ACCURATE CALIBRATION OF THE BEAM ENERGY IN A STORAGE RING BASED ON MEASUREMENT OF SPIN PRECESSION FREQUENCY OF POLARIZED PARTICLES*

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A method is described for measuring the particle energy in an electron-positron storage ring by means of resonance depolarization by a high frequency field. The measurement accuracy is discussed taking into account energy spread and synchrotron oscillations. It is found that in practice the limitation in accuracy is due to the irregular pulsations of the magnetic guide field. As a result, the electron beam energy in the storage ring VEPP-2M has been measured with an accuracy of ±2·10^{-5}.

I. INTRODUCTION

A set of experiments to study vector mesons with electron-positron storage rings has demonstrated the advantages of the method of colliding beams. Among these is high energy resolution: This resolution is restricted by the natural energy spread in the beam (about 10^{-3} in the φ-meson region). Methods used up to now for the absolute calibration of the particle energy in storage rings (measurement of the magnetic-field distribution, phase-oscillation frequencies, etc.) have provided an accuracy slightly better than 10^{-2}. An accuracy of 10^{-4}, an order higher than the energy spread, is of practical interest.

In addition, the energy-spread contribution to the reaction-energy uncertainty can markedly be decreased by energy decomposition of the beam particles at the interaction point. The energy decomposition should be fairly strong to eliminate "mixing" of the particles due to betatron (transverse) oscillations. If the direction of decomposition for both particles coincides (more energetic electrons collide with more energetic positrons), reaction identification requires a high accuracy of the interaction-point coordinates in the direction of decomposition. Provided that the direction of decomposition for electrons and positrons is opposite, the collision energy will be the same over the whole cross section of the colliding beams with an accuracy limited only by betatron mixing and corrections of the order of (Δγ/γ)^2.

This assumption results in a desire for absolute calibration of the particle energy in a storage ring with an accuracy much better than 10^{-4}. The urgency of this problem has grown in connection with the discovery of new narrow resonances (Gypsy-mesons).

Progress in experiments on beam-radiation polarization,1,2 which has been recently achieved on several electron-positron storage rings,3,4 provides a new method of absolute energy measurement. This method is based on measuring the energy-dependent spin precession frequency of relativistic particles by means of beam depolarization with an external high-frequency electromagnetic field resonant with a spin frequency. The accuracy of this method is not related to the energy spread of the beam particles to first approximation and even in the first experiments reached the value 10^{-4}.

II. EVALUATION OF METHOD ACCURACY

For a relativistic electron, the frequency of spin precession around the direction of the guiding field H, after averaging over betatron oscillations can be written in the form

Ω = ω_s(1 + γq'/q_0),
where \( \omega_s(\gamma) = eH_z/\gamma mc \) is the particle revolution frequency, \( \gamma \) is the relativistic factor, and \( q', q_0 \) are the anomalous and normal parts of the gyromagnetic ratio.

To measure the precession frequency, a method of beam resonance depolarization by a radio-frequency longitudinal magnetic field \( H_z \) is used.\(^2\)\(^3\)

The resonance condition is of the form

\[
\omega_s(1 + \gamma q'/q_0) = \omega_d + \kappa \omega_s,
\]

where \( \omega_d \) is the frequency of the external \( H_v \) and \( \kappa \) is an integer.

In the presence of an accelerating rf voltage the particle energy and hence the mistuning \( \epsilon(\gamma) = \omega(1 - \kappa + \gamma q'/q_0) - \omega_d \) oscillate with the synchrotron frequency \( \omega_s, \epsilon = 0 + \Delta \cos(\omega t + \phi). \) The modulation amplitude \( \Delta = (d\epsilon/d\gamma)\Delta \gamma \) is

\[
\Delta = \left[ \alpha(\kappa - 1) + (1 - \alpha)\gamma \frac{q'}{q_0} \right] \frac{\Delta \gamma}{\gamma} \omega_s,
\]

where \( \alpha \) is the momentum compaction factor.

As a result of this modulation, the spin-motion spectrum will have a central frequency and side bands at a distance \( \pm n\omega_s \), from the former, where \( n \) is an integer. The phase-motion averaged mistuning \( \bar{\epsilon} = \epsilon_s + \delta \Omega \), where \( \epsilon_s \) is the frequency shift of the synchronous particle, has a spread

\[
\delta \Omega = \omega_s q' q_0 \left[ \frac{\delta \gamma}{\gamma} \alpha - \frac{\alpha}{2} \left( \frac{\Delta \gamma}{\gamma} \right)^2 \right],
\]

where \( \delta \gamma = \bar{\gamma} - \gamma_s \) is the mean particle energy shift with respect to the equilibrium particle, proportional to the squared amplitudes of betatron and synchrotron oscillations.

