ACHROMATISM IN FINAL FOCUSING SYSTEMS FOR HIGH-CURRENT HEAVY-ION BEAMS

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Focusing driver beams onto a small focal spot is critical to the success of heavy-ion fusion. Undesired increase in spot size can result from chromatic aberration caused by velocity tilts near beam ends. These tilts are caused by the electrostatic force along the direction of propagation of a high-current beam. We demonstrate that this aberration can be reduced by designing the final focusing system so that it has some symmetry with respect to its longitudinal midpoint and by realizing the symmetric and anti-symmetric properties of the trajectories and velocities of the beam particles.

1 INTRODUCTION

The ability to focus a beam onto a small focal spot (a few millimeter in radius) is critical to the success of heavy-ion fusion (HIF). This is because, for a fixed driver energy, the target gain increases as the spot size decreases (up to a certain point). Chromatic aberrations, which can arise from two sources, can increase spot size. The first source, which is well known, is due to beam particles having finite temperature in longitudinal direction, i.e., the direction of beam propagation. The second source, which can be severe for high-current beams (in the kiloampere range), is due to the velocity tilts caused by electrostatic repulsion in the longitudinal direction.

Here, we address the chromatic aberration caused by the second source. In Section 2, this aberration is explained in more detail and a method to reduce it for an idealizing final focusing system is illustrated. In Section 3, we point out that an actual focusing system can be designed so that it has features that resembles the idealized system.

2 PRINCIPLE OF REDUCING CHROMATIC ABERRATION CAUSED BY VELOCITY TILTS INDUCED BY THE SPACE-CHARGE FORCE

When an ion beam passes through a final focusing quadrupole system, there must be no longitudinal velocity tilt (i.e., the beam must have uniform momentum); otherwise there will be chromatic aberrations. However, to focus high-current
heavy-ion beams onto millimeter-size spots, the final focusing quadrupole system is usually long (roughly 10 meters or more). Thus, there is sufficient time for the velocity tilts induced by the longitudinal space-charge force to develop at the beam ends. The peaks of the velocity tilts can be a few percent of the beam center-of-mass velocity. If the tilts are left uncompensated, severe chromatic aberrations can result.

We now demonstrate that for an idealized final focusing system that is symmetric with respect to the longitudinal midpoint of the system, the chromatic aberration caused by velocity tilts can be suppressed.

Figure 1a shows such a focusing system. This system has a waist-to-waist configuration, i.e., its envelope has zero slope at the starting point and at the focal spot. Figure

![Diagram](image-url)

**FIGURE 1** Schematics of an ideal final focusing system that consists of five quadrupoles and has a waist-to-waist configuration. The beam is assumed to move from left to right. (a) Beam envelopes. (b) Beam line-charge density profiles at the two waists and the midpoint of the focusing system. (c) Velocity-tilt profiles in the beam frame at the same locations.
Figure 1a shows the beam envelopes in the transverse $x$ and $y$ directions. The envelopes and the quadrupole arrangement are symmetric with respect to the longitudinal midpoint of the system. Figures 1b and 1c show the evolution of the line-charge density and longitudinal velocity tilt $v_z$ (in the beam frame) at three locations (points i, ii, and iii as marked in Figure 1a) along the final focusing system. We assume that the beam is moving from left to right. The line-charge density is decreasing toward the right end of the beam to satisfy the isentropic compression requirements on pellets for inertial confinement fusion.

The central figure in Figure 1c shows zero velocity tilt. This is the ideal beam configuration, which should be maintained throughout the final focusing system to avoid chromatic aberration. However, in long final focusing systems carrying high-current beams, the beam end expansion due to longitudinal space-charge force in the longitudinal direction launches rarefaction waves that can propagate, over a considerable fraction of the beam length, toward the beam center while the beam is traversing through the focusing system.

