Benchmarks for the new-physics search through CP violation in $B^0 \to \pi^0 K_S$

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Using isospin relations, we predict the standard model correlation between $S_{\pi^0 K_S} = (\sin 2\beta)_{\pi^0 K_S}$ and $A_{\pi^0 K_S}$ the mixing-induced and direct CP asymmetries of $B^0 \to \pi^0 K_S$. The calculation uses flavor SU(3) only to fix the isospin-3/2 amplitude through the $B^+ \to \pi^+ \pi^0$ branching ratio, and thus has a small irreducible theoretical error. It can reach percent level precision thanks to expected future lattice-QCD progress for the calculation of the relevant SU(3)-breaking form-factor ratio, and serves as a benchmark for new-physics searches. We obtain an interesting picture in the $A_{\pi^0 K_S} - S_{\pi^0 K_S}$ plane, where the current experimental data show a discrepancy with the standard model, and comment on the direct CP asymmetries of $B^0 \to \pi^0 K^+$ and $B^+ \to \pi^0 K^+$. A modified electroweak penguin with a large new CP-violating phase can explain the discrepancy and allows us to accommodate also the corresponding data for other $b \to s$ penguin-dominated decays.

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Intriguing experimental results for observables of nonleptonic $b \to s$ decays [1] have been receiving considerable attention for several years, where the “$B \to \pi K$ puzzle” is an important example (see, e.g., [2–7]). The challenge is to disentangle possible signals of new physics (NP) from uncertainties that are related to strong interactions. In this context, a particularly interesting probe is offered by the time-dependent CP asymmetry in $B^0 \to \pi^0 K_S$,

$$\Gamma(B^0(t) \to \pi^0 K_S) - \Gamma(B^0(t) \to \pi^0 K_S)$$

$$\Gamma(B^0(t) \to \pi^0 K_S) + \Gamma(B^0(t) \to \pi^0 K_S)$$

$$= A_{\pi^0 K_S} \cos(\Delta M_{dt}) + S_{\pi^0 K_S} \sin(\Delta M_{dt}), \quad (1)$$

where $S_{\pi^0 K_S}$ arises from interference between mixing and decay, and $A_{\pi^0 K_S}$ is the “direct” CP asymmetry. In the standard model (SM), we have—up to doubly Cabibbo-suppressed terms—in the following expressions [8]:

$$A_{\pi^0 K_S} = 0, \quad S_{\pi^0 K_S} = (\sin 2\beta)_{\pi^0 K_S} = \sin 2\beta, \quad (2)$$

where $\beta$ is one of the angles in the standard unitarity triangle of the Cabibbo-Kobayashi-Maskawa matrix. The current world average is [1]

$$\langle \sin 2\beta \rangle_{\pi^0 K_S} = 0.58 \pm 0.17, \quad (3)$$

which should be compared with the “reference” value following from $B^0 \to J/\psi K_S$ and similar modes

$$\langle \sin 2\beta \rangle_{J/\psi K_S} = 0.681 \pm 0.025. \quad (4)$$

The search for NP signals in the CP asymmetries of $B^0 \to \pi^0 K_S$ requires a reliable SM prediction of $S_{\pi^0 K_S}$ and/or $A_{\pi^0 K_S}$. In this paper, we show that $S_{\pi^0 K_S}$ can be calculated in the SM as a function of $A_{\pi^0 K_S}$, with projected irreducible theoretical errors at the percent level. The starting point is the isospin relation [9]

$$\sqrt{2}(A(B^0 \to \pi^0 K^0) + A(B^0 \to \pi^0 K^0))$$

$$= -[(\hat{T} + \hat{C})e^{\gamma'} + \hat{P}_{ew}] = 3A_{3/2}; \quad (5)$$

a similar relation holds for the CP-conjugate amplitudes, with $A_{3/2} \to \bar{A}_{3/2}$ and $\gamma \to -\gamma$. Here, $\hat{T}, \hat{C}$ and $\hat{P}_{ew}$ are, respectively, the color-allowed tree, color-suppressed tree, and electroweak penguin (EWP) contributions [10]. The subscript of $A_{3/2}$ reminds us that the $\pi K$ final state has isospin $I = 3/2$, so that the individual QCD penguin contributions cancel in (5). $S_{\pi^0 K_S}$ can be written as