In practice, the value of \( \delta \Omega \) will be governed by the squared nonlinearity of the magnetic guide field \( (\partial^2 H_z/\partial x^2) \).

\[
\delta \Omega = \omega_s q' q_0 \left( \frac{X^2 (\partial^2 H_z)}{H_z^2 (\partial x^2)} \right),
\]

where \( X^2 \) is the squared radial size. Estimation for the VEPP-2M storage ring gives \( \delta \Omega \approx 10^{-6} \). \( \omega_s \). With squared nonlinearity compensation, the spin frequency spread can be additionally reduced.

In an experiment the effective resonance width measured is characterized by the frequency band \( \delta \epsilon \), where the depolarization rate is about maximum. The central resonance width \( \delta \epsilon_0 \) is equal to the spin-frequency spread in the beam \( \delta \Omega \), if \( \delta \Omega \) exceeds the decrement of the radiative damping of particle oscillations \( \lambda \). Otherwise \( \delta \Omega \approx \lambda \), due to radiation effects, stochastic averaging of precession frequency will occur leading to an additional decrease of the effective band width to \( \delta \epsilon_0 = (\delta \Omega)^2/\lambda \).

The side resonance widths \( \epsilon_s = n\omega_s \) are determined by the synchrotron frequency spread \( \omega_s(\delta \omega_s \gg \lambda) \) and are usually much higher than that of the central one. In our case \( \lambda = 10^{-3}\omega_s \) and \( \delta \epsilon = 10^{-7}\omega_s \), while \( \Delta \gamma/\gamma = 10^{-3} \). Thus, in spite of the spread in the beam particle energy, the spin dynamics is such that the measurement of the central frequency of the spectrum, in principle, enables us to determine the absolute value of the mean energy to the limiting accuracy preset by the known value of the anomalous magnetic moment of the electron\(^7\)

\[
q' /q_0 = (11596524.1 \pm 2.0) \times 10^{-7}.
\]

In practice, however, the energy-measurement accuracy is restricted by harmonic and irregular pulsation of the magnetic field, leading to "smearing" of the mean frequency of spin precession. For example, for the VEPP-2M storage ring these restrictions give \( \delta \epsilon \approx 2 \times 10^{-7} \omega_s \).

### III. THE PRECESSION FREQUENCY MEASUREMENT

For the present study, a radio-frequency magnetic field on the orbit was produced by a depolarizer in the form of a current-carrying loop around a ceramic section of the vacuum chamber. The loop is a part of the resonance circuit excited by an external generator at a frequency \( \omega_d = \omega_s (2 - \gamma q'/q_0) \). A quick search for a resonance was initially done by frequency modulation at \( \omega_d = \omega_0 + \Delta \omega_0 \cos \Omega \cdot d \). Provided that the successive resonance crossings are uncorrelated and rapid, the depolarization time \( \tau_\text{d} = \Delta \omega_0 /w_0^2 \), where \( w_0 = \omega_s (H_v/H_z)(l/2L) \) is the frequency of the spin precession in the direction \( H_v \), and \( l/L \) is the effective length of the longitudinal field.

In our case the synchrotron-modulation index \( \Delta \omega_s = 0.1 \approx 1 \). Under these conditions the depolarization rate at the side frequencies \( (\epsilon_s = \pm n\omega_s) \) is small, and the central spectrum line is
readily separated by measuring the depolarization time versus mistuning $\epsilon_r$.

The measurements were made in the following way: an electron beam, after radiative polarization at high energy, was switched to the experimental energy; the polarization-dependent counting rate $N$ of particles escaping from the beam by elastic scattering within a bunch was measured at time $T$ a depolarizer was switched on. The process of beam depolarization is characterized by an increase of the counting rate divided by the squared beam current (Fig. 1).

The results of the depolarization measurement for the central and side resonances are given in Fig. 2, from which it is seen that the qualitative resonance pattern corresponds to that expected. Between the resonances depolarization was not observed.

The depolarization bandwidth $\Delta f_d = \Delta \omega_d/2\pi$ thus measured was equal to about 30 kHz. Then that band was reduced to 2 kHz. This enabled the mean particle energy to be determined by the central line depolarization with accuracy $\Delta \gamma / \gamma = 10^{-4}$, which was an order lower than the energy spread.

A more accurate measurement was made in the slow scanning regime with a depolarizer frequency in the form of a narrow line ($\Delta f_d = 30$ Hz) at the rate = $5$ Hz/sec (Fig. 3). An accuracy of a single measurement was achieved to be = $2 \cdot 10^{-5}$. The accuracy is mainly determined by the time (100 sec) required for statistic taking ($\tau_d = 10$ sec).

These results were used for performing a number of high-precision metrological experiments

![FIGURE 1 Dependence of the normalized counting rate on the time in the beam depolarization process.](image1)

![FIGURE 2 Dependence of the depolarization time on a depolarizer frequency.](image2)

![FIGURE 3 Dependence of the normalized counting rate on a depolarizer frequency.](image3)
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