We review the present electroweak precision data constraints on the mediators of the three types of see-saw mechanisms. Except in the see-saw mechanism of type I, with the heavy neutrino singlets being mainly produced through their mixing with the Standard Model leptons, LHC will be able to discover or put limits on new scalar (see-saw of type II) and lepton (see-saw of type III) triplets near the TeV. If discovered, it may be possible in the simplest models to measure the light neutrino mass and mixing properties that neutrino oscillation experiments are insensitive to.

1 Introduction

As it is well known, the original see-saw mechanism \(^1\), nowadays called of type I, explains the smallness of the light neutrino masses \(|m_\nu| \sim 1\text{ eV}\) invoking a very heavy Majorana neutrino \(M_N \sim 10^{14}\text{ GeV}\):

\[
|m_\nu| \approx \frac{v^2 |\lambda|^2}{M_N} \approx |V^*|^2 M_N,
\]

where \(|\lambda| \sim 1\) is the corresponding Yukawa coupling and \(v \approx 246\text{ GeV}\) the electroweak vacuum expectation value. For reviews see \(^2,3\). Alternatively, if the heavy scale is at the LHC reach \(M_N \sim 1\text{ TeV}\), it requires a very small heavy–light mixing angle \(|V| \sim 10^{-6}\). In its simplest form the model cannot be tested at large colliders, because the heavy neutrino \(N\) is a Standard Model (SM) singlet and only couples to SM gauge bosons through its mixing \(V\). Hence it is produced through the vertex \(-g/\sqrt{2} \bar{\ell} \gamma^\mu V_{\ell N} P_L N W^-_\mu\), with \(\ell\) a charged lepton, with a cross section proportional to \(|V_{\ell N}|^2\), which is strongly suppressed. See Fig. 1-(I). There are two other types of see-saw mechanism giving tree level Majorana masses to the light neutrinos \(\nu\), as shown
Figure 1: Examples of production diagrams for same-sign dilepton signals, $l^+l'^+X$, mediated by the three types of see-saw messengers.

Figure 2: See-saw mechanisms of type I, II and III. $\lambda_N$, $\lambda_\Delta$ and $\lambda_\Sigma$ are the Yukawa coupling matrices in the Lagrangian terms $-\overline{\ell}_L\phi\lambda^\dagger_N N_R$, $\overline{\ell}_L\lambda_\Delta(\bar{\nu} \cdot \bar{\Delta})\nu_L$ and $-\overline{\Sigma}_R\lambda_\Sigma(\bar{\phi} \cdot \bar{\Sigma})\nu_L$, respectively, with $\overline{\ell}_L = -i\gamma^\mu C\sigma_2 \ell_L$ and $C$ the spinor charge conjugation matrix. Whereas $\mu_\Delta$ is the coefficient of the scalar potential term $\bar{\phi} \cdot \bar{\Delta} \phi$.

In Fig. 2. In all cases the extra particles contribute at low energies to the dimension 5 lepton number (LN) violating operator $^4$

$$ (O_5)_{ij} = (\overline{\ell}_L)^c \bar{\phi} \phi^c \nu^i_L - \frac{\nu^2}{2} (\nu^i)^c \nu^j \quad \text{(with } l = \begin{pmatrix} \nu \\ \ell \end{pmatrix} \text{ and } \bar{\phi} = i\sigma_2 \phi^* \text{)} \ , \quad (2) $$

which gives Majorana masses to light neutrinos after spontaneous symmetry breaking. The see-saw of type II $^5$ in Fig. 2 is mediated by an $SU(2)_L$ scalar triplet $\Delta$ of hypercharge $Y = 1$, implying three new complex scalars of charges $Q = T_3 + Y$: $\Delta^{++}, \Delta^+, \Delta^0$. The see-saw of type III $^6$ exchanges an $SU(2)_L$ fermion triplet $\Sigma$ of hypercharge $Y = 0$, assumed to be Majorana and containing charged leptons $\Sigma^\pm$ and a Majorana neutrino $\Sigma^0$. The main difference for LHC detection is that the see-saw messengers for these last two mechanisms can be produced by unsuppressed processes of electroweak size (Fig. 1). Their decay, even if suppressed by small couplings, can take place within the detector due to the large mass of the new particle. All three types of see-saw messengers produce LN conserving as well as LN violating signals, but the former have much larger backgrounds. On the other hand, same-sign dilepton signals, $l^\pm l'^\pm X$, do not have to be necessarily LN violating. Thus, in the example in Fig. 1–(II), the decay
coupling $\lambda_\Delta$ needs not be very small because it is only one of the factors entering in the LN violating expression for $\nu$ masses (see Table 1). In fact, this process is LN conserving as we can