The rarefaction wave fronts travel toward the beam center at approximately the ion sonic velocity $v_s$ which can be expressed as:

$$v_s = \sqrt{\frac{Ze g \lambda}{4\pi\varepsilon_0 m}} \tag{1}$$

Here, $Z$ is the ion charge state; $e$ is the electronic unit of charge; $\varepsilon_0$ is the permittivity of free space; $g$ is a geometric factor that has a value in the order of unity; $\lambda$ is the line-charge density; and $m$ is the ion mass. The maximum velocity of the velocity tilts generated by the rarefaction wave at the beam ends is about twice the sonic velocity.

The chromatic aberration induced by the velocity tilts can be suppressed by ensuring that the beam is at the longitudinal center of the focusing system when the beam velocity tilt is zero (as shown by the middle figures of Figures 1b and 1c). Since the propagation of the rarefaction waves is time reversible, the line-charge density and velocity tilt profiles must be symmetric and anti-symmetric, as shown by the left and right figures in Figures 1b and 1c, with respect to the midpoint of the focusing system. (A detailed description of how to obtain the specific line-charge density and velocity-tilt profiles at the entrance of the focusing system is presented in Ref. 1).

For a cold beam with an initially uniform density profile, the particle trajectories are simply given by the product of the transverse dimension of the envelope and a constant that depends on particle initial position. And the transverse dimension of the envelope is a function of the line-charge density. Beam particle trajectories are therefore symmetric with respect to the midpoint of the system. The symmetry of the focusing system and symmetric and anti-symmetric properties of the trajectories and velocities of the beam particles lead to the suppression of the chromatic aberration caused by the velocity tilts.

This suppression of the chromatic aberration can be seen easily by inspecting the expression for the chromatic aberration of a single particle. To obtain this expression, we start with the single-particle equation of motion. This equation in the transverse
direction $x$ has the form

$$x'' + \frac{V'_z}{V_z} x' + (k_x^2 - k_{xx}^2)x = 0,$$

(2)

where $k_x^2$ and $k_{xx}^2$ are the linear external magnetic focusing force and space-charge force per unit mass divided by the square of the particle longitudinal velocity $V_z$ in the laboratory frame, respectively, and the double prime represents the second derivative with respect to $z$. Both $k_x^2$ and $k_{xx}^2$ are symmetric if the beam is at the longitudinal center of the focusing system when the velocity tilt is zero.

We now expand $k_x$ and $k_{xx}$ in Eq. (2) in the small parameter $\delta \equiv \Delta P/P_0$, where $P_0$ is the (constant) momentum of the beam center and $\Delta P$ is the deviation from $P_0$ caused by the velocity tilts. At the beam ends, we have $\delta \approx \pm 2 \frac{mv_o}{P_0}$. After expansion, we obtain

$$k_x^2 = k_{x0}^2(1 - \delta),$$

(3)

$$k_{xx}^2 = k_{xx0}^2(1 - 2\delta),$$

where $k_{x0}^2$ and $k_{xx0}^2$ are $k_x^2$ and $k_{xx}^2$ with $\delta = 0$. Thus, Eq. (2) to zeroth order in $\delta$ is

$$x''_i + (k_{x0}^2 - k_{xx0}^2)x_i = 0,$$

(4)

whose solution is

$$x_i = c_i(z)x_0 + s_i(z)x'_0,$$

(5)

where $c_i$ and $s_i$ are cosine- and sine-like functions and the quantities with subscript zero are initial conditions.

Equation (2) to first order in $\delta$ is

$$\eta'' + (k_{x0}^2 - k_{xx0}^2)\eta = f.$$  

(6)

Here, $\eta$ is the correction to $x_i$ due to $f$. At the focal spot, $\eta$ represents the magnitude of the chromatic aberration. The inhomogenous term $f$ is given by

$$f \equiv (k_{x0}^2 - 2k_{xx0}^2)x_i \delta - \frac{V'_z}{(P_0/m)} x'_i,$$

(7)

Note that $V'_z/(P_0/m)$ has the order of $\delta'$. The Green's function solution to Eq. (6) is

$$\eta = -c_i(z_f) \int_0^{z_f} f(\tau)s_i(\tau) \, d\tau + s_i(z_f) \int_0^{z_f} f(\tau)c_i(\tau) \, d\tau,$$

(8)

where $z_f$ is the length of the focusing system. Here $k_x$, $k_{xx}$, and $v'_x$ are symmetric while $\delta$ is anti-symmetric. Also, $c_i$ and $s_i$ are symmetric for a symmetric focusing system and thus $x_i$ and $x'_i$ are symmetric and anti-symmetric, respectively. Consequently, both the integrals in Eq. (8) vanish and the chromatic aberration is therefore zero. Chromatic aberration in the $y$ direction is zero as well.

