Institute for Theoretical and Experimental Physics of the State Committee for Atomic Energy of the USSR.

L. L. Gol'din, Ju. P. Sivkov

BUNCHING OF PARTICLES BEFORE THE BEGINNING OF ACCELERATION

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Abstract

Beam bunching in the strong focusing proton synchrotrons is considered. Methods of the bunching of the beam in the accelerators with high betatron oscillations frequencies are discussed.

1. Introduction

Self-phasing in an accelerator makes it possible to have stable acceleration also of particles whose initial phase and revolution frequency differ from the values as determined by the accelerating voltage. The boundary of the stability region in phase space \((\phi, \dot{\phi})\), i.e. the separatrix, can be expressed by the equation

\[ \ddot{\phi} = 2 \left( \cos \phi + \cos \phi_s \right) + 2 \sin \phi_s \left( \phi + \phi_3 - \pi \right) \]

where \(\phi_s\) is the stable phase, counted from the moment at which the accelerating voltage goes through zero, \(\phi\) the phase of an accelerated particle when it passes the accelerating gap, \(\dot{\phi} = \frac{d\phi}{d\tau}\) with \(\tau = 2\pi F_{\text{synchr}}\), where \(F_{\text{synchr}}\) is the frequency of small synchrotron oscillations, corresponding to \(\phi_s = 0\).

A separatrix for \(\phi_s = \frac{\pi}{6}\) is shown on Fig. 1. Particles with \(\phi\) and \(\dot{\phi}\) outside the loop of the separatrix (bucket) will sooner or later hit the chamber walls.

The energy spread of the particles leaving the injector (an electrostatic generator) is very small in comparison to the spread which is compatible with the conditions of phase stability. Therefore, frequency matching consists simply in making the accelerating frequency \(f_{\text{acc}}\) equal to the revolution frequency multiplied by a harmonic number, \(q_f\). If that has been achieved, the injected particles occupy in phase space a narrow band along the \(\phi\) axis (Fig. 1). The capture efficiency, which is the number of particles captured into stable acceleration divided by the total number of particles, is then determined by the diameter of the bucket measured on the \(\phi\) axis, divided by \(2\pi\).
Another condition, which must be fulfilled when optimising the capture, requires that the real revolution frequency of the particles multiplied by the harmonic number, $q_f$, is close to the synchronous revolution frequency $f_H$, which corresponds to the magnetic field on the axis of the vacuum chamber. A deviation of this frequency means that the momentum of the particles does not correspond to the magnetic field. This causes the appearance of a $\Psi$ function and larger betatron oscillation amplitudes than their optimum values.

As one can see from Fig. 1, the capture efficiency for $\phi_s = \frac{n}{6}$ hardly exceeds 0.5 and only a small fraction of the phase space inside the bucket is used.

By prebunching the beam of injected particles one can distort the distribution of particles in $\phi$ and $\dot{\phi}$ in such a way that it better matches the shape of the bucket. As Johnsen 1) has shown, such a distortion occurs when the particles move outside the bucket. After the prebunching, one must then displace the bucket on to the bunched beam.

Johnsen 1) in his paper discusses in detail prebunching by a non-accelerating field $\phi_s = 0$ and presents some numerical results concerning the capture into the CERN Proton Synchrotron for $\phi_s = 0$ and $f_{\text{acc}} > q_f$. Calculations for $f_{\text{acc}} < q_f$ are given in a paper by Courant 2).

The present report presents the calculation of beam prebunching adapted to the parameters of the ITEF proton synchrotron. The focusing of this accelerator is considerably stronger than that of the Geneva and Brookhaven synchrotrons. A large range of initial conditions has been considered and some peculiarities have been found which are of general interest.
2. Calculation of Phase Motion during Prebunching

The phase motion of particles are described by the well known equation

\[ \frac{d^2 \phi}{dt^2} = \sin \phi_s - \sin \phi \]  \hspace{1cm} (2)

The phase trajectories \( \dot{\phi}(T) \), \( \ddot{\phi}(T) \) according to eqn. (2) and for the case \( \phi_s = \text{const} = \frac{\pi}{6} \) have been calculated on an electronic computer.