Table 1: Coefficients of the operators up to dimension 6 arising from the integration of the heavy fields involved in each see-saw model. The parameters $\lambda_1$ and $\lambda_3$ are the coefficients of the scalar potential terms $-\langle \phi^\dagger \phi \rangle (\Delta^\dagger \Delta)$ and $-\langle \Delta^\dagger T, \Delta \rangle (\phi^\dagger \phi)$, respectively, and $\lambda_{ij}$ are the diagonalised SM charged-lepton Yukawa couplings. The remaining parameters are defined in the caption of Fig. 2.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Type I</th>
<th>Type II</th>
<th>Type III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_4$</td>
<td>$\frac{(\alpha_3)_{11}}{\Lambda}$</td>
<td>$\frac{1}{2} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>$\frac{1}{2} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
<td>$\frac{1}{8} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>$\frac{1}{4} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
<td>$\frac{3}{16} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
</tr>
<tr>
<td>$\alpha_7$</td>
<td>$\frac{1}{3} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
<td>$\frac{1}{3} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
</tr>
<tr>
<td>$\alpha_8$</td>
<td>$\frac{1}{2} (\lambda_{ij}^2)<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
<td>$\frac{3}{16} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
</tr>
<tr>
<td>$\alpha_9$</td>
<td>$\frac{1}{3} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
<td>$-2 \frac{\mu_{ij}}{M_{\Delta}}$</td>
<td>$\frac{1}{3} (\frac{\lambda_{ij}^2}{M_{N_a}})<em>{1a} (\frac{\lambda</em>{ij}^2}{M_{N_a}})_{a_j}$</td>
</tr>
</tbody>
</table>

conventionally assign LN equal to 2 to $\Delta^-$. There are other processes that do violate LN, e.g. when one of the doubly-charged $\Delta$ in Fig. 2–(II) decays into WW. Then, what does violate LN is the corresponding $\Delta WW$ vertex, which is proportional to the coupling of the only LN violating term in the fundamental Lagrangian $\tilde{\phi}^\dagger (\tilde{\sigma} \cdot \Delta) \tilde{\phi}$, with total LN equal to 2. In the examples in Fig. 1–(1, III) LN is violated in the decay (mass) of the heavy neutral fermion.

In conclusion, all the three mechanisms produce same-sign dilepton signals, but only the last two are observable at LHC [7,8,9,10,11,12,13] in minimal models. Heavy neutrino singlets in particular non-minimal scenarios could also be observed, as described in Section 3.

In the following we first review the experimental constraints on the parameters entering the three see-saw mechanisms, and then the LHC reach for the corresponding see-saw messengers. Complementary reviews on this subject have been presented by other speakers at this Conference (see F. Bonnet, T. Hambye and J. Kersten in these Proceedings).

2 Electroweak precision data limits on see-saw messengers

The low energy effects of the see-saw messengers can be described by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_4 + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \ldots,$$

where $\Lambda$ is the cut-off scale, in our case of the order of the see-saw messenger masses $M$, and the different terms contain gauge-invariant operators of the corresponding dimension. The non-zero terms up to dimension 6 are $^{14,15}$

$$\mathcal{L}_4 = \mathcal{L}_{SM} + \alpha_4 \left( \phi^\dagger \phi \right)^2,$$

(4)
\[ \mathcal{L}_5 = (\alpha_5)_{ij} \overline{\phi^i} \gamma^\mu \phi^j L^\mu + \text{h.c.,} \]
\[ \mathcal{L}_6 = \left[ (\alpha_{\phi \phi}^{(1)})_{ij} \left( \phi^i \gamma^\mu \phi^j \right) \left( \overline{\phi^i} \gamma^\mu \phi^j L^\mu \right) \right. \]
\[ + (\alpha_{\phi \phi}^{(2)})_{ij} \left( \phi^i \gamma^\mu \phi^j \right) \left( \overline{\phi^i} \gamma^\mu \phi^j L^\mu \right) + \left( \alpha_{\phi \phi}^{(3)})_{ij} \right) \left( \phi^i \gamma^\mu \phi^j \right) \left( \overline{\phi^i} \gamma^\mu \phi^j L^\mu \right) + \text{h.c.} \]
\[ + \alpha_{\phi} \left( \phi^i \phi^j \right) \left( \phi^j \phi^i \right) + \alpha_{\phi}^{(3)} \left( \phi^i \gamma^\mu \phi^j \right) \left( \phi^j \gamma^\mu \phi^i \right) + \alpha_{\phi} \frac{1}{3} \left( \phi^i \phi^j \right)^3, \]

