LABORATORY MEASUREMENT OF RF RESONANCES AND IMPEDANCES

IN ISR VACUUM CHAMBER COMPONENTS

by

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Abstract

Changes of section along the ISR vacuum chamber can form radio-frequency circuits which may interact with the beam. These were studied and treated where necessary during machine construction. Installation of nuclear physics experiments as well as improvements in the machine necessitate a continued watch on vacuum-chamber components. This report describes low-power measurement and calculation methods applied to such components.

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Introduction

Each new nuclear physics experiment in the ISR normally requires modifications to the vacuum chamber surrounding the beam. Modifications are also needed to accommodate the continual improvements in beam handling, pumping and diagnostics in the machine. Thus the question of interaction between the beams and radio-frequency circuits formed by the vacuum chamber which had to be studied during the construction of the ISR, continues to be important during the use and development of the machine. All changes and additions to the vacuum chamber have to be presented to the "Beam Equipment Interaction Committee" (BEIC) for approval. This committee recommends modifications where necessary and may insist on RF measurements being made on either the equipment itself or a model. From these measurements a factor is deduced indicating the amount of beam-equipment coupling. If this factor is so high that instabilities are to be feared, then either changes must be made in the design, or sufficient RF damping must be put in to make the equipment safe.

Stability criteria

It has been shown (ref. 1) that the stability criterion for a coasting beam passing through a cavity can be expressed as a maximum permitted value of Z/n, where

- Z is the "coupling impedance" between cavity and beam, and is equal to the shunt impedance of the cavity as usually defined,
- n is the ratio of the frequency at which Z is quoted or measured (e.g. a cavity resonant frequency), to beam revolution frequency in the ISR.

Z in fact relates the voltage induced by the cavity in the beam (or from the beam to the cavity) to the power loss in the cavity. If the interaction takes place over an appreciable distance, it will include a transit-time factor.

A factor commonly quoted for maximum allowable coupling impedance in the ISR is Z/n = 10 ohms. This is based on the stability criterion (ref. 1) for the limit of longitudinal stability in the ISR. The derivation is valid for a circulating current of 76 mA with full momentum spread $\Delta p = 1.3 \times 10^{-3}$ $\text{m}/\text{c}$ at half height. It is limited to certain shapes of beam momentum distribution, in particular without large off-centre tails and without a flat top.

K. Hübner (ref. 2) has shown that the critical impedance for longitudinal stability of a stack consisting of more than 10 pulses is higher than that for a single pulse. Thus there should be a safety margin of stability at the higher currents, once the above criterion is satisfied.
Transverse instabilities have been considered by W. Schnell (ref. 3) and by D. Möhl (ref. 4). W. Schnell concluded that, for certain assumed conditions, resonant cavities would not cause transverse instability as long as the longitudinal coupling impedance was low enough to prevent longitudinal instability. D. Möhl presents calculations of growth rates for transverse instabilities from given impedance characteristics, with special reference to the CERN PS. A later paper by Schnell and Zotter (ref. 5) derives a simplified criterion for transverse stability in terms of the longitudinal coupling impedance. This criterion is applied to known machine performance at 26 GeV and gives a figure for coupling impedance which agrees well with that calculated from chamber wall resistance and inductance.

The stability of bunched beams has been investigated by F. Sacherer (ref. 6). The theory he works out applies to beams with equidistant bunches and gives growth rates of oscillations which are excited by a resonator having a given shunt impedance. Thus if a certain instability growth rate can be tolerated one can derive a criterion for maximum allowable shunt impedance of cavities.

Application of this theory to the ISR is discussed in reference 7, where comparison between theory and experimental excitation of instabilities is presented. Theoretical and experimental work is continuing.

