Gauged System Mimicking the Gürsey Model

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Abstract

We comment on the changes in the constrained model studied earlier when constituent massless vector fields are introduced. The new model acts like a gauge-Higgs-Yukawa system, although its origin is different.

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1 Introduction

There are two endeavors which are studied by many physicists over and over again, and often the solutions sought for the two problems intersect. The first one is to write a theory which is shown to have a nontrivial limit, or zero for the beta function of the renormalization group for a non zero value of the coupling constant, as the renormalization cut-off goes to infinity. The perturbatively nontrivial φ⁴ theory in four dimensions was shown to go to a free theory as the cut-off was lifted a while ago [1, 2]. This example shows that perturbative expansions are not decisive in obtaining a nontrivial model. There is continuing research on this subject [3].

The other ongoing endeavor is to use only fermions to build a model of nature, where all the observed bosons are constructed as composites of these entities. In solid state physics the basic fields, the electrons come together to form bosons to explain superconductivity [4]. Heisenberg spent years to formulate a "theory of everything" for particle physics, using only fermions [5]. The Nambu-Jona-Lasinio model [6], which was constructed based on an analogy with the BCS theory of superconductivity [4], is a model which satisfies both ambitions, which is written in terms of fermions only and perturbatively non-renormalizable. This model also was shown to go to a trivial model [7, 8].

There are new attempts to make sense of these theories either as an effective model at low energies, which will give valuable information in QCD [9, 10], for example for the studies of hadron mass generation through spontaneous symmetry breaking. Another attempt is gauging the model and investigating
whether the new coupling gives rise to a nontrivial theory \[11, 12, 13, 14\]. These examples show that the search for non-trivial models using only fermions may be an interesting endeavor.

Another attempt in writing a model using only fermions came with the work of Gürsey \[15\]. Here a non-polynomial but conformal invariant Lagrangian was written to describe self-interacting fermions with the intention of remedying some of the problems of the Heisenberg model \[5\]. To be able to write a conformally invariant lagrangian, Gürsey had to use a non-polynomial form. Kortel found solutions to this conformal invariant theory \[16\] which were shown to be instantons and merons much later \[17\].

One of us, with collaborators, tried to make quantum sense of this model a while ago \[18, 19, 20\], finding that even if these attempts were justified, this model went to a trivial model as the cut-off is removed. Several processes were calculated \[21\] involving incoming and outgoing spinors which gave exactly the naive quark model results, missing the observed logarithmic behavior predicted by QCD calculations.

We tried to give a new interpretation of our old work in \[22\]. In that work we saw that the polynomial form of the original model really did not correspond to the original Gürsey model in the exact sense. The two versions obey different symmetries. This was shown explicitly in reference \[22\]. We went to higher orders in the calculation, beyond one loop for scattering processes. By using the Dyson-Schwinger and Bethe-Salpeter equations we could calculate higher order processes. We saw that while the non-trivial scattering of the fundamental fields was not allowed, bound states could scatter from each other with non-trivial amplitudes.

The essential point in our analysis was the fact that, being proportional to \[\frac{1}{p^2}\], the composite scalar field propagator cancelled many of the potential infinities that arise while calculating loop integrals. As a result of this cancellation, only composite fields participate in physical processes such as scattering and particle production. The scattering and production of elementary spinor fields were not allowed. This phenomena was an example of treating the bound states, instead of the principal fields, as physical entities.

A further point will be to couple an elementary vector field to the model described in reference \[22\], in line with the process studied for the Nambu-Jona-Lasinio model \[11, 12\]. Coupling the same elementary field to the model described in reference \[23\] will be similar, giving a model with two vector fields, one composite, the other one elementary. Our final goal is to investigate if we get a non-trivial theory when we couple a Yang-Mills system with color and flavor degrees of freedom, like it is done in \[13, 14\]. Here we study the abelian case, as an initial step.

In this note we summarize the changes in our results when this elementary vector field is coupled to the model described in reference \[22\]. We outline the model as is given in Refs. \[18\] and \[22\] in next section and give our new results in subsequent sections. The main conclusion is that our original model, in which only the composites take part in physical processes like scattering or particle production, is reduced to a gauged–Higgs-Yukawa model, where both
the composites and the fundamental spinor and vector fields participate in all the processes.