where we choose the basis of B"{u}chmuller and Wyler to express the result. \( l_L \) stands for any lepton doublet, \( e_R \) for any lepton singlet, and \( \phi \) is the SM Higgs doublet. In Table 1 we collect the explicit expressions of the coefficients in terms of the original parameters for each type of see-saw (see Fig. 2 and the table caption for definitions).

Only the dimension 6 operators can give deviations from the SM predictions for the electroweak precision data (EWPD). The operators of dimension 4 only redefine SM parameters. The one of dimension 5 gives tiny masses to the light neutrinos, and contributes to neutrinoless double \( \beta \) decay. An important difference is that the coefficient \( \alpha_5 \) involves LN-violating products of two \( \lambda \)'s or of \( \mu \) and \( \lambda \), while the other coefficients depend on \( \lambda^* \lambda \) or \( |\mu|^2 \). Therefore, it is possible to have large cancellations in \( \alpha_5 \) together with sizeable coefficients of dimension six. Type I and III fermions generate the operators \( \mathcal{O}_{\phi \phi}^{(1,3)} \), which correct the gauge fermion couplings. Type II scalars, on the other hand, generate 4-lepton operators and the operator \( \mathcal{O}_{\phi}^{(3)} \), which breaks custodial symmetry and modifies the SM relation between the gauge boson masses. EWPD are sensitive to all these effects and put limits on the see-saw parameters.

There are two classes of processes, depending on whether they involve neutral currents violating lepton flavour (LF) or not. The first class puts more stringent limits, but only on the combinations of coefficients entering off-diagonal elements. The second class is measured mainly at LEP and constrains the combinations in the diagonal entries. The LF violating limits are satisfied in types I and III if \( N \) and \( \Sigma \) mainly mix with only one charged lepton family. In Table 2 we collect the bounds from EWPD on the \( N \) and \( \Sigma \) mixings with the SM leptons \( V_{LN,\Sigma} \), and in Table 3 their product including the LF violating bounds. These

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Only with ( \epsilon )</th>
<th>Only with ( \mu )</th>
<th>Only with ( \tau )</th>
<th>Universal</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ V_{LN} = \frac{v(\lambda_1^L)_{1N}}{\sqrt{2} M_N} ] ( &lt; )</td>
<td>0.055</td>
<td>0.057</td>
<td>0.079</td>
<td>0.038</td>
</tr>
<tr>
<td>[ V_{\Sigma} = \frac{v(\lambda_1^\Sigma)<em>{1N}}{2 \sqrt{2} M</em>{\Sigma}} ] ( &lt; )</td>
<td>0.019</td>
<td>0.017</td>
<td>0.027</td>
<td>0.016</td>
</tr>
</tbody>
</table>

values update and extend previous bounds on diagonal entries for \( N \) (see also). Their dependence on the model parameters entering in the operator coefficients in Table 1 is explicit in the first column of Table 2. All low energy effects are proportional to this mixing, and the same holds for the gauge and Higgs couplings between the new and the SM leptons, responsible of the heavy lepton decay (and \( N \) production if there is no extra NP). An interesting by-product of a non-negligible mixing of the electron or muon with a heavy \( N \) is that the fit to EWPD prefers a Higgs mass \( M_H \) higher than in the SM, in better agreement with the present direct limit. This is so because their contributions to the most significative observables partially cancel, so that
both the mixing and $M_H$ can be relatively large without spoiling the agreement with EWPD. The new 90 % CL on $M_H$ increases in this case up to $\sim 260$ GeV (see also\cite{25,26}). In all other cases the limit stays at $\sim 165$ GeV.