**Non-resonant impedance**

An enlarged section of vacuum chamber will behave as a cavity with certain resonant modes, some of which may couple with the beam. The abrupt change in surrounding wall diameter at each end of such a chamber will also present an impedance to the beam, in this case inductive. This impedance will be presented over a wide range of frequencies, going far below the frequencies of resonance. Other shapes such as bellows sections, and plates inside the chamber, parallel to the beam, will also present inductive impedances. In all cases the magnetic field associated with the beam is able to penetrate a volume from which the electric field is partially excluded.

Methods have been proposed for measuring the impedance seen by the beam, using a wire to simulate the beam (refs. 8, 9, 10). Herward describes a method in which one measures the electrical length of an inner line which represents the beam. The impedance is then determined from the difference between the electrical length and the physical length. If, as is often the case, the chamber has end-to-end symmetry, this can be done by short-circuiting one end of the line and measuring the quarter-wave resonant frequency. Since the result will be the same
as if the voltage-maximum and current-maximum were interchanged, it will be valid for travelling-wave conditions without the necessity of a second measurement.

If the difference between electrical and physical length is \( \Delta l \), and the beam is relativistic, the inductance introduced by irregularities in the chamber wall is

\[
L = \frac{Z_0 \Delta l}{c} \text{ Henry}
\]

where \( Z_0 \) is the characteristic impedance of the coaxial line.

The ratio \( Z/\pi \) is given by

\[
j\omega L = 2j\omega Z_0 \frac{\Delta l}{c} = 2jZ_0 \frac{\Delta l}{R_0} \text{ ohms},
\]

where \( 2\pi R_0 \) is the machine circumference.

The non-resonant impedance may result from inductance deliberately introduced into the chamber wall. An example is the "Wideband Accelerating Gap" (SEIC note 162). This consists of a ceramic insulator in the beam pipe, with a ferrite ring alongside and a current return clamped around. Four strips arranged round the gap join to coaxial connectors. With the connectors unterminated, there is a resonance at 30 MHz of the inductance due to the ferrite with the capacitance of the ceramic, giving an impedance of 30 ohms. This resonance disappears when the connectors are terminated, but the slope impedance at frequencies below 2 MHz gives a \( |Z|/\pi \) ratio of about 3 ohms. This can be reduced to about 0.25 ohm, when the gap is not being used for experiments, by short-circuiting the coaxial connectors.

In the structure just mentioned, the impedance at low frequencies could be measured directly by connecting the head of a HP 4815A Vector Impedance Meter across the ceramic insulator.

**Coupling impedance of a resonator**

The coupling impedance between a cavity and a beam passing through it is defined as

\[
R = \frac{2P}{I_0^2} \text{ ohms}
\]
where $P$ is the power coupled into (and dissipated in) the cavity by a beam having peak RF component of current $I_0$ at cavity frequency.

The cavity will at the same time have a peak gap voltage $V_0$, related to the power loss by the shunt impedance $R_{sh}$ of the cavity. This is defined by

$$R_{sh} = \frac{V_0^2}{2P} \text{ ohms}$$

Since the power taken from the beam is $\frac{1}{2}I_0V_0$ and this is dissipated in the cavity, it is seen that

$$P = \frac{1}{2}I_0V_0$$

and

$$\text{coupling impedance } R = \text{ shunt impedance } R_{sh}.$$

The coupling and shunt impedance values referred to here are related to the peak RF voltage appearing on the beam path. The value $Z$ used when considering beam stability refers to the effective voltage induced in the cavity from the beam and vice versa. Thus $R$ and $Z$ as defined here can be related by a transit-time factor:

$$Z = RT^2$$

where $T$ is the transit-time factor relating effective voltage during the time the beam and cavity interact, with the peak value of the voltage.

Some authors use a somewhat different definition of shunt impedance. To distinguish, we shall call this $r$:

$$r = \frac{E_0^2}{P}$$

("linac" ohms per unit length)

where $E_0$ is the peak value of electric field on the beam path, $P$ is the power dissipation per unit length.
In this case, \( r \) is not in true ohms, since it relates peak voltage (in time) to power.