$$S_{\pi^0 K_S} = \frac{2|\bar{A}_{00}|A_{00}}{|\bar{A}_{00}|^2 + |A_{00}|^2} \sin(2\beta - 2\phi_{\pi^0 K_S}), \quad (6)$$

with $A_{00} = A(B^0 \to \pi^0 K^0)$ and $\bar{A}_{00} = A(B^0 \to \pi^0 K^0)$ [11]. If $A_{3/2}$ and $\bar{A}_{3/2}$ are known, $2\phi_{\pi^0 K_S} = \text{arg}(\bar{A}_{00}A_{00}^*)$ can be fixed through (5), as shown in Fig. 1. In order to determine $A_{3/2}$, we first rewrite the lower line of (5) as

$$3A_{3/2} = -(\hat{T} + \hat{C})(e^{\gamma'} - qe^{\gamma''}). \quad (7)$$

In the SM, the ratio $qe^{\gamma''} = -\hat{P}_{ew}/(\hat{T} + \hat{C})$ is given by

$$qe^{\gamma''} = -\frac{3}{2\lambda^2 R_b} \frac{C_0(\mu) + C_{10}(\mu)}{C_1(\mu) + C_{22}(\mu)} R_q = 0.66 \times \frac{0.41}{R_b} R_q, \quad (8)$$

where $\alpha = |V_{ub}| = 0.22$, $R_b = 0.41 \pm 0.04 \times |V_{ub}/V_{cb}|$ is a unitarity triangle side (value follows from [12]), and the Cs are Wilson coefficients. If we assume exact $SU(3)$ flavor symmetry and neglect penguin contractions, we have $R_q = 1$ [11,13], while we shall use $R_q = 1 \pm 0.3$ for the numerical analysis (results are robust with respect to
FIG. 1. The isospin relations (5) in the complex plane. The magnitudes of the amplitudes, $|A_{ij}| = |A(B \to K' \pi^0)|$ and $|\bar{A}_{ij}| = |\bar{A}(B \to K' \pi^0)|$, can be obtained from the corresponding branching ratios and direct CP asymmetries listed in Table I, while $A_{3/2}$ and $\bar{A}_{3/2}$ are fixed through (8) and (9).

to the strong phase $\alpha_*$. Since $\rho e^{i\alpha_*}$ factorizes at leading order in the $1/m_b$ expansion, $R_K$ can be well predicted using factorization techniques and future input from lattice QCD.

$SU(3)$ flavor symmetry allows us furthermore to fix $|\tilde{T} + \tilde{C}|$ through the $b \to d$ decay $B^+ \to \pi^+ \pi^0$ [14]

$$|\tilde{T} + \tilde{C}| = R_{T+C}[V_{ud}/V_{us}]\sqrt{2}|A(B^+ \to \pi^+ \pi^0)|,$$  \hspace{1cm} (9)

where the tiny EWP contributions to $B^+ \to \pi^+ \pi^0$ were neglected, but could be included using isospin [11,15]. We stress that (9) does not rely on further dynamical assumptions. For the $SU(3)$-breaking parameter $R_{T+C} \sim f_K/f_\pi$, we use the value $1.22 \pm 0.2$, where the error is quite conservative, as discussed below.

Relations (7)–(9) allow us to determine $A_{3/2}$ and $\bar{A}_{3/2}$, thereby fixing the two isospin triangles in Fig. 1. Since the triangles can be flipped around the $A_{3/2}$ and $\bar{A}_{3/2}$ sides, we encounter a fourfold ambiguity (not shown). Using (6), $S_{K^0}$ is determined as well. The corresponding prediction is shown in Fig. 2, where we keep $A_{\mu K^*_s}$ as a free parameter. For the implementation of this construction, we express the curves in Fig. 2 in parametric form [2] as functions of a strong phase $\delta_c$, defined through

$$r_c e^{i\delta_c} = (\tilde{T} + \tilde{C})/\tilde{P},$$  \hspace{1cm} (10)

where $\tilde{P}$ is the $B^0 \to \pi^- K^+$ penguin amplitude [10]. We find that no solutions exist for certain ranges of $\delta_c$, separating the full $[0^\circ, 360^\circ]$ range into two regions. They contain $\delta_c = 0^\circ$ or $180^\circ$ and correspond to the left and right panels of Fig. 2, respectively. As one circles the trajectory in either panel by changing $\delta_c$, each value of this strong phase in the respective interval is attained twice. In order to illustrate this feature, we show—for central values of the input data/parameters—points corresponding to various choices of $\delta_c$. The bands show the $1\sigma$ variations obtained by adding in quadrature the errors due to all input data/parameters. Moreover, we assume $\gamma = 65^\circ \pm 10^\circ$ [16,17]. This angle will be determined with excellent accuracy thanks to CP violation measurements in pure $B$ decays at the LHCb experiment (CERN).