In type II see-saw a crucial phenomenological issue is the relative size of $(\lambda\Delta)_{ij}$ and $\mu_\Delta$ for $M_\Delta \sim 1$ TeV. The $\nu$ masses are proportional to their product, $(m_\nu)_{ij} = \frac{2v^2\mu_\Delta(\lambda\Delta)_{ij}}{M_\Delta^2}$, which gives the strength of the LN violation. If $\mu_\Delta$ is small enough, $(\lambda\Delta)_{ij}$ can be relatively large and saturate present limits on LF violating processes, eventually showing at the next generation of experiments. If instead $(\lambda\Delta)_{ij}$ are very small, the flavour structure appears only in the $\nu$ mass matrix. The present limits are reviewed in\cite{15}. Neglecting LF violating bounds (i.e., assuming that $(\lambda\Delta)_{ee}$ is small enough not to give a too large $\mu \rightarrow e\bar{e}e$ decay rate), $\mu_\Delta$ and $\lambda_\Delta$ are constrained by the $T$ oblique parameter and four-fermion processes, respectively. From a global fit to EWPD (see\cite{20} for details on the data set used) we obtain the following limits at 90 % CL:

$$\frac{|\mu_\Delta|}{M_\Delta^2} < 0.048 \text{TeV}^{-1}, \quad \frac{|(\lambda\Delta)_{ee}|}{M_\Delta} < 0.100 \text{TeV}^{-1}.$$  \hspace{1cm} (7)

### 3  Dilepton signals of see-saw messengers

The previous limits apply to any particle transforming as the corresponding see-saw messenger, independently of whether it contributes or not to light neutrino masses. As indicated above, in minimal models the tight restriction imposed by $\nu$ masses (Eq. 1) gives much more stringent limits for the mixings of TeV-scale see-saw messengers. However, these limits can be avoided if additional particles give additional contributions to neutrino masses that cancel the previous ones, for instance if the fermionic messengers are quasi-Dirac, i.e. a nearly degenerate Majorana pair with appropriate couplings\cite{27}. The EWPD limits are in this case relevant for production and detection of type I messengers $N$, but the signals are different because they conserve LN to a very large extent\cite{14,28}. On the other hand, type II and III messengers with masses near the TeV can be produced and detected at LHC even in minimal models. Let us discuss the three types of see-saw mechanism in turn.

#### 3.1  Type I: Fermion singlets $N$

As already explained, a type I heavy neutrino $N$ with a mixing saturating the EWPD limit cannot be Majorana, unless extra fields with a very precise fine tuning keep the $\nu$ masses small enough\cite{29}. Unnatural cancellations allowing for LN-violating signals are also possible in principle. In this case a fast simulation shows that LHC can discover a Majorana neutrino singlet with $M_N \simeq 150$ GeV for $|V_{\mu N}| > 0.054$ (near the EWPD limit)\cite{8}, assuming an integrated luminosity $L = 30$ $fb^{-1}$.

Such a signal can be also observed for much smaller mixings and larger masses if there is some extra NP\cite{30}, especially if the extra particles can be copiously produced at LHC\cite{31}. This is
the case, for instance, if the gauge group is left-right symmetric and the new $W'_R$ has a few TeV mass. Then $pp \rightarrow W'_R \rightarrow t\bar{t}N \rightarrow t\ell\ell'W$ is observable, even with negligible mixing $V_{tN}$, for $M_N$ and $M_{W'_R}$ up to 2.3 TeV and 3.5 TeV, respectively, for an integrated luminosity $L = 30 \, fb^{-1}$. Similarly, if the SM is extended with a leptoquark $Z'$, the process $pp \rightarrow Z' \rightarrow NN \rightarrow t\ell\ell'WW$ can probe $Z'$ masses up to 2.5 TeV, and $M_N$ up to 800 GeV.