It is convenient to obtain the shunt or coupling impedance \( R \) from separate determination of ratio \( R/Q \) and quality factor \( Q \). \( R/Q \) is independent of surface and dielectric losses and can be measured by perturbation techniques, or in many cases calculated. The \( Q \) is then measured using a square-law or calibrated detector coupled into the cavity.

**Calculation of \( R/Q \) for a resonator**

Since \( Q \) is defined as \( \omega U/P \), where \( U \) is the stored energy in the cavity and \( P \) the power loss, we can write

\[
R/Q = \frac{V_o^2}{2\omega U} \quad \text{(ohms)}.
\]

For simple shapes of cavity or line, the stored energy \( U \) can be calculated in terms of gap voltage by integrating either electric or magnetic field over the volume of the resonator. When the electric field is not uniform across the gap, it is usual to take \( V_o \) as the peak value (in space and time) of electric field \( E_0 \) multiplied by gap length. The correction factor for the effective voltage applied to particles crossing the gap is then included in the transit-time factor.

As an example, let us consider a closed cylinder (fig. 1a) resonating in \( TM_{210} \) mode. The electric field is everywhere parallel with the axis and varies with radius \( r \) as \( J_0 (kr) \) where \( k = 2.405/a \) and \( a \) is the outer radius. We can write

\[
U = \frac{1}{2} \int_0^a \frac{E_z^2}{2} 2\pi r \, dr = \tau \varepsilon \varepsilon_0 \int_0^a r J_0^2 (kr) \, dr
\]

\[
= \frac{1}{4} \pi \varepsilon a^2 \varepsilon_0 \frac{E_0^2}{2} J_1^2 (ka)
\]

where \( l \) is the length of the cylinder.

Substituting this in the \( R/Q \) expression gives

\[
R/Q = \frac{\frac{\eta k}{\pi \varepsilon_0}}{rka^2J_1^2(ka)} = 0.982 \varepsilon \frac{\lambda}{2a} \quad \text{where} \quad \eta = \frac{\varepsilon_0}{\varepsilon_0}
\]

\[
= 370.1 \frac{\lambda}{2a} \quad \text{ohms}.
\]
The table below gives some calculated values of $R/Q$, all expressed in terms of highest voltage developed across the resonator. The values only apply for particles crossing the resonator at the point where this voltage is developed.

<table>
<thead>
<tr>
<th>Cavity shape</th>
<th>Mode</th>
<th>$R/Q = V^2/2aU$</th>
<th>Referred to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical</td>
<td>TM$_{010}$</td>
<td>$\frac{370}{2a}$</td>
<td>Total maximum voltage between end plates</td>
</tr>
<tr>
<td>Fig. 1a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular</td>
<td>TM$_{110}$</td>
<td>$\frac{480a}{\sqrt{a^2+b^2}}$</td>
<td></td>
</tr>
<tr>
<td>Fig. 1b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical</td>
<td>TE$_{111}$</td>
<td>$\frac{640}{\sqrt{1 + \left(\frac{2l}{3.41a}\right)^2}}$</td>
<td>Peak maximum transverse voltage across cylinder</td>
</tr>
<tr>
<td>Fig. 1c</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical</td>
<td>TM$_{110}$</td>
<td>$193.1 \frac{2}{2a}$</td>
<td>Integrated electric field equivalent to magnetic deflecting field (relativistic particles)</td>
</tr>
<tr>
<td>Fig. 1a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular</td>
<td>TE$_{011}$</td>
<td>$\frac{480b}{\sqrt{a^2+b^2}}$</td>
<td>Peak maximum voltage across centre of waveguide</td>
</tr>
<tr>
<td>Fig. 1d</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coaxial</td>
<td>TEM, $\frac{1}{4}\lambda$</td>
<td>$\frac{240}{\pi} \frac{2ln \frac{b}{a}}{}$</td>
<td>Peak voltage at open end</td>
</tr>
<tr>
<td>Fig. 1e</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coaxial</td>
<td>TEM, $\frac{3}{4}\lambda$</td>
<td>$\frac{80}{\pi} \frac{2ln \frac{b}{a}}{}$</td>
<td>do.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
These calculated R/Q values are for closed cavities. In practical cases the pipe through which the beam enters and leaves the cavity has diameter comparable with cavity dimensions so the calculated values are only valid as approximations. E. Keil and B. Zotter have computed coupling impedance, Q and transit-time factor for TM_{01} modes in cylindrical cavities in an infinitely long beam pipe (refs. 11, 12). Applying the worst case (i.e., highest value) transit-time factor to the ratio R/Q from the computations gives the curve plotted in Fig. 2.

