PENGUIN AMPLITUDES IN HADRONIC B DECAYS:
NLO SPECTATOR SCATTERING

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We present results on the NLO ($\alpha_s^2$) spectator-scattering corrections to the topological penguin amplitudes for charmless hadronic two-body $B$-decays in QCD factorization. The corrections can be sizable for the colour-suppressed electroweak penguin amplitudes $\alpha_s^4_{\text{EW}}$ but otherwise are numerically small. Our results explicitly demonstrate factorization at this order. To assess the phenomenological viability of the framework, we consider penguin-to-tree ratios in the penguin-dominated $\pi K$ system and find agreement to the expected precision (i.e., a power correction).

Keywords: QCD; Factorization; $B$-meson decays.

1. Introduction

Branching ratios and CP asymmetries in $B$-decays into two light mesons provide access to the flavour structure of the Standard Model (CKM matrix elements) and its possible extensions. Within the Standard Model, the theoretical expressions always involve two terms with a relative weak phase,

\[ \mathcal{A}(\bar{B} \to M_1 M_2) = T_{M_1 M_2} e^{-i\gamma} + P_{M_1 M_2}. \] (1)

Direct CP asymmetries are then governed, besides $\gamma$, by the imaginary part $\text{Im} P/T$. A nontrivial theoretical task is to evaluate the strong amplitudes $P$ and $T$. Integrating out the weak scale by means of the weak effective Hamiltonian, this reduces to the computation of hadronic matrix elements of local operators $Q_i$, a task currently not feasible on the lattice. Fortunately, at leading power in an expansion in $\Lambda_{\text{QCD}}/m_b$ they obey

\[ \langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{BM_1}(0) \int du \, T^{|i|u}(u) \phi_{M_2}(u) + (M_1 \leftrightarrow M_2) \] (2)

\[ + \int d\omega \, du \, dv T^{\bar{B}}(\omega, v, u) \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u). \]

\footnote{We restrict ourselves to flavour-$SU(3)$-nonsinglet mesons in this note.}

\[ = F^{BM_1}(0) \int du \, T^{|i|u}(u) \phi_{M_2}(u) + \langle M_1 \leftrightarrow M_2 \rangle \]

The hard-scattering kernels $T^{|i|u}$ are perturbatively calculable as series in $\alpha_s$, while the form factors $F^{BM_1}$ and the light-cone distribution amplitudes (LCDAs) $\phi$ encapsulate universal nonperturbative properties of the initial- and final-state particles. An important outcome is that all strong phases are contained in the hard kernels. $T^{|i|u}$ is currently known to $O(\alpha_s^4)$ while the computation of $T^{\bar{B}}$ has recently been completed at $O(\alpha_s^2)$ by evaluating the one-loop spectator-scattering corrections to the (topological) penguin amplitudes. As stated above, the latter are crucial for any direct CP asymmetry in the Standard Model. Moreover they are important for the branching fractions particularly of the penguin-dominated $\Delta S = 1$ modes such as $\bar{B} \to \pi K$.

Spectator-scattering contributions to their strong phases and effects proportional to the large Wilson coefficient $C_1$ appear first at this order, similarly to the colour-suppressed tree amplitude considered in Ref. 2.

2. Effective theory and matching

The kernels $T^{\bar{B}}$ receive contributions from two hard scales $\mu_b \sim m_b, \mu_{\text{sc}} \sim \sqrt{m_b \Lambda}$. They...
correspond to hard and hard-collinear terms in an expansion by momentum regions.\cite{5} The result is further factorization $T_{II}(\omega, v, u) = \int dz H_{II}^{I}(\mu_{z}; u, z) J(\mu_{bc}; \omega, v, z)$.\cite{6} The coefficient functions are conveniently obtained in a two-step matching onto soft-collinear effective theory, SCET$\rightarrow$SCET$_I$, integrating out subsequently the hard and hard-collinear scales. The hard coefficients $T_I$ and $H_{II}^I$ are interpreted as Wilson coefficients in SCET$_I$ and are found by solving the matching equation

$$Q_i = \int dt T_I^i(t) [\bar{\chi}(tn_+)\chi(0)] [C_{A0}[\xi(0)h_v(0)] \right.$$

$$\left. - \frac{1}{m_b} \int ds C_{B1}(s) [\xi(0)D_{Ih_c1}(sn_+)h_v(0)] \right.$$  