The deviation of particles in \( \dot{\phi} \) is proportional to the difference between \( qf \) and \( f_{\text{acc}} \). In fact

\[ \dot{\phi} = \frac{d\phi}{dT} = \frac{1}{2 \pi F_{\text{synchr}}} \frac{d\phi}{dt} = \frac{f_{\text{acc}} - qf}{F_{\text{synchr}}} \]  \hspace{1cm} (3)

From eqn. (3) follows that, if for a particle \( \dot{\phi} > 0 \), so \( f_{\text{acc}} > qf \) and if on the contrary \( \dot{\phi} < 0 \), \( f_{\text{acc}} < qf \).

The initial distribution of the injected monoenergetic particles is homogeneous in \( \phi_0 \) (\( \dot{\phi}_0 \) being the initial phase of an individual particle) and is characterised by a certain \( \dot{\phi}_0 = \text{const} \). With the help of Fig. 2, curves \( \dot{\phi}_0 = \text{constant} \) have been plotted for different times \( T \). From many curves, plotted for different \( \phi_0 \) and different times \( T \), the most suitable ones for prebunching have been selected. In order to determine the prebunching efficiency, a drawing of the bucket on transparent paper has been put on the plots. If \( \Delta \phi_0 \) is the interval on the curve \( \dot{\phi}_0 = \text{constant} \) which lays inside the bucket, the capture efficiency is obviously \( \frac{\Delta \phi_0}{2\pi} \).

In Figs. 3, 4 and 5 are presented some variants of particle prebunching both for \( \phi_0 > 0 \) and \( \phi_0 < 0 \). The lines are drawn through all points which have different \( \phi_0 \) and the same \( \phi_0 \) and \( T \). All points having the same \( \phi_0 \) are, in intervals of \( \frac{\pi}{6} \), marked by the same numbers. The curves have been
displaced in $\phi$ and $\dot{\phi}$ for convenience. The real coordinates of the points #4 are indicated on the graphs. Also indicated is the capture efficiency $K_j$ for all curves.

The "jump" of the bucket on the bunched beam at the end of the prebunching period is realised by changing the frequency and the phase of the accelerating voltage. This change can be carried out by means of an automatic feedback system connecting the accelerating field with the radius and the phase of the beam, as it has been proposed by Johnsen. However, it seems to be possible, to design a prebunch programme in such a way that it matches itself both frequency and phase of the accelerating field and the beam. A feedback system is then switched on only for refining and maintaining the obtained result.

3. Discrepancy between the momentum of particles and the magnetic field during the prebunching.

The synchronous (in respect to the magnetic field) revolution frequency

$$\frac{1}{q} f_H$$

rises linearly with the magnetic field according to the well known relationship

$$\frac{q f - f_H}{f_H} = \left( \frac{E_0^2}{E^2} - \alpha \right) \frac{\Delta P}{P_H} \approx \frac{\Delta P}{P_H}$$

where $E_0$ is the proton rest energy, $E$ its total energy, $\alpha$ the momentum compaction factor (for the orbit length) and $\frac{\Delta P}{P_H}$ the relative momentum error in respect to the value $P_H$, determined by the magnetic field. At injection $E_0 \approx E$, $\alpha \approx 1$ and $\frac{E_0^2}{E^2} - \alpha \approx 1$.

Because $f_{acc}$ is very different from $q f$ during the prebunching (for the ITEF accelerator $(f_{acc} - q f)/f_{acc} = 1 \pm 3\%$) the particles happen to be alternatively in an accelerating and a decelerating RF field.

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Consequently, their momentum and revolution frequency change only very little, while \( f_H \) rises. Therefore \( \Delta p_{PH} \) according to eqn. (4) decreases during the prebunching. For our accelerator it would be the best to have at the end of the prebunching \( \Delta p_{PH} = 0 \). The betatron frequency would then be close to 12.75 at the beginning of the acceleration, and the closed orbit deviations would be negligible. At the moment of injection, however, \( \Delta p_{PH} \) must then be greater than zero, that means the betatron frequency would then be considerably smaller than 12.75.