In order to resolve the fourfold ambiguity in Fig. 2, we need further information on $r_c$, $\delta_c$: i) $r_c$ can be determined if we fix $|\tilde{T} + \tilde{C}|$ through $BR(B^+ \to \pi^+ \pi^0)$ [see (9)] and $|\tilde{P}|$ through $BR(B^+ \to \pi^+ K^0) \propto |\tilde{P}|^2 + \ldots$, where the dots represent negligible doubly Cabibbo-suppressed terms that are already strongly constrained by data [18]. In the left panel of Fig. 3, the corresponding $r_c$ constraint is shown at the “charged” circle. ii) Using the $SU(3)$ flavor symmetry and other plausible dynamical assumptions [2], a fit to all available $B \to \pi\pi$ data yields the $\pi\pi$ curves. Since BaBar and Belle do not fully agree on the measurement of the direct CP asymmetry in $B^0 \to \pi^+ \pi^-$ [1], we show in the right panel of Fig. 3 the corresponding allowed regions separately. We observe that the data imply $\delta_c \sim (0-30)^\circ$, in agreement with the heavy-quark expansion.
analyses in [4,19,20], differing in their treatment of non-perturbative charm-penguin contributions. Consequently, we can exclude the solutions shown in the right panel of Fig. 2. Right panel: $B \rightarrow \pi\pi$ constraints for the BaBaR and Belle data for $A_{\pi^+\pi^-}$ and the HFAG average. The solid and dotted lines refer to 1σ and 90% C.L. ranges, respectively.

which is about 2 standard deviations away from the experimental result in (3). It should be noted that (11) depends on the input data collected in Table I.

In Fig. 4, we show the future theory error benchmark for the SM constraint in the $A_{\pi^0K_S}$ plane. Both $R_q$ (8) and $R_{T+C}$ (9) factorize at leading order in the $1/m_b$ expansion, and can be well predicted using input from lattice QCD. It should be stressed that “charming penguins” do not enter these ratios. As a working tool, we use the approach of Beneke, Buchalla, Neubert, and Sachrajda (BBNS)[4,19], but similar conclusions can be reached using Ref. [20] (where also derivatives of form factors would be needed). The key parameter is $R_q$, which dominates the current theoretical error (11). Its uncertainty is governed by the $SU(3)$-breaking form-factor ratio $\xi_{\pi K} = F_{B^+K}(0)/F_{B^-\pi}(0)$. If we assume $\xi_{\pi K} = 1.2(1 \pm 0.03)$, i.e., a 20% determination of the $SU(3)$-breaking corrections, as an optimistic—but achievable—goal for lattice QCD, we obtain the BBNS result $R_q = (0.908^{+0.057}_{-0.043})e^{(0.127)}$ to be compared with the present value $R_q = (1.02^{+0.27}_{-0.22})e^{(0.127)}$ [21]. Similarly, we find $R_{T+C} = 1.23^{+0.02}_{-0.03}$, where the increase of precision is very mild as the form-factor dependence essentially cancels out. Setting, moreover, the uncertainties of the experimental inputs to zero, while keeping central values fixed, we obtain a prediction of $S_{\pi^0K_s}$ with errors at the percent level, as shown in Fig. 4. Consequently, the irreducible theory

![FIG. 4 (color online). The correlation in the $A_{\pi^0K_S}$ plane for a future benchmark scenario (narrow band) in comparison with the current situation (wider band), as explained in the text.](image_url)
error of our proposed method for predicting $S/C_{25}^{0}$ in the SM is much smaller than in calculations using only the $1/m_b$ expansion, and makes it promising for a future $e^+e^-$ super-$B$ factory (for a review, see, e.g., Ref. [22]).

