MIXING OF PRIMORDIAL GAS IN LYMAN BREAK GALAXIES

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ABSTRACT
Motivated by an interpretation of $z \sim 3$ objects by Jimenez and Haiman (2006), we examine processes that control the fraction of primordial ($Z = 0$) gas, and so primordial stars, in high-SFR Lyman break galaxies. A primordial fraction different from 1 or 0 requires microscopic diffusion catalyzed by a velocity field with timescale comparable to the duration of star formation. The only process we found that satisfies this requirement for LBGs without fine-tuning is turbulence-enhanced mixing induced by exponential stretching and compressing of metal-rich ejecta. The time-dependence of the primordial fraction for this model is calculated. We show that conclusions for all the models discussed here are virtually independent of the IMF, including extremely top-heavy IMFs.

$\text{Subject headings: ISM: evolution–galaxies: abundances–galaxies: evolution–galaxies: ISM–turbulence}$

1. INTRODUCTION

Galaxies begin their lives with entirely primordial ($Z = 0$) gas. As they age, metal production and mixing can only reduce the primordial gas fraction. We have explored the expected time dependence of the primordial fraction for various types of mixing and chemical evolution processes using a kinetic equation for the evolution of the abundance distribution. The present Letter addresses the narrower question of whether any models for mixing and chemical evolution predict this transformation occurs at an accessible redshift.

Jimenez and Haiman (2006) (hereafter JH) showed that several UV properties of a variety of objects, mostly Lyman break galaxies (LBGs), at redshift $z \sim 3$, can all be understood if these objects contain a substantial fraction, about 10 to 50 percent, of massive stars with essentially zero metallicity ($Z \lesssim 10^{-5} Z_\odot$; we use $Z$ and metallicity indiscriminately here). These UV properties cannot together be explained by a top-heavy IMF, and require $Z = 0$ stars; a top-heavy IMF is not required. Massive stars have short lifetimes, so their metal abundances reflect that of the concomitant gas. Thus, if JH are correct, a substantial fraction of the interstellar medium of these galaxies, with star formation ages a few hundred million years, has not been polluted by any products of nucleosynthesis.

Motivated by JH and other suggestions for $Z = 0$ stars (Malhotra & Rhoads 2002; Shimasaku et al. 2006), and the fact that during some early period in the lives of all galaxies a transformation from primordial to non-primitive must occur, leaving spectrophotometric signatures (JH; see Schaerer 2003), we examined the viability of a number of models for this transformation.

A change in the primordial fraction requires microscopic diffusion enhanced by a complex velocity field, such as instabilities in swept-up supernova shells or a turbulent interstellar medium (ISM), along with dispersal of nucleosynthesis products over many kpc, involving a scale range of $\sim 10^5$. Existing hydrodynamic simulations in a cosmological context (e.g., Governato et al. 2004; Scannapieco et al. 2005) therefore cannot represent the transition of the primordial gas or mixing in general, since they adopt mixing rules that arbitrarily spread metals among nearest neighbor cells or SPH particles. The only hydrodynamic simulations of true mixing of tracers in galaxies were concerned either with turbulent dispersal of mean metallicity, not mixing (Klessen & Lin 2003), or mixing of initial spatially periodic inhomogeneities by numerical diffusivity in a turbulent galaxy with no continuing source of metals (de Avillez & Mac Low 2002).

Instead of simulations, the arguments given here are phenomenological, in order to clarify the essentials for each physical process. A detailed discussion is given elsewhere (Pan and Scalo 2006, hereafter PS) using a formal probability distribution evolution equation. Here we report that most processes we examined are either far too fast or slow to partially erase the primordial fraction (Sec. 2), except for mixing by turbulence-enhanced diffusivity based on exponential stretching of blobs of nucleosynthesis products, discussed in Sec. 3.2.