It is not dangerous, if the betatron frequency at injection comes close to a half-integer value, because \( \Delta p_{PH} \) changes rapidly during the prebunching. Experience shows that in this case even the crossing of a half-integer resonance does not cause a particle loss. Apart from this, in the majority of the practically interesting cases, the resonance \( Q = 12.5 \) will not be reached.

Let us calculate the momentum deviation of the particles at injection. We require that at the end of the prebunching \( \frac{p}{p_H} = 0 \) and \( qf = f_H \). We assume, furthermore, that the deviation of the acceleration frequency \( f_{acc} \) is constant all over the prebunching period, but in general not equal to \( f_H \). According to eqn. (3) the velocity of phase variation of a particle, which is, at the end of the prebunching, in the centre of a bunch, \( \phi_{end} \), corresponds then to the deviation of the acceleration frequency \( f_{acc} \) from the frequency \( f_H \), that is

\[
\phi_{end} = \frac{f_{acc} - f_H}{f_{synchr}} = \frac{\Delta f}{f_{synchr}}
\]  

(5)

The frequency deviation \( \Delta f \) is by definition maintained over the whole prebunching period.

The discrepancy between the momentum of a particle and the magnetic field at the beginning of the prebunching is, as can be deduced from equations (3), (4) and (5)
\[ \frac{\Delta P_o}{P_H} \approx \frac{qf_o - f_{H0}}{f_{H0}} = \frac{F_{\text{synchr}}}{f_H} (\dot{\phi}_{\text{end}} - \dot{\phi}_o) \]  \hspace{1cm} (6)

It is (as has been mentioned above), necessary to inject the particles into the accelerator with an advance in time in respect to injection onto the synchronous orbit.

It can also be seen from eqn. (6) that if all other conditions are equal - those prebunching variants must be chosen, for which \( \dot{\phi}_{\text{end}} - \dot{\phi}_o \) has the smallest value.

On Figs. 6 and 7 the variation of the frequencies \( f_{\text{acc}} \), \( qf \) and \( f_H \) is presented as functions of time. Figure 6 corresponds to the case \( \Delta f > 0 \), and Fig. 7 to the case \( \Delta f = 0 \). With the help of eqn. (6) one can easily find a formula for \( \Delta t_o \), the advance of the injection time,

\[ \Delta t_o = \frac{H}{P_H} \Delta n = \frac{H}{f_H} F_{\text{synchr}} (\dot{\phi}_{\text{end}} - \dot{\phi}_o) = 67 (\dot{\phi}_{\text{end}} - \dot{\phi}_o) \mu s \]  \hspace{1cm} (7)

In computing \( \Delta t_o \), it was assumed that,

\[ H = 90 \text{ gauss} \]
\[ \dot{H} = 6.3 \times 10^3 \text{ gauss/sec} \]
\[ f_H = 7.8 \times 10^5 \text{ hertz} \]
\[ F_{\text{synchr}} = 3.67 \text{ khertz} \]

The optimal duration of the prebunching, \( \Delta t_{\text{3}} \), is to be determined by a comparison of the different values of the prebunching parameters in the phase space \((\phi, \dot{\phi})\). For the transition from \( \tau \) to \( t \) we have, obviously,

\[ \Delta t_{\text{3}} = \frac{\tau_{\text{opt}}}{2F_{\text{synchr}}} = 43 \tau_{\text{opt}} \text{ (in \mu s)} \]  \hspace{1cm} (8)

where \( \tau_{\text{opt}} \) corresponds to an optimal prebunching.
4. The calculation of prebunching parameters

a) The case $\dot{\phi}_0 > 0$.

