains demonstrate enhanced magnetic susceptibility (e.g. are super-paramagnetic) (see Jones & Spitzer 1967). Potentially, this process that also aligns grains with long axes perpendicular to magnetic field can enhance the alignment and therefore polarization. However, we do not know about the abundance of the required grains within the disks.

For the largest grains, a particular mechanical alignment, which was termed in Lazarian (1994) “weathercock mechanism” is applicable. In the presence of gas-grain motions large irregular grains would tend to get aligned with long dimension along the flow as their center of pressure and center of mass do not coincide. However, the mechanism requires substantial relative velocities of gas and grain, which is not certain in the protostellar disks.

All in all, while further studies of alternative alignment mechanisms seems necessary, at present the discussed RT mechanism provides the safest bet.

6.2. Observational Prospects

Multifrequency observations of protostellar disks have become a booming field recently. They have advanced substantially our knowledge of the disks and allowed theoretical expectations to be tested.

Magnetic fields are an essential components of the protostellar disks. They are likely to be responsible to accretion
Therefore observational studies of them are essential. In this respect our paper is the first, as far as we know, attempt to provide the expectations of the polarization arising from accretion disks that is based on the predictions of the grain alignment theory.

Our study reveals that multifrequency polarimetry is very important for the protostellar disks. The synthetic observations that we provide explicitly show that observations at wavelength less than 100 \( \mu m \) mostly test magnetic fields of the skin layers, while at longer wavelengths test magnetic fields of the bulk of the disk. Therefore polarimetry can, for instance, test theories of layered accretion (Gammie 1996).

Combining the far-infrared polarimetry with polarimetric measurements at different frequencies may provide additional insight into the magnetic properties of protostellar accretion disks. For instance, circular polarization arising from scattering of starlight from aligned grains (see Lazarian 2003) and polarization in emission lines arising from the aligned atoms (see Yan & Lazarian 2006) can provide additional information about the magnetic in the outer parts and above the accretions disks.

Most of the present day polarimetry will be done for unresolved protostellar disks. The size of the T Tauri disks is usually less than \( \sim 300 \) AU (see, for example, C01). If we take the distance to proto-stars to be around \( \geq 100 \) pc, then the angular sizes of the disks are usually smaller than 6\(^\prime\). The angular resolution of SCUBA polarimeter (SCUPOL) is around 14\(^\prime\) (Greaves et al. 2000) and that of SHARC II polarimeter (SHARP; Novak et al. 2004) at 350\( \mu m \) is around 9\(^\prime\). Therefore it is not easy to obtain plots like Figures[11]. The angular resolution of the intended SOFIA polarimeter is around 5\(^\prime\) at 53\( \mu m \), 9\(^\prime\) at 85\( \mu m \), and 22\(^\prime\) at 215\( \mu m \). We see that the intended SOFIA polarimeter will be at the edge of resolving structure of close-by disks, while other instruments will not resolve typical T Tauri disk. Therefore for most of the near future observations our predictions in Fig. 13 and 14 are most relevant.

Higher resolution polarimetry is expected in future, however. This will make our predictions of polarization the resolved accretion disks in Fig. 11 and 12 testable. Note, that the actual structure of magnetic fields may be much more complex than the one in our simple model.

While in the paper we dealt with the protostellar accretion disks, our results are suggestive of importance of polarimetric studies of magnetic fields in the disks of evolved stars. In a broader context, the present paper is one of the first studies that make use of the advances in grain alignment theory to extend the utility of polarimetric studies of magnetic field beyond its traditional interstellar domain.

**7. SUMMARY**

Making use of the recent advances in grain alignment theory we calculated grain alignment by RTs in a magnetized T Tauri disk. Based on this, we calculated polarized emission from the disk. Our results show that
Fig. 12.— Spectral energy distribution for four different viewing angles. When $\theta = 90^\circ$ (i.e. for edge-on disk), inner part of the disk (i.e. region close to the star) is invisible because it is occulted by the outer part of the disk. Therefore the spectral energy distribution truncates for $\lambda < 10\mu m$. When $\theta = 0^\circ$ (i.e. for face-on disk), the polarized emission is zero as expected. This is because the assumed magnetic field configuration is perfectly azimuthal.

Fig. 13.— Degree of polarization vs. viewing angle. Left panel shows the degree of polarization for total (i.e. interior + surface) emission, while the right panel shows that for radiation from disk interior only.

- Polarization arising from aligned grains reveals magnetic fields of the T Tauri disk.
- Grain size distribution is the most important factor that determine the degree of polarization.
- Disk interior dominates polarized emission in FIR/sub-millimeter wavelengths. When there are many grains with maximum grain size of $\sim 1000\mu m$, the degree of polarization is around or less than $\sim 2\%$ in these wavelengths. However, when the maximum grains are smaller, we expect higher degree of polarization.
- Disk surface layer dominates polarized emission in mid-IR wavelengths. The degree of polarization is very sensitive to the maximum size of grain in the disk surface layer. When the maximum grain size is as large as...
\[ \sim 1 \mu m, \text{ we expect } \sim 10\% \text{ of polarization at } \lambda \sim 50 \mu m. \] However, when the maximum size is smaller the the value will drop.

- Our study of the effect of the disk inclination predicts substantial changes of the degree of polarization with the viewing angle. The coming mid-IR/FIR polarimeters are very promising for studies of magnetic fields in protostellar disks.

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