2 The Model

Our initial model is given by the Lagrangian

\[ L = i \bar{\psi} \gamma^\nu \partial_\nu \psi + g \bar{\psi} \psi \phi + \xi (g \bar{\psi} \psi - a \phi^3). \] (1)

Here the only terms with kinetic part are the spinors. \( \xi \) is a Lagrange multiplier field, \( \phi \) is a scalar field with no kinetic part, \( g \) and \( a \) are coupling constants. This expression contains two constraint equations, obtained from writing the Euler-Lagrange equations for the \( \xi \) and \( \phi \) fields. Hence, it should be quantized using the Dirac constraint analysis as performed in reference [22].

The Lagrangian given above is just an attempt in writing the original Gürsey Lagrangian

\[ L = i \bar{\psi} \gamma^\nu \partial_\nu \psi + g' (\bar{\psi} \psi)^{4/3}, \] (2)

in a polynomial form.

We see that the \( \gamma^5 \) invariance of the original Gürsey Lagrangian is retained in the form written in equation (1). This discrete symmetry prevents \( \psi \) from acquiring a finite mass in higher orders. We also see that the two models given by lagrangians in equations (1) and (2) are not equivalent since the former does not obey one extra symmetry obeyed by the latter one. This was carefully studied in reference [22]. We, therefore, take the first model as a model which only approximates the original Gürsey model, without claiming equivalence and study only that model in this work.

To quantize the latter system consistently we proceed via the path integral method. This procedure was carried out in reference [22]. At the end of these calculations we found out that we can write the constrained lagrangian given in equation (1) as

\[ L'' = i \bar{\psi} \gamma^\nu (\gamma^\mu - \gamma^\nu g \Phi) \gamma_\mu \psi - \frac{a}{16} (\Phi^4 + 2 \Phi^3 \Xi - 2 \Phi \Xi^3 - \Xi^4) + \frac{i}{4} c^* (\Phi^2 + 2 \Phi \Xi + \Xi^2) c, \] (3)

where the effective lagrangian is expressed in terms of scalar fields \( \Phi \), and \( \Xi \), ghost fields \( c \), \( c^* \) and spinor fields only.

The fermion propagator is the usual Dirac propagator in lowest order, as can be seen from the Lagrangian. After integrating over the fermion fields in the path integral, we obtain the effective action. The second derivative of the effective action with respect to the \( \Phi \) field gives us the induced inverse propagator for the \( \Phi \) field, with the infinite part given as

\[ \text{Inf} \left[ \frac{ig^2}{(2\pi)^4} \int \frac{d^4p}{p(p + g)} \right] = \frac{g^2 q^2}{4\pi c} \] (4)
Here dimensional regularization is used for the momentum integral and $\epsilon = 4 - n$. We see that the $\Phi$ field propagates as a massless field.

When we study the propagators for the other fields, we see that no linear or quadratic term in $\Xi$ exists, so the one loop contribution to the $\Xi$ propagator is absent. Similarly the mixed derivatives of the effective action with respect to $\Xi$ and $\Phi$ are zero at one loop, so no mixing between these two fields occurs. We can also set the propagators of the ghost fields to zero, since they give no contribution in the one loop approximation. The higher loop contributions are absent for these fields.

In reference [22] we also studied the contributions to the fermion propagator at higher orders and we found, by studying the Dyson-Schwinger equations for the two point function, that there were no new contributions. We had at least one phase where the mass of the spinor field was zero.

In reference [23] we studied a similar model where the composite vector field replaced the composite scalar field, with similar results.

### 3 New Results and Higher Orders

Here we couple an elementary vector field to the model described in reference [22], in a minimal way, with a new coupling constant $e$, acting in accordance of the work in references [11, 12, 13, 24, 25]. The new lagrangian is given as

$$L' = i\bar{\psi} \left( \partial - ig\Phi \right) \psi - \frac{a}{4} (\Phi^4 + 2\Phi^3\Xi - 2\Xi^3\Phi - \Xi^4)$$

$$+ \frac{i}{4} e^* (\Xi^2 + 2\Phi\Xi + \Phi^2) e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} e A\psi. \quad (5)$$

Here $A^\mu$ is the elementary vector field and $F^{\mu\nu}$ is defined from $A^\mu$ in the usual way. We take the vector field propagator in the Feynman gauge in our explicit calculations. This lagrangian reduces to the effective expression given below, since the $\Xi$ and the ghost fields decouple.

$$L' = i\bar{\psi} \left( \partial - ig\Phi \right) \psi - \frac{a}{4} \Phi^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} e A\psi. \quad (6)$$

In this section we summarize the changes in our results for this new model. If our fermion field had a color index $i$ where $i = 1...N$, we could perform an $1/N$ expansion to justify the use of only ladder diagrams for higher orders for the scattering processes. Although in our model the spinor has only one color, we still consider only ladder diagrams anticipating that one can construct a variation of the model with $N$ colors.