The above result holds for a cold beam with a linear space-charge force. In real beams, the space-charge force deviates from linearity near the focus because of
nonzero emittance. But as shown in Section 3, the space-charge contribution to $\eta$ is smaller than the contribution from the external magnetic force. Thus, a linear space-charge force is a good approximation in obtaining results for the above equations.

For the scheme described here to work, zero velocity tilt must occur at the midpoint of the focusing system. This cannot happen for every part of the beam because the beam has finite length. Figure 1c indicates that the velocity tilt at the left end of the beam is the greatest because the line-charge density is maximum in the left part of the beam and falls off abruptly at the left end when the velocity tilt is zero. To minimize the overall chromatic aberration, one should therefore require that the left end of the beam passes the midpoint of the focusing system when the velocity tilt is zero.

3 SAMPLE FINAL FOCUSING SYSTEM

In this section, we present the design of a final focusing system for HIF drivers that has some degree of symmetry with respect to the longitudinal midpoint of the system. We then outline the method for determining the location $L$, near the mid-point of the final focusing system, to put the beam end when the velocity-tilt is zero so that the chromatic aberration can be suppressed.

As shown in Figure 2, the design has a waist-to-waist configuration for a 5 kA, 10-GeV beam with an unnormalized emittance of 20 mm·mrad. The beam is composed of ions of charge state 3 with atomic mass 210. The relativistic $\beta$ of the beam is approximately $1/3$. At the entrance of the focusing system, the beam has a circular cross section having a 3.46-cm radius. The beam then undergoes free-space expansion and is then focused by five quadrupoles onto a 3 mm radius spot. This final focusing system is designed using the envelope code TRACE.\(^3\) Note that the integrated value of $k_{xx}x$, over the length of the system, is only about 20% of $k_{xx}$.

The distance between the waists of the focusing system is 31.7 m. It takes about 0.15 $\mu$s for the beam to traverse half of the length of the focusing system and $v_x$ is about $8 \times 10^5$ m/s from Eq. (1). Thus, the part of the beam eroded by the rarefaction

FIGURE 2 Final focusing system for charge-state-3, 5-kA, 10-GeV beam with atomic mass 210 and normalized emittance 20 mm-mrad.
wave is approximately 0.12 m. This is a considerable fraction of the beam length if the main pulse (the part of the beam having the maximum current) is about 0.5 to 1 m long.

Because the envelope has some symmetry in both transverse directions, placing the beam end at a location $L$ close to the longitudinal midpoint of the focusing system, when the velocity tilt is zero, should greatly reduce the aberration in one direction and reduce it in the other direction.

To find the location $L$, we use Eq. (8). Since $\delta$ changes sign at $L$, there exists a location $L$ such that the integrals vanish in Eq. (8). The exact location of $L$ can be obtained from Eq. (8) using trial and error, placing $L$ at an arbitrary location near the midpoint of the system and then seeing if the integrals in Eq. (8) vanish. Usually the chromatic aberration is worse in one transverse direction than the other; we can thus require that the integrals vanish for the direction that gives the worse aberration.

4 CONCLUSION

Chromatic aberrations caused by velocity tilts near beam ends of heavy-ion beams in the kiloampere range can be reduced by proper design of the final focusing system. We have demonstrated that the chromatic aberration can be reduced by designing the final focusing system so that it has some symmetry with respect to its longitudinal midpoint, by proper placement of the beam end when the velocity tilt is zero at a location somewhere near the midpoint of the system, and by recognizing that the line-charge density and particle velocity are symmetric and anti-symmetric, respectively, with respect to this location.

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REFERENCES