3.2 Type II: Scalar triplets $\Delta$

$SU(2)_L$ scalar triplets can be produced through the exchange of electroweak gauge bosons with SM couplings, and then they may be observable for masses near the TeV scale (see for reviews[21,31]). Although suppressed, their decays can occur within the detector for these large masses. In Fig. 1-(II) we display one of the possible processes. The search strategy and LHC potential depend on the dominant decay modes. These are proportional to the $\Delta$ vacuum expectation value $|<\Delta^0>| \equiv v_{\Delta}$, as for example $\Delta^{\pm\pm} \rightarrow W^\pm W^\pm$, or to $(\lambda_{\Delta})_{ij}$, as $\Delta^{\pm\pm} \rightarrow \ell^\pm\ell^\pm$. $\Delta^{\pm\pm}$ can also decay into $W^\pm W^{\pm\pm}$ if kinematically allowed (see[10]). All these different decay channels make the phenomenological analysis of single and pair $\Delta^{\pm\pm}$ production quite rich[12]. The EWPD limit in Eq. 7 translates into the bound $v_{\Delta} = \frac{v^2|\mu_\Delta|}{\sqrt{\Delta_{\Delta}} < 2 \, GeV$. This is to be compared with $|m_{\nu}| = 2\sqrt{2}v_{\Delta}|\lambda_{\Delta}| \sim 10^{-9} \, GeV$, which gives a much more stringent constraint for non-negligible $\lambda_{\Delta}$. Dilepton (diboson) decays are dominant for $v_{\Delta} < (>) v_{\Delta}' \sim 10^{-4} \, GeV$. If for instance $\lambda_{\Delta}$ is of the same size as the charged lepton Yukawa couplings $\sim 10^{-2} - 5 \times 10^{-6}$, $v_{\Delta}$ varies from $5 \times 10^{-8}$ to $10^{-4} \, GeV$, below the critical value $v_{\Delta}'$, and $\Delta$ decays mainly into leptons. In this case the LHC reach for $M_{\Delta^{\pm\pm}}$ has been estimated, based on statistics, to be $\sim 1 \, TeV$ for an integrated luminosity $L = 300 \, fb^{-1}$. In Fig. 3 we plot the invariant mass distribution $m_{\ell\ell}$ of same-sign dilepton pairs containing the lepton of largest transverse momentum for $M_{\Delta} = 600 \, GeV$. As this fast simulation analysis shows, the SM background is well separated from the signal, and the LHC discovery potential strongly depends on the light neutrino mass hierarchy. For the simulated sample we find $4 \ (44)$ signal events for the normal $\nu$ mass hierarchy IH (inverted IH), well separated from the main backgrounds: $tt\nu j$ (1007 events), $Zb\bar{b}n j$ (91 events), $tW$ (68 events), and $Zt\bar{t}n j$ (51 events). We get rid of other possible backgrounds like $ZZn j$ requiring no opposite-sign dilepton pairs with an invariant mass in the range $M_{Z} \pm 5 \, GeV$. For larger $v_{\Delta}$ values, with dominant non-leptonic decays, the corresponding reach estimate based on statistics is $\sim 600 \, GeV$. Note that only in the leptonic case LHC is sensitive to the see-saw flavour structure. Near the critical value, one could in principle extract information on the structure and on the global scale of the see-saw.

Tevatron Collaborations have already established limits on the scalar triplet mass assuming that $\Delta^{\pm\pm} \rightarrow \ell^\pm\ell^\pm$ 100% of the time: At the 95% CL $M_{\Delta^{\pm\pm}} > 150 \, GeV$ for $\Delta^{\pm\pm}$ only decaying to muons[34], and an integrated luminosity $L = 1.1 \, fb^{-1}$.

3.3 Type III: Fermion triplets $\Sigma$

Not so much attention has been paid to the study of the LHC reach for $SU(2)_L$ fermion triplets $\Sigma$. Up to very recently a similar electroweak process, the production of a heavy vector-like lepton doublet[35], had to be used to guess that LHC could be sensitive to $M_{\Sigma} \sim 500 \, GeV$. A dedicated study[13] estimates that an integrated luminosity $L = 10 \, fb^{-1}$ should allow to observe LN violating signals (see Fig. 1-(III) for a relevant process) for $M_{\Sigma} < 800 \, GeV$. Vector-like fermion triplets couple to SM leptons proportionally to its mixing $V_{\ell\Sigma}$, which is $\leq 10^{-6}$ according to Eq. 1 if $\Sigma$ is at the LHC reach $\sim 1 \, TeV$. So, one can eventually improve the analysis using the displaced vertex signatures of their decays.
4 Conclusions

Same-sign dilepton signals l±l'±X will allow to set significative limits on see-saw messengers at LHC, as illustrated in Table 4. The estimates for $M_\Delta$ and $M_\Sigma$ are mainly based on statistics,

<table>
<thead>
<tr>
<th>LHC reach (in GeV)</th>
<th>$M_N$</th>
<th>$M_\Delta$</th>
<th>$M_\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150</td>
<td>600 – 1000</td>
<td>800</td>
</tr>
</tbody>
</table>

and a more detailed analysis is needed to confirm them.

Acknowledgments

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