#### 3.1 Renormalization Group Analysis

In the models given in references [22] and [23], we had two coupling constants, $g$ and $a$ in reference [22] and only one, which we rename as $g'$, in reference
In the model described in reference [23], there is no need for infinite coupling constant renormalization, since the spinor box diagram is finite when the incoming and outgoing particles are vectors [26, 27]. In the model described in reference [22], the coupling constant $a$ needs renormalization. In these models there is no need for infinite renormalization for $g$ and respectively for $g'$ since the diagrams for the $<\bar{\psi}\psi\phi>$ and $<\bar{\psi}\psi A_\mu>$ vertices are finite.

Using the language of renormalization group analysis, the first order for this vertex is given by

$$\mu \frac{dg_0}{d\mu} = 0,$$

(7)

since the diagram given in Figure 1.a is finite, due to the presence of $\epsilon$ in the scalar propagator. Higher order calculations using the Bethe-Salpeter equation verify that the right hand side of the equation does not change in higher orders. This process was studied in reference [22].

We see that in the original model the only infinite renormalization is needed for the four $\phi$ vertex; hence the coupling constant for this process runs. The first correction to the tree diagram is the box diagram, shown in Figure 1.b. This diagram has four spinor propagators and gives rise to a $\frac{1}{\epsilon}$ type divergence. The renormalization group equation written for this vertex is

$$16\pi^2 \mu \frac{da}{d\mu} = -d g_0^4,$$

(8)

Here the right hand side of the equation is equal to a constant, since $g_0$ does not run. Since we include the four $\phi$ term in our original lagrangian, we can renormalize the coupling constant of this vertex to incorporate this divergence. There are no higher infinities for this vertex. The two loop diagram contains, as shown in Figure 1.c, a $\phi$ propagator which makes this diagram finite. The three-loop diagram is made out of eight spinor and two scalar lines, Figure 1.d. At worst we end up with a first order infinity of the form $\frac{1}{\epsilon}$ using the dimensional regularization scheme. Higher order ladder diagrams give at worst the same type of divergence. This divergence for the four scalar vertex can be renormalized using standard means.

In the new model, where an elementary vector field is added to the model described in reference [22], we add a new coupling constant $e$ which describes
the coupling of the vector field to the spinors. Here all three coupling constants are renormalized.

We can write the three first order renormalization group equations for these three coupling constants similar to the analysis in [28].

\begin{align}
16\pi^2\mu \frac{de}{d\mu} &= be^3, \\
16\pi^2\mu \frac{dg}{d\mu} &= -cge^2, \\
16\pi^2\mu \frac{da}{d\mu} &= -dg^4,
\end{align}

where \(b, c, d\) are numerical constants. These values are given as \(b = 2, c = 4, d = 4\). These processes are illustrated in diagrams shown in Figure 2. below.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The three coupling constant corrections in one loop}
\end{figure}

Our equations differ from those in reference [28], since the interaction of the composite scalar field with the spinors does not result in infinite terms due to the presence of the factor \(\epsilon\) in the scalar propagator. Here \(\epsilon\), the parameter in the dimensional regularization scheme, is inversely proportional to \(\ln \frac{\Lambda}{\Lambda_0}\), where \(\Lambda\) is the cut-off parameter. These equations have the immediate solutions

\begin{align}
e^2 &= \frac{c_0^2}{A}, \\
g &= g_0 A^{c/2b}, \\
a &= a_0 + \frac{dg_0^4}{2(2c + b)c_0^2} A^{\frac{2}{c}+1},
\end{align}

where

\begin{equation}
A = 1 - \frac{2bc_0^2}{16\pi^2} \ln \frac{\mu}{\mu_0}
\end{equation}

If we use diagrammatical analysis, we see that only the spinor-vector field coupling gives infinite contribution to the first two equations. The third equation diverges because of the contribution of the box diagram, which is infinite even
in the absence of the vector field. For this coupling constant, at one loop level, there is no difference from its behavior in the original model.