2. ESTIMATES OF TIMESCALES FOR MIXING PROCESSES

2.1. Star Formation Age

The existence of a primordial gas fraction $P(t)$ that is not nearly unity or zero, i.e., both $P(t)$ and $1 - P(t)$ are significantly larger than zero, a condition we refer to as a “significant” or “intermediate” primordial fraction, requires a mixing or depletion process whose characteristic timescale is comparable to the star formation (SF) age, the time since SF began. If the mixing timescale is much smaller, $P(t)$ will be nearly zero; if it is much

$\text{1 The existence of } Z = 0 \text{ gas requires an IMF deficient in stars with } M \lesssim 1 \ M_\odot. \text{ Otherwise the number of low-mass stars observable today would be large, contradicting observational limits on the star fraction with very small } Z \text{ (see Oey 2003) by several orders of magnitude. We discuss the effects of different IMFs below, where we show that all our conclusions are independent of the IMF as long as it satisfies the requirement } M \geq 1 \ M_\odot.$

$\text{2 A recent estimate of the effect of SN mixing (sec. 2.4 below) on the primordial fraction at high redshift (Tumlinson 2006) apparently used a shell mass much larger than found in analytical and numerical calculations of supernova remnant evolution (see Thornton et al. 1998; Hanawama & Tomisaka 2008).}$
larger, $P(t)$ will remain near unity. SF ages for LBGs and likely-related objects at somewhat different redshifts have been estimated by Papovich et al. (2001), Shapley et al. (2001), Erb et al. (2006) and others, using galaxy evolution models that assume an exponentially decreasing SFR that began at some time in the past. Although there is much variation, median SF ages are $3 \times 10^8$ yr.

These SF ages cannot be significantly different for a number of reasons. The lookback times of about 11.5 Gyr for $z \sim 3$ imply a strict upper limit to the SF age of 2 Gyr, and a more likely upper limit of 1.5 Gyr (corresponding to $z \sim 8$). The irregular morphologies of the LBGs (Giavalisco 2002) suggest that LBGs are in the process of formation, accumulating large fragments of gas and stars through mergers (Conselice 2000), and probably undergoing one of their first major bursts of SF; starburst populations have durations, estimated from statistics (Kennicutt et al. 1987; Nikolic et al. 2004), modeling of integrated light features (Marcillac et al. 2000), and theoretical arguments (see Leitherer 2001), that are similar to the estimates in LBGs. Finally, ages much greater than $3 \times 10^8$ yr would produce greater metallicity than observed (see Giavalisco 2002).

We assume star formation has been a continuous function of time. If instead the SFR preceding or during the present episode consists of bursts of shorter duration, most of our arguments remain unchanged if the SF age is replaced by the accumulated duration of SF.

2.2. Depletion of the Primordial Fraction by Sources

The JH result of 10-50% primordial at $z \sim 3$ seems surprising, but actually galaxies should remain almost completely primordial for billions of years in the absence of microscopic diffusivity. SN metal production slowly depletes primordial gas by transferring it from a $Z = 0$ delta function in the $Z$ probability distribution (pdf) to another delta function at a much larger $Z$, the source metallicity $Z_s$ averaged over the IMF ($\sim 0.1$ assuming the hot ejecta are well mixed). Intermediate values of $Z$ cannot be reached without diffusivity.

This suggests the simplest explanation for an intermediate primordial fraction in LBGs: 50 to 90% of the primordial gas passed through stars that became SNe. The timescale for this process is the source timescale, $\tau_{src} = M_{gas}/B R_{SN}$, where $B$ is the star formation rate and $R_{SN}$ is the returned fraction from SNe averaged over the IMF. We take the total gas mass $M_{gas}$ as $5 \times 10^{10} M_\odot$, extrapolated from gas masses estimated in the $z \sim 2$ UV-selected sample of Erb et al. (2006). From a number of studies, we adopt a median SFR of 100 $M_\odot/yr$ for LBGs at $z \sim 3$ assuming the IMF lower limit $M_l = 0.1$ (Papovich et al. 2001; Shapley et al. 2001; Giavalisco 2002; Erb et al. 2006; Yan et al. 2006). For this $M_l$, we calculate $R_{SN} \approx 0.1$ for various IMFs using the ejected masses given in Woosley & Weaver (1995), Meynet & Maeder (2002) and Nomoto et al. (2006), finding little dependence on metallicity, including $Z = 0$, and 20-30% variation between studies. Variations in the form of the IMF change the SFR by $\sim 50\%$, with only a slight effect on $R_{SN}$. The source timescale is then $\tau_{src} = 5$ Gyr, within a factor of a few considering the uncertainty in the SFR and $M_{gas}$, too large to affect the primordial fraction.

