Solution of the dynamic vacuum equation

1. We solve the dynamic vacuum equation

\[ VP = F P I - (P - P_0) S' \]  \hspace{1cm} (1)

under 3 conditions:
1) linear current rise
2) constant current
3) no current

\( P_0 \) is the base pressure, without current. \( V \) is the volume and \( S \) is the pumping speed, both taken per unit length. \( F \) is the gas load per unit length, pressure and current.

2. Linear current rise

We put \( I = \dot{I} t \), and introduce scaled variables \( p = P/P_0 \), and \( t = A \tau = \frac{S}{F} \tau \), and get a one-parameter equation:

\[ \frac{dp}{d\tau} = a \left[ p (\tau - 1) + 1 \right] \]  \hspace{1cm} (2)

where \( a = ASV^{-1} = S^2/FIV \). We notice from the stationary solution of (1) that \( A \) is the time required to accumulate the critical current.

3. Equation (2) has been solved numerically for a few values of \( A \), up to \( \tau = 0.95 \) when the pressure is \( p_1 \). The result is shown in the attached figures.
4. **Constant current**

In the interval $\tau_1 < \tau < \tau_2$ we solve the following equation

\[
\frac{dp}{d\tau} = a \left[ p (\tau_1 - 1) + 1 \right]
\]

(3)

with $p = p_1$ at $\tau = \tau_1$. The solutions are for $\tau \neq 1$

\[
p = (p_1 - \frac{1}{1 - \tau_1}) \exp \left[- a (1-\tau_1)(\tau - \tau_1)\right] + \frac{1}{1 - \tau_1}
\]

(4)

and for $\tau_1 = 1$

\[
p = p_1 + a (\tau - \tau_1)
\]

(5)

We notice that for $\tau_1 < 1$, $p \rightarrow (1 - \tau_1)^{-1}$ for $\tau \rightarrow \infty$ i.e. the pressure tends towards some constant. This may be a way of measuring the critical current by just measuring the pressure rise for one single current. For $\tau_1 > 1$, the pressure goes beyond all limits. This shows again that $\tau_1 = 1$, or $t = A$ is the time required to accumulate the critical current.

5. **Pressure drop after dumping**

Let the pressure at $\tau = \tau_2$ be $p_2$. For $\tau > \tau_2$ we solve the equation.

\[
\frac{dp}{d\tau} = a (1 - p)
\]

(6)

with the solution

\[
p = 1 + (p_2 - 1) \exp \left[- a (\tau - \tau_2)\right]
\]

(7)

6. **Experimental procedure**

From the equilibrium pressure rise one can find $\tau_1$, and hence $A$. This fixes the time scale. From the pressure decay one can then find $a$, and, knowing $A$, one gets $S/V$. Then one can inspect $A$ again, and, knowing $I$, one is able to find $S/F$. 


One division gives \( \frac{S/V}{S/F} = F/V \). Since \( V \) is fairly accurately known, the gas load can be calculated.

7. Conclusion

The pressure was calculated as a function of time, assuming an additional gas load proportional to the pressure and to the current, and taking the pumping speed as independent of pressure.

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