In the original model we need an infinite renormalization for only one of the coupling constants, the one with the four scalar field. Further renormalization may be necessary at each higher loop, like any other renormalizable model. The difference between our original model and other renormalizable models lies in the fact that, although this model is a renormalizable one using naive dimensional counting arguments, we have only one set of diagrams which is divergent. We need to renormalize only one of the coupling constants by an infinite amount. This set of diagrams, corresponding to the scattering of two bound states to two bound states, has the same type of divergence, i.e. $\frac{1}{\epsilon}$ in the dimensional regularization scheme for all odd number of loops. The contributions from even number of diagrams are finite, hence require no infinite renormalization.

When additional the vector particle contributions are added, this expression is modified. The process where two scalar particles goes to two scalar particles gets further infinite contributions from the box type diagrams with vector field insertions, where one part of the diagram is connected to the non-adjacent part with a vector field as shown in the Figure 3.a. All these diagrams go as $\frac{1}{\epsilon}$ where $\epsilon$ is the parameter in dimensional regularization scheme. There are no higher divergences for this process. Note that mixed scalar and vector insertions do not give additional infinities, since the scalar propagator reduces the degree of divergence. Also note that the diagram where the internal photon is connecting adjacent sides, as shown Figure 3.b, will be a contribution to the coupling constant renormalization of one of the vertices. Since this is not a new contribution, we will not consider it separately.

![Figure 3: (a) The vector particle correction to the fermion box diagram, (b) The box diagram with one vertex correction](image)

3.2 Propagators and Vertices

We have no essential change in the spinor propagator. In reference [10] Miransky explains how for coupling constant $\alpha$ less than $\pi/3$, there is no mass generation in the quenched approximation. Here $\alpha = \frac{e^2}{4\pi}$. J.C.R. Bloch, in his Durham thesis, [29], explores the range where this result is valid when the calculation is done without this approximation. He states that the quenched and the rainbow approximations, used by Miransky and collaborators, have non-physical features,
namely they are not gauge invariant, making the calculated value wildly vary depending on the particular gauge used. Bloch, himself, uses the Ball-Chiu vertex, \[30\], instead of the bare one, where the exact longitudinal part of the full QED vertex, is uniquely determined by the Ward-Takahashi identity relating the vertex with the propagator. The transverse part of the vertex, however, is still arbitrary. Bloch then considers a special form of the Curtis-Pennington vertex \[31\] in which the transverse part of the vertex is constructed by requiring the multiplicative renormalizability of the fermion propagator with additional assumptions.

Bloch claims that for the different gauges used with this choice, he gets rather close values for the critical coupling \[32\]. He also performs numerical calculations where the approximations are kept to a minimum. The results are given in the table on pg. 202 of \[hep-ph/0208074\].

Using on the arguments in the Bloch’s thesis, also using the results of his numerical calculations, we conclude that at least for \(\alpha < 0.5\) we can safely claim that there will be no mass generation or the assumed \(\gamma_5\) symmetry will be not broken. Since we do not study heavy ion processes, the numerical value we have for \(\alpha\) will be much smaller than this limit. Hence, our results will be valid. Note that in QCD mass generation occurs at relatively low energies, where the coupling constant has already increased.

Miransky \[10\] also explains how in the Landau gauge we can take the coefficient of the momentum term as unity. Using these arguments we can conclude that there are no additional contributions to the spinor propagator used in reference \[22\], at least in the Landau gauge.

The photon propagator also will be the similar as the one given in QED, with only additional \(1/\epsilon\) contributions from the scalar particle insertions. The lowest order diagram for this process is shown in Figure 4.a. The dominant contribution will be from the vector insertions, which are studied in QED.

![Figure 4](image-url)

Figure 4: (a) Scalar contribution to the vector propagator, (b)The vector particle correction to the scalar propagator

The additional contribution to the scalar propagator can be calculated using diagrammatical analysis. If we take only the planar diagrams which connect two different spinor lines, as shown in Figure 4.b, the scalar field contributions are only of order \(1/\epsilon\), the same as the one loop initial contribution. Higher order divergences come from the vector field insertions.