We examined the IMF-dependence of $\tau_{src}$. There are two nonstandard IMFs that are especially relevant. 1. Intermediate primordial fractions require that the IMF lower limit $M_l \gtrsim 1 M_\odot$ to avoid too many $Z = 0$ stars observable today. Such a cutoff does not affect the source timescale: The empirical SFRs are based on integrated light from massive stars corrected for the rest of the IMF, and the resulting decrease of the SFR due to the cutoff is exactly compensated by the increase of the mass ejected by supernovae per unit mass of stars formed $R_{SN}$. This IMF-independence of $\tau_{src}$ holds for any cutoff smaller than the lower mass limit for SNe, $\sim 8 M_\odot$. By the same argument, such a cutoff does not overproduce metallicity.

2. A perrennially popular IMF for $Z = 0$ star formation consists of only very massive stars (VMS) due to the Jeans mass resulting from $H_2$ cooling (Hutchins 1976; see Bromm and Larson 2004) although it has been questioned on a number of grounds (Silk & Langer 2000). Comparing $H_\alpha$ emission per unit SFR for a VMS IMF (50-500) $M_\odot$ of $Z = 0$ stars in Schaerer (2003) with the same quantity for a (0.1-100) $M_\odot$ IMF in Kennicutt (1998), both for a Salpeter IMF (for illustration only), the SFR for a VMS IMF is 26 times smaller. Assuming only stars in the range 130-260 $M_\odot$ explode as pair instability SNe (Woosley et al. 2002), we find $R_{SN} = 0.27$, so $\tau_{src} = 50$ Gyr, an order of magnitude larger than the normal IMF case.

These results strengthen our conclusion that the formation of massive stars is far too slow to deplete primordial gas significantly in the available time.

2.3. Filling the Gap by Diffusion

Without a process to spread metals into the “gap” between the $Z_s$ peak and the primordial $Z = 0$ peak, $P(t)$ would remain near unity for billions of years. The only physical process that can fill this gap is microscopic diffusivity. However an estimate of the rate at which diffusivity from random sources could pollute primordial gas in LBGs, using diffusion lengths for the cold neutral, warm neutral, and warm ionized ISM similar to those in Oey (2003), shows that the fraction of the mass of a galaxy mixed in time $t$ is only $\sim 10^{-4} (A/0.5$ Gyr)$^{1/2}$, so diffusivity by itself cannot pollute more than a tiny fraction of the primordial gas over the estimated SF ages.

To reduce the primordial fraction, a velocity field is required to catalyze diffusivity. However a velocity field cannot by itself affect the global metallicity distribution or primordial fraction, or mix at all: Displacement of fluid parcels of different $Z$ by the velocity field conserves their volumes and thus volume fractions (or mass fractions for compressible flows), replacing one by another in space, having no effect on the $Z$ distribution. This can be shown rigorously using a metallicity pdf equation (PS). A velocity field can only enhance mixing by spatially ramifying the $Z$ field for diffusion to operate on small scales. Models that mix by sweeping of gas by SN or SB shells, cloud motions, differential rotation, or “turbulent diffusion” are unphysical without recognition that they are implicit models for microscopic diffusion.