$$+ \frac{1}{m_b} \int dt ds H_{II}^{I}(t, s) [\bar{\chi}(tn_+)\chi(0)]$$

$$\times [\xi(0)D_{Ih_c1}(sn_+)h_v(0)] \right), \quad (3)$$

where the (schematic) rhs involves SCET$_I$ collinear fields for the directions of motion of $M_2$ ($\chi, \bar{\chi}$) and $M_1$ ($\xi, A_{Ih_c1}$) as well as soft fields $h_v$ and $\bar{q}_s$ suitable for interpolating the $B$-meson, sandwiched between suitable partonic states. The peculiar form of the second bracket in the first convolution is designed to reproduce full-QCD heavy-to-light form factors as in (2). Of interest to us is the second term in (3), which includes all hard interactions of $M_2$ with the spectator quark. Decoupling properties of SCET$_I$ suggest that its hadronic matrix element factorizes into a light-cone distribution amplitude $\langle M_2 | [\bar{\chi}\chi]|0\rangle \propto \phi_{M_2}$ and a nonlocal object $\Xi_{BM_1}(s) = \langle M_1 | [\bar{\xi}(0)D_{Ih_c1}(sn_+)h_v(0)] | B \rangle$. This expectation is indeed confirmed by the finiteness of the convolutions, found in all currently available computations.

Several fermion-line topologies occur in the full-QCD amplitude differing in how the quark fields in $Q_i$ are contracted with the fields interpolating the external states and/or with each other. Each pair $(Q_i, \text{topology})$ contributes to one of a few operators in the effective theory that are distinguished, besides the chirality of the light fields, only by their flavour content. Their matrix elements define scale- and scheme-independent amplitude coefficients $\alpha_i$. One-loop spectator-scattering corrections to the colour-allowed and colour-suppressed topological “trees” $\alpha_1, \alpha_2$ have been computed in Refs.\cite{2,3} and the corresponding QCD penguin amplitudes $\alpha_{3}^{P}, \alpha_{4}^{P} (p = u, c)$ as well as the electroweak penguin amplitudes $\alpha_{3}^{P,EW}, \alpha_{4}^{P,EW}$ are given in Ref.\cite{4}.

To arrive at the final form (3) one has to perform a second matching step corresponding to the matching equation

$$\int d^4x T \left( \mathcal{L}_{I\text{SCET}_1}^{(1)}(x) [\bar{\xi}(0)D_{Ih_c1}(sn_+)h_v(0)] \right)$$

$$= \int dwdr J(w, r) [\bar{\xi}(rn_+)\xi(0)][\bar{q}_s(wn_+)h_v(0)].$$

The jet function $J$ containing the hard-collinear physics appears identically in the factorization formula for form factors\cite{10} and is known to NLO\cite{11}, while the brackets $[\bar{\xi}\xi], [\bar{q}_s h_v]$ result in LCDAs for $M_1$ and the $B$-meson once hadronic matrix elements are taken.

3. Penguin amplitudes

Some diagrams related to computing the coefficients $H_{II}^{I}$ relevant to the penguin amplitudes are shown in Fig.\cite{1} In each diagram, the line to the left is the $b$-quark, the upgoing lines are collinear with $M_2$, the right-going line (hard-)collinear with $M_1$, and the external vector boson is a hard-collinear-1 gluon or photon. Of the first row in the Figure, the left diagram contributes to the (colour-suppressed) electroweak penguin amplitude $\alpha_{4}^{P,EW} (p = u, c)$, due to the photon exchanged between quark lines. Tadpole diagrams like the one on the right vanish when summed. On the other hand,

\footnote{Alternatively, one could try to extract $\Xi_{BM_1}(s)$ from experiment.\cite{12} This is not feasible beyond LO because the full $s$-dependence is needed.}
Fig. 1. Sample diagrams related to the penguin amplitudes. The upper row contributes to $\alpha_{p,4}^{4}$, while the lower row introduces isospin breaking in $\alpha_{p,4}^{4}$.

The diagrams on the second row induce the flavour structure of the QCD penguin amplitude $\alpha_{p,4}^{4}$ due to the exchanged gluon, but the hard-collinear photon implies isospin breaking. Such small electromagnetic corrections to the QCD penguin amplitudes are omitted here.\(^\text{4}\)