The higher order planar insertions will be the dominant ones if we allow \(N_f\) flavors for the fermions, where \(N_f\) is large, and perform an \(1/N_f\) expansion. We will assume that the same approximation can be done in our case too. The diagrams where there are \(n-1\) nonadjacent and planar vector field contributions,
go as \((\frac{D}{\varepsilon})^n\), where \(D = \frac{4\varepsilon^2}{16\pi^2}\) is a numerical constant. Naively the planar vector field contributions can be summed up as a geometric series \([33]\). The same result is true also for the planar vertex corrections as in Figure 5.a.

The vector- spinor- antispinor vertex do not get infinite contributions from our composite scalar particle. A typical diagram is given in Figure 5.b. Here the infinities coming from the integrations are cancelled by the \(\varepsilon\) factors in the scalar propagators. That vertex, for the purely electromagnetic case, Figure 5.c, is vastly studied in the literature \([30\ 31]\).

![Figure 5:](image)

(a) (b) (c)

3.3 Scattering and Production Processes

The process where two composite scalars scattering from each other was studied above. The scattering of two scalars producing four, or to any higher even number of scalars is finite, as expected to have a renormalizable model. The process where two scalars create an odd number of scalars is forbidden by the \(\gamma^5\) invariance of the theory, hence two scalar \(\phi\) particles can only go to an even number of scalar particles. This assertion is easily checked by diagrammatic analysis.

We also note that in the original model the four spinor kernel was of order \(\varepsilon\). The lowest order diagram, shown in Figure 6.a, vanishes as \(\varepsilon\) due to the presence of the scalar propagator. In higher orders this expression can be written in the quenched ladder approximation \([10]\), where the kernel is seperated into a scalar propagator with two spinor legs joining the proper kernel. If the proper kernel is of order \(\varepsilon\), the loop involving two spinors and a scalar propagator can be at most finite that makes the whole diagram in first order in \(\varepsilon\). This fact also shows that there is no nontrivial spinor-spinor scattering.

As a result of this analysis, in the ungauged version, we end up with a model where there is no scattering of the fundamental fields, i.e. the spinors, whereas the composite scalar fields can take part in a scattering process. The coupling constant for the scattering of the composite particles runs, whereas the coupling constant for the spinor-scalar interaction does not run. The processes giving this conclusion are carefully studied in reference \([22]\).

This result changes drastically when the gauged model is studied instead of the original one. This process, which is prohibited in the previous model, \([22]\).
now is possible due to the presence of the vector field channel. In lowest order this process goes through the tree diagram given in Figure 6.b.

The process is finite though, since at the next higher order the QED box diagram with two spinors and two vector particles, Figure 6.c, is ultra violet finite from dimensional analysis, and is calculated in reference [34]. Higher orders do not give new type of ultra violet divergences.

We also allow spinor production from the scattering of scalar particles, since now we can use vector particles as intermediaries, Figure 6.d.

![Figure 6:](image)

(a) (b) (c) (d)

Figure 6: (a) Two fermion scattering through the scalar particle channel, (b) Two fermion scattering through the vector particle channel, (c) Higher order diagram for two spinor scattering, (d) Spinor production from scattering of scalars

4 Conclusion

In this note we discussed the differences between the new model, introduced in this paper, and the model studied in reference [22]. We found out that many of the features of the original model are not true anymore. As far as renormalizations are concerned, we have essentially QED, with corrections coming from the scalar part mimicking the Yukawa interactions with the $\Phi^4$ term added. We end up with the gauge-Higgs-Yukawa system, although our starting point is gauging a constrained model.

We also have scattering processes where two scalar particles go to an even number of scalar particles, or scattering of spinor particles from each other. In the one loop approximation all these diagrams give finite results, like the case in the standard Yukawa coupling model. We also have creation of spinor particles from the interaction of scalars, as well as scattering of spinors with each other, and all the other processes in the gauge-Higgs-Yukawa system.

If we consider the model described in reference [23], we see that the same differences prevail. The main results are the same. The only difference from the scalar model is the finiteness of the spinor box diagram with incoming and outgoing vector particles [26, 27], both in the new model and the one in reference [23].

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