2.4. Several Catalyzing Velocity Fields

Expanding SNRs and superbubbles (SBs) are the main agents of mixing in many sequential enrichment inhomogeneous chemical evolution models (Reeves 1972; see...
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3. TURBULENCE-ENHANCED DIFFUSIVE MIXING

Turbulence deforms large-scale fluid elements and the tracers they contain into filaments and sheets, bringing tracers closer together until scales are reached on which diffusivity can homogenize faster than the strain timescale of the fluid. The process is described by the general equation for the evolution of the metallicity field in an arbitrary velocity field \( \mathbf{u}(\mathbf{x}, t) \),

\[
\frac{\partial Z(\mathbf{x}, t)}{\partial t} + \mathbf{u}(\mathbf{x}, t) \cdot \nabla Z(\mathbf{x}, t) = \frac{1}{\rho} \nabla \cdot (\rho \kappa \nabla Z(\mathbf{x}, t)) + S \tag{1}
\]

where \( \kappa \) is the diffusivity and \( S \) denotes the sources.

In our model a straining event on the scale of the sources \( L_s \) takes \( Z \) to a critical scale \( L_{diff} \), small enough for diffusivity to operate, at constant mean strain rate in a single step, by exponential stretching of line elements (Batchelor 1952; Voth et al. 2002). This “short circuit” of the scalar cascade [Villermaux et al. 2001] is supported by experiments (Villermaux 2004; Voth et al. 2002) and simulations (Girimaji & Pope 1990; Goto & Kida 2003), and is similar to the scalar turbulence field theory of Shramm and Sigg (2000). We assume that in supersonic turbulence compressions are analogous to stretching in the sense of bringing tracers to the critical scale \( L_{diff} \).

The scale \( L_{diff} \) below which the diffusivity term exceeds the advection term in eq 1 is obtained by equating the two terms and replacing spatial derivatives by \( L_{diff}^{-1} \) and \( \mathbf{u} \) by the velocity at scale of \( L_{diff} \). Assuming \( \tau_s \approx \tau \) scaling appropriate for exponential stretching, the result is \( L_{diff} = \left[ \kappa (U/L_s)^3 \right]^{1/2} \), where \( U \) is the rms turbulent velocity on the scale of the sources. A residence-time average diffusivity, assuming the WNM contains more than 20% of the ISM mass, is \( \kappa \sim 10^{20} \text{ cm}^2 \text{s}^{-1} \). Then the average diffusivity scale in the ISM is \( L_{diff} \sim 0.06 (L_{100}/U_{100})^{1/2} \) pc, where \( L_{100} = L_s/100 \) pc, and \( U_{100} = U \) in units of 10 km/s (Kulkarni & Heiles 1987). We assume the same scale for \( z \sim 3 \) LBGs, noting that \( L_{diff} \) only enters the mixing time (below) logarithmically.

The mixing time follows from the assumed exponential stretching, in which scales change as \( dl/dt = -\gamma l \), where \( \gamma = U/L_s \) is the large scale strain rate. This gives a time to bring tracers from \( L_s \) to \( L_{diff} \) as \( \tau_{mix} = (L_s/L_{diff}) \ln(L_{diff}/L_s) = (L_s/U) \ln(U L_s/\kappa) \). The quantity \( U L_s/\kappa \) is the diffusivity analogue of the Reynolds number, called the Peclet number \( P_e \). Numerically \( P_e \sim 3 \times 10^7 (U_{100} L_{100}/\kappa_{20}) \), giving \( \tau_{mix} = (L_s/U)^2 \ln(P_e) \sim 75 \) Myr. The result could be smaller if the velocity dispersion increases with the SFR, as in turbulence driven by SNe (e.g., Dib & Burkert 2005).

The primordial fraction is the cumulative pdf of the gas metallicity \( \int_0^Z f(Z', t) dZ' \), where \( f(Z, t) \) is the differential pdf of metallicity, as the integration limit \( Z \) approaches zero. We can obtain the equation for \( P(t) \) heuristically, without details of the integral closure (Janicka et al. 1973) we used to derive the full pdf equation corresponding to the advection-diffusion equation eq 1 (PS), a turbulent mixing closure that gives the same timescale for exponential variance decay as found in simulations by de Avillez and Mac Low (2002, eq. 17), and the same dependence of mixing time on initial size and SN rate (their Fig.7), if velocity dispersion \( U \) scales as the square root of the SN rate (Dib and Burkert 2005).