When the photons in the upper row are replaced by gluons, contributions to $\alpha_{p,4}^{p}$ arise. There are more diagrams, including ones without quark loops but with insertions of penguin operators from the weak Hamiltonian. With the input parameter ranges given in Ref. 4 we obtain for the leading-power contribution $a_{c,4}^{c}$ to the amplitude $\alpha_{c,4}^{4}$:

$$a_{c,4}^{c}(\pi\pi) = -0.029$$

$$- [0.002 + 0.001i]_{V} - [0.001 + 0.007i]_{P}$$

$$+ \left[ \frac{r_{sp}}{0.485} \right] \left[ \begin{array}{c} [0.001]_{LO} + [0.001 + 0.001i]_{HV} \\ + [0.000 - 0.000i]_{HP} + [0.001]_{tw3} \end{array} \right]$$

$$= -0.028 + 0.005 + (-0.006 + 0.003i).$$

(4)

4. Phenomenological implications

Because of the smallness of the corrections compared to uncertainties due to hadronic input parameters, we do not give updated numbers for the branching fractions here. Instead we consider a penguin-to-tree ratio, for which part of the nonperturbative uncertainties cancel out, but which nevertheless can be related to experimental data.

"HP" denote the newly computed one-loop spectator-scattering corrections. The term "tw3" denotes an estimate of the twist-3 tree-level spectator scattering contribution, which while being a power correction is by convention included in $a_{10}^{4}$. The numbers show that the impact of the new corrections is very small. This is somewhat surprising as the large Wilson coefficient $C_{1}$ is involved in the "HP" terms. Closer inspection shows a numerical cancellation between diagrams carrying different colour factors, the origin of which is unclear. The corrections to the colour-allowed electroweak penguin amplitude $\alpha_{3,EW}$ are also very small. The colour-suppressed electroweak penguin amplitude $\alpha_{4,EW}$ receives a larger correction. The correction to its leading-power part $a_{10}^{4}$ can be $O(100\%)$ with respect to the $O(\alpha_{s})$ result\(^4\). This is because, like the colour-suppressed tree amplitude $\alpha_{2, 4,EW}$ is especially sensitive to spectator scattering due to a numerical cancellation between naive factorization and the 1-loop correction to the first term in \(^2\). For the same reason, it is also more sensitive to uncertainties in hadronic input parameters, most importantly the inverse moment $\lambda_{-1}^{B}$ of the $B$-meson light-cone distribution amplitude. Altogether, perturbation theory appears to be well behaved and significant changes in the predictions for branching fractions and CP asymmetries at this time will be due mainly to changes of hadronic input parameters such as form factors and light-cone distribution amplitudes.

\(^4\)Including them consistently would necessitate taking into account isospin-breaking effects in the form factors, LCDAs, and decay constants, among other complications.
correction that factorizes at numerically large "charming penguin" power data on
ring and yellow wedge can be inferred from magnitude for this power correction). The grey
blue one corresponding to the expected mates of the annihilation contribution, the
plitude. The cross shows the theoretical pre-
tion) incomputable penguin annihilation am-
\[ \beta C_r \bar{\beta} \]
We observe that theory and experiment,
opening to the right is disfavoured by data.
M pseudoscalar-pseudoscalar (PP) final state
\[ M_1 M_2 = \pi K. \] Here \( \hat{\alpha}_c(\pi K) = \alpha_c(\pi K) + r^c_k \alpha^c_0(\pi K) + \beta^c_0(\pi K), \) where \( r^c_k \alpha^c_0(\pi K) \) is a numerically large “charming penguin” power correction that factorizes at \( O(\alpha_s) \) and \( \beta^c_0(\pi K) \) models the (within QCD factoriza-
tion) incomputable penguin annihilation amplitude. The cross shows the theoretical pre-
diction with errors combined in quadrature (the onion-shaped regions are various esti-
mates of the annihilation contribution, the blue one corresponding to the expected mag-
nitude for this power correction). The grey ring and yellow wedge can be inferred from
data on \( \bar{B} \to \pi^+ \pi^- \) and \( \bar{B} \to \pi^+ \bar{K}^- \) with very lit-
tle theory input, where the lighter-coloured areas also including a generous uncertainty
on modulus and phase of \( V_{ub}. \) The wedge opening to the right is disfavoured by data.
We observe that theory and experiment, which includes \( A_{CP}(\bar{B}^0 \to \pi^+ K^-) \), agree
within errors, which is nontrivial. Some annihilation contribution is needed, but at the
level expected for a power correction. It is conveivable that a large one-loop spectator-
scattering correction to \( \alpha^c_0 \) might have a sim-
ilar impact.

In conclusion, factorization works after inclusion of NLO spectator-scattering effects.
The corrections are small except for the colour-suppressed electroweak penguin am-
plitude. The penguin-to-tree ratios relevant to \( \Delta S = 1 \) decays are consistent with data
at the level of a power correction.

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