The mode of prebunching allows theoretically the achievement of a capture efficiency of the order of $0.8 \pm 0.9$. One of the very best occurs in the case of $\dot{\phi}_0 = 2.5$. Fig. 4 shows that the capture efficiency for this case does not depend very much on $\tau_3$ and that the velocity of phase change $\dot{\phi}$ at the end of the bunching is great. It is, therefore, possible to select such a $\tau_3$ that the phase of the centre of the bunch is, at the end of the prebunching, $\pi + 2\pi n$. The frequencies $f_{acc}$ and $f_H$ are then, at the end of the prebunching, well enough matched. Having found that the center of the future bucket (point No. 4) was, at the beginning of the prebunching, at $\phi_0 = -1.4$, one can determine $\dot{\phi}_{end}$ for the time when $\phi_{end} = \frac{\pi}{6} + 2\pi n = 19.4$.

Using the first integral of eqn. (2)

$$\dot{\phi} = \sqrt{\dot{\phi}_0^2 + 2(\cos \phi - \cos \phi) + 2 \sin \phi (\phi - \phi_0)}$$

we find $\dot{\phi}_{end} = 5.32$.

We obtain from eqn. (5)

$$\Delta f = f_{synch} \dot{\phi}_{end} = 19.5 \text{ kHz}$$
$$\tau_{opt} = 5.4$$
$$\Delta t_3 = 43 \times 5.4 = 230 \mu s$$

The time by which the injection must be advanced, is to be determined from eq. (7),

$$\Delta t_o = 67 (5.32 - 2.5) = 190 \mu s.$$
The momentum mismatch at the beginning of the prebunching is in this case
\[
\frac{\Delta p}{p_H} = 1.3 \text{o/o}
\]

If the undisturbed betatron frequency is equal to 12.75, at the beginning of the prebunching, it should be
\[
12.75 - 16 \cdot \frac{\Delta p}{p_H} = 12.54
\]

The somewhat quieter case with
\[
\tau = 4
\]
gives
\[
\phi_{\text{end}} = 4.7
\]
\[
\Delta f = 17.2 \text{ kHz}
\]
\[
\Delta t_3 = 200 \text{ } \mu\text{s}
\]
\[
\Delta t_0 = 150 \text{ } \mu\text{s}
\]
\[
\Delta Q_0 = -0.17
\]
\[
\Delta Q_0 = 12.58
\]

b) The case \( \dot{\phi}_o < 0 \)

One of the most favourable cases for prebunching is that obtaining to \( \dot{\phi}_o = -3.0 \). The results of a calculation for \( \phi_{\text{end}} = \frac{\pi}{6} \) and \( \dot{\phi}_o = -3 \) are presented in Table I. The capture efficiency for \( \Delta f > 0 \) is equal to \( \sim 0.65 \), while the momentum mismatch at the beginning of the prebunching is only 0.62 o/o. With \( \Delta f > 0 \) it is possible to make \( K_3 = 0.8 \) by the sacrifice of an increase in the momentum mismatch up to 2.4 o/o. The table also presents the case \( \dot{\phi}_o = -4.0 \).

Of special interest is prebunching without frequency shift (\( \Delta f = 0 \)). In this case prebunching is obtained only by advancing the injection in time.
### Table I

<table>
<thead>
<tr>
<th>Case</th>
<th>$\dot{\phi}_o$</th>
<th>$\phi_{\text{end}}$</th>
<th>$\Delta t_{3\mu s}$</th>
<th>$\Delta t_{o\mu s}$</th>
<th>$\dot{\phi}_{\text{end}}$</th>
<th>$\Delta f_{\text{kHz}}$</th>
<th>$\Delta f_{p/o/o}$</th>
<th>$\Delta q_o$</th>
<th>$\Delta r_{o\text{mm}}$</th>
<th>$K_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3,0</td>
<td>$\pi/6$</td>
<td>87</td>
<td>150</td>
<td>-1,7</td>
<td>-6,2</td>
<td>0,62</td>
<td>0,10</td>
<td>6</td>
<td>0,65</td>
</tr>
<tr>
<td>2</td>
<td>-3,0</td>
<td>$\pi/6$</td>
<td>335</td>
<td>285</td>
<td>2,0</td>
<td>7,3</td>
<td>2,40</td>
<td>0,385</td>
<td>36</td>
<td>0,8</td>
</tr>
<tr>
<td>3</td>
<td>-4,0</td>
<td>$\pi/6$</td>
<td>145</td>
<td>240</td>
<td>-1,85</td>
<td>-6,8</td>
<td>1,03</td>
<td>0,165</td>
<td>15,5</td>
<td>0,70</td>
</tr>
</tbody>
</table>