Before turning to the interpretation of the current experimental data in terms of NP, let us briefly comment on the difference of direct $CP$ asymmetries $A/C_{25}^{0}$, which recently received quite some attention as a possible sign of NP [23]. Figure 5 shows the SM correlation between this difference and the $CP$ asymmetry $A/C_{25}^{0}$, keeping $A/C_{25}^{0}$ fixed. It depends on $CP$-averaged $B \rightarrow \pi K$ branching ratios and $\gamma$, and becomes equivalent to the sum rule for rate differences [24] when neglecting higher orders in subleading amplitudes. We see that current data (cross) can be accommodated in the SM within the error on $A/C_{25}^{0}$, although hadronic amplitudes then deviate from the $1/m_b$ pattern (see also Ref. [7]). It would be desirable to reduce this uncertainty in the future.

Let us now consider a NP scenario, which allows us to resolve the discrepancy between (3) and (11). Following [2], we assume that NP manifests itself effectively in the data as a modified EWP with a $CP$-violating NP phase $\gamma_{ei}/C_{30}$, i.e., $q \rightarrow q e^{i \phi}$ in (7). Here, $q$ can differ from the SM value in (8). Since $\delta_{c}$ is rather small, the impact of this type of NP on $A_{\pi^{0}K_{S}}$ and $A_{\pi^{0}K_{S}^{+}}$ is suppressed. In Fig. 6, we show constraints on $q e^{i \phi}$ from two $\chi^2$ fits, using only the $B \rightarrow \pi K$ data or both the $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ data. The latter have a strong impact on the allowed region of $q e^{i \phi}$ [2,7].

FIG. 5. The SM correlation between $A_{\pi^{0}K_{S}} - A_{\pi^{0}K_{S}^{+}}$ and $A_{\pi^{0}K_{S}}$ for central values of inputs, with hadronic parameters fixed as for Fig. 2 (solid), or following from the sum rule for rate differences [24] (dashed). The dependence on $\delta_{c}$ is as in Fig. 2 and is constrained to SM values (upper curve in Fig. 2(a)).

FIG. 6 (color online). Constraints on $q e^{i \phi}$. Left panel: $\chi^2$ fit, using only the $B \rightarrow \pi K$ data. Right panel: $\chi^2$ fit, using both the $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ data. The inner and outer regions correspond to $1\sigma$ and $90\%$ C.L., respectively, while the stars denote the minima of the fits. The $90\%$ C.L. regions with 10 times more data lie inside the dotted lines (see also the text).
yielding two almost degenerate minima, $q = 1.3 \pm 0.4$, 
$\phi = (63^{+9}_{-8})^\circ$ and $q = 0.8^{+0.3}_{-0.3}$, $\phi = (45^{+18}_{-28})^\circ$. We also show the 90% C.L. regions (dashed curves) that correspond to a future scenario, assuming the benchmark value of $R_q$
used in Fig. 4 and ten times more data, with central values fixed to the present $\chi^2$ minimum. In the $\chi^2$ fits we allow all ratios of $SU(3)$-related amplitudes to fluctuate flatly around $f_K/f_\pi$ within 30% in magnitude and 30° in phase.

The possibility of resolving the discrepancy between (3) and (11) through a modified EWP is intriguing. We next illustrate that the observed pattern of the mixing-induced CP asymmetries in other penguin-dominated $b \to s$ decays [1] can also be accommodated in the same NP scenario. In Fig. 7, we show the results of a BBNS calculation of the $S$ parameters for four channels of this kind: we assume that all electroweak Wilson coefficients are re-scaled by the same factor $q e^{i \phi}$, and use as input the preferred data set “G” of [21]. The value of $q e^{i \phi}$ is then varied along a contour that runs vertically through the preferred region in Fig. 6. Unlike the SM, the modified EWP scenario allows us to accommodate the data well (see also, e.g., [7,25]).

The same is true for a more specific scenario where the effective FCNC couplings of the Z boson at the weak scale are suitably modified. Since $S_{\eta' K_s}$ receives a tiny, negative shift from $2 \beta$, in agreement with the data, we do not show this in Fig. 7.

In conclusion, we have demonstrated that the SM correlation in the $A_{\eta' K_s} - S_{\eta' K_s}$ plane can be predicted reliably in the SM, with small irreducible theoretical errors, and have shown that the resolution of the present discrepancy with the data can be achieved through a modified EWP sector, with a large CP-violating NP phase.

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[10] We are using a notation very similar to [2], with $\tilde{T} = |V_{ub}V_{ts}|T'$, $\tilde{C} = |V_{cb}V_{td}|T'$ and $P = |V_{ub}V_{cd}|(P'_{L} - P'_{R})$, while the quantities $q$, $\omega$, $r_c$ and $\delta_c$ agree with [2].