The primordial fraction decreases whenever fluid elements with primordial mass fraction \( P \) and gas that has been polluted by sources or previous mixing events, with mass fraction \( 1 - P \), are stretched sufficiently to result in diffusive mixing. This interaction occurs with an average frequency \( \tau_{mix}^{-1} \). \( P \) is also gradually depleted by recycling through massive stars that inject metals when they ex-
Most mixing processes predict a primordial fraction that is either unity or zero at $z \sim 3$ because they mix on a timescale that is much larger or smaller than the empirical SF ages. $P(t)$ should be zero in almost all galaxies if stellar explosions mix as efficiently as assumed in sequential enrichment models (or much more efficiently, Tumlinson 2006). Our turbulence-enhanced diffusivity model naturally preserves primordial gas from rapid mixing for a few times the mixing time, which itself depends only weakly on parameters, in particular the assumed IMF or the averaged diffusivity, and gives an intermediate primordial fraction in galaxies with SF ages $\sim 1 - 3 \times 10^8$ yr. That this timescale happens to match the star formation ages of these galaxies is no coincidence if star formation is driven by turbulence (Mac Low & Klessen 2004) powered by stellar explosions. Future systematic investigations of the spectrophotometric signatures of primordial gas in galaxies could distinguish these possibilities.

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Fig. 1.— Primordial fraction as a function of SF duration for combinations of the mixing timescale $\tau_{mix}$ and the source timescale $\tau_{src}$ (see sec 2.2). The empirical value for $\tau_{src}$ is about 5 Gyr within a factor of a few for an IMF with $M_1 = 0.1$. $\tau_{src}$ is independent of IMF, much larger but does not affect the result significantly, as discussed in the text. The range suggested for $z \sim 3$ LBGs (JH) is indicated by the arrow. Also shown is an infall model with infall rate equal to the SFR.

plode, on a timescale $\tau_{src}$ (sec 2.2), which we assume is constant for the times of interest. The equation for the primordial fraction is then

$$dP/dt = -P(1 - P)/\tau_{mix} - P/\tau_{src}$$

(2)

whose solution is, using the fact that $\tau_{mix} \ll \tau_{src}$,

$$P = (1 + \tau_{mix}/\tau_{src} \exp(t/\tau_{mix}))^{-1}$$

(3)

$P$ decreases exponentially on timescale $\tau_{mix}$, but only after a delay time $\tau_{mix} \ln(\tau_{src}/\tau_{mix})$, which is $3 \times 10^8$ yr for $\tau_{mix} = 75$ Myr and $\tau_{src} = 5$ Gyr assuming an IMF with $M_1 \lesssim 8 M_\odot$. The delay time is the time for sources to provide enough non-primordial gas to make $P$ depart from unity, but the dependence on $\tau_{src}$ is only logarithmic, and so is nearly independent of IMF, even for the extreme case of a VMS IMF (see sec 2.2).

The behavior of $P$ as a function of time for different $\tau_{mix}$ and $\tau_{src}$ is illustrated in Fig. 1. The smallest mixing timescale shown, 0.025 Gyr, corresponds to a larger velocity dispersion $\sim 30$ km/s, not unreasonable for galaxies with large SFRs. Our major result is that $P(t)$ declines on a timescale similar to SF ages inferred from empirical modeling, without adjustment of parameters.

The effect of $Z = 0$ infall can be understood by adding to eq 2 a term $P(1 - P)/\tau_{in}$ where $\tau_{in} = M_{gas}/$infall rate is the infall timescale. An example is shown in Fig. 1. Infall allows intermediate values of $P(t)$ for a longer time, but only for infall timescales close to $\tau_{mix}$, implying a huge infall rate. Therefore it is unlikely that infall modifies the primordial fraction predicted by turbulent mixing. Galactic winds with large rates (Erb et al. 2006) have no effect if the winds sample the full pdf of metallicity. If the galaxies have undergone previous episodes of SF with an accumulated duration as large as $\sim 1$ Gyr, even the turbulent model cannot explain the intermediate primordial fractions claimed by JH.
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