### Table II

Bunching parameters for some cases with $\Delta f = 0$

<table>
<thead>
<tr>
<th>$\dot{\phi}_o$</th>
<th>$\tau_{-3}$</th>
<th>$\phi_{\text{end}}$</th>
<th>$K_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3,0</td>
<td>5,5</td>
<td>-1,6</td>
<td>0,62</td>
</tr>
<tr>
<td>-2,75</td>
<td>3,0</td>
<td>-4,0</td>
<td>0,68</td>
</tr>
<tr>
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<td>3,2</td>
<td>-2,9</td>
<td>0,70</td>
</tr>
<tr>
<td>-2,5</td>
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<td>-2,9</td>
<td>0,79</td>
</tr>
<tr>
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<td>2,7</td>
<td>-2,9</td>
<td>0,74</td>
</tr>
<tr>
<td>-2,25</td>
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<td>-2,9</td>
<td>0,70</td>
</tr>
<tr>
<td>-2,25</td>
<td>2,2</td>
<td>-2,9</td>
<td>0,58</td>
</tr>
<tr>
<td>-2,0</td>
<td>2,2</td>
<td>-2,1</td>
<td>0,64</td>
</tr>
<tr>
<td>-2,0</td>
<td>2,0</td>
<td>-2,8</td>
<td>0,58</td>
</tr>
</tbody>
</table>
Fig. 8 shows the phase-space curves as a function of $\dot{\phi}_o$ and Fig. 9 the dependence of the optimal capture efficiency on $\dot{\phi}_o$. Table II presents calculated prebunching parameters for $\Delta f = 0$.

5. Some thoughts on the adjustment of prebunching parameters

In order to adjust a prebunching programme, it is necessary to choose the correct values for the three main parameters: the frequency shift $\Delta f$, the time $\Delta t_o$ by which the injection has to be advanced and the duration of the prebunching $\Delta t_3$. Before one starts to adjust the prebunching, one must adjust the normal injection and acceleration programme, by which the particles are injected onto the synchronous orbit, and in which $f_{\text{acc}} = qf$. The orbit deviations from the centre of the vacuum chamber are at the beginning very small, and the initial frequency is so chosen that the capture is optimal.

One must then set the value $\Delta t_o$, according to the chosen prebunching programme. Furthermore, the RF frequency must be shifted by $\Delta f$ and at the end of the prebunching (after the time $\Delta t_3$) it must return to the value which is in agreement with the magnetic field.

When setting the parameters $\Delta f$ and $\Delta t_3$, one must keep the following in mind: The prebunching time depends strongly on $\Delta f$, whereas a change of $\Delta t_3$, when $\Delta t_o$ is kept constant, results in a discrepancy between $f_H$ and $qf$ at the end of the prebunching. The capture efficiency does not depend very much on $\Delta t_3$, but the phase difference between the beam and the accelerating field changes rapidly. It is, therefore, very likely that for a given $\Delta f$, one can find a value $\Delta t_3$ with a reasonable capture efficiency. If that is not the case, one would change $\Delta f$ slightly (by 0.3 ± 0.5 kHz) and continue the search.
If one does not succeed in finding a good capture efficiency by varying \( \Delta t_3 \) in the limits of, say, \( \pm 30 \pm 5 \), one must change \( \Delta t_o \) and shift correspondingly the range of possible variation of \( \Delta t_3 \). In the case of \( \dot{\Phi}_o > 0 \), the adjustment of the parameters is easier, because a variation of \( \Delta t_3 \), which is sufficient for bringing the phase of the centre of the bunch onto the stable phase of the acceleration voltage, does not cause a noticeable discrepancy between \( f_H \) and \( qf \).

The stability requirements for \( \Delta t_o \) are not high. Obviously, an instability of \( \Delta t_o \) of the order of \( \pm 10 \mu s \) can easily be tolerated. The instability of \( \Delta f \) should be small compared with the diameter of the bucket, for instance \( \pm 5 \times 10^{-4} f_{acc} \), that is \( \pm 0.4 \text{ kHz} \). Most serious are the stability requirements on \( \Delta t_3 \), especially for \( \dot{\Phi}_o > 0 \). In this case \( \dot{\Phi} \) reaches at the end of the bunching a value of \( 5 \times 6 \) and an error in \( t_3 \) of \( 10 \mu s \) causes a phase discrepancy of \( 0.25 \) radian. It seems possible to adjust \( \Delta t_3 \) and to keep it stable with an accuracy of not less than \( \pm 3 \mu s \). For \( \dot{\Phi}_o \leq \dot{\Phi} \) the accuracy of \( \Delta t_3 \) can be reduced by a factor of \( 2 \times 3 \). Much bigger errors can be tolerated if at the end of the prebunching a feedback system between the beam and RF field starts to operate.

**Conclusion**

A noticeable prebunching can be obtained, if the particles are allowed to move for a short time outside the bucket. The calculated capture efficiency of 0.5 can be increased up to 0.65 \( \pm 0.85 \). The synchronisation of both phase and frequency of the particles with the accelerating field at the end of the prebunching can be achieved by controlling amplitude and timing of the frequency modulation. The automatic control system for the frequency of the accelerating field based on radial and phase position of the beam can accept only small errors in the frequency programme system of the bunching period.
The capture efficiency comes out somewhat better for $f_{\text{acc}} > qf$ than for the case where the opposite inequality holds. The frequency deviation is in the first case $\sim 20$ kHz and a rather high stability of the bunching time is required. In the case $f_{\text{acc}} < qf$, where the capture efficiency is small, the tolerances for the setting of the parameters are less critical. Furthermore, the momentum deviation of the particles relative to the synchronous particle turns out to be less important for the second case.

Prebunching makes it necessary to inject the particles somewhat earlier, that means at a smaller magnetic field strength than without prebunching. The experience from running the accelerator shows that a momentum deviation of $+1.5$ o/o is fully acceptable. On the 7-GeV ITLF accelerator prebunching of the particles is, therefore, possible with RF frequencies either greater or smaller than the synchronous frequency. Finally, prebunching is even possible without changing the RF frequency at all by only injecting the particles somewhat earlier. The particles are bunched, however, in this case not in the neighbourhood of the stable phase and it is, therefore, necessary to change the RF phase at the end of the prebunching in a time interval which is small compared to a period of the synchrotron oscillation.

If a beam with a small energy spread is captured in the conventional way, a momentary bunching of the particles into a narrow phase interval occurs at the end of the first quarter of the first synchrotron oscillation. As a result of this the space charge density in the bunch is sharply increased and the beam intensity, consequently, limited. If prebunching is applied, the beam is always sufficiently spread in phase and the maximal space charge density turns out to be considerably smaller than with conventional injection.

12.9.63

References:
2) E. D. Courant, Ibid, p. 201.
Fig. 1. Separatrices in phase space and initial position of the injected beam.
Fig. 2. Graph of phase displacements.
The numbers on the curves indicate
the time in units of $\tau$. 
Fig. 3. Phase trajectories for $\phi_0 = +3$
Fig. 4. Phase trajectories for $\dot{\phi}_0 = +2.5$
Fig. 5. Phase trajectories for $\dot{\phi}_o = -3$

Fig. 6. Variation of the frequencies for $\Delta f > 0$
Fig. 7. Variation of the frequencies for $\Delta f = 0$.

Fig. 8. The position of the particles in phase space in the case of optimal prebunching (variant $\hat{\phi}_K = 0$, without shift of the accelerating frequency).
Fig. 9. The dependence of the optimal capture coefficient on the initial momentum discrepancy ($\Delta f = 0$)