Cavities encountered are often of approximately elliptical section. Cutoff wavelengths for many modes are given by Kretzschmar (ref. 13), so cavity resonant frequencies can be calculated. Coupling impedance or R/Q ratio can then be calculated for an equivalent rectangular or circular section. Another paper by Kretzschmar (ref. 14) plots a function relating electric field to power loss in the walls for certain TM modes in elliptical section cavities. The coupling impedance can be calculated from this function. Fig. 3 shows a normalized expression \( R \frac{\delta}{L} (1 + \frac{A}{L}) \), where \( \delta \) is the skin depth in the wall material at the frequency of interest, plotted against the ratio of diameters.

**Cavity R/Q from perturbation measurements**

If the R/Q ratio of a component cannot be calculated with sufficient accuracy, it may be determined experimentally by perturbation measurement.

The relative magnitude and the direction of the electric field can be found at various points in a cavity by inserting a short piece of wire carried on a styrofoam or thin teflon rod, and observing a swept display of resonant frequency (Fig. 4). The change of resonant frequency is proportional to the square of the electric field component in the direction of the wire. The peak electric field along the wire \( E_p \) is given by (ref. 15)

\[
\frac{E_p^2}{2\omega U} = \frac{8}{\pi} \frac{2n(q/p)-1}{\varepsilon_q q^2 \omega} \left| \frac{\delta f}{f} \right| \text{ ohms/metre}^2
\]

where
- \( U \) is the stored energy in the cavity
- \( q \) is the length of the wire
- \( \varepsilon_q \) is the radius of the wire
- \( \varepsilon_0 \) is the dielectric constant of free space
- \( \omega/2\pi = f \) is the cavity resonant frequency
- \( \delta f \) is the frequency change produced by the wire.
If $E_p$ is constant along the path of length $l$ followed by the beam, the gap voltage is $E_p l$, so

$$R/Q = \frac{V_0^2}{\omega U} = \frac{8\pi^2}{\pi} \frac{\delta f}{\omega} \frac{q(p/q^2)-1}{\varepsilon_0 q^3} \frac{\delta f}{f^4} \text{ ohms.} \quad (1)$$

If $E_p$ in fact varies across the gap, this is taken into account separately in the transit-time factor.

The Q value can be quickly determined by measurement, and the effective coupling impedance $Z$ obtained by applying a transit-time factor to $R$.

Alternatively the field may be perturbed by a dielectric rod along the path to be followed by the beam. Then the rms value of electric field along the rod (peak in time) $E_0$ is given by

$$\frac{E_0^2}{2\omega U} = \frac{2}{(\varepsilon-1) \varepsilon_0 A / l} \frac{\delta f}{f} \text{ ohms/metre}^2$$

where $A$ is the cross-sectional area of the rod
$\varepsilon$ $\varepsilon_0$ is its dielectric constant
$l$ is the length of rod in the field.

If the field along the beam path is constant we have

$$R/Q = \frac{E_0^2 l^2}{2\omega U} = \frac{2l}{(\varepsilon-1) \varepsilon_0 A \omega} \frac{\delta f}{f} \text{ ohms} \quad (2)$$

This expression applies only for electric field in the direction of the rod axis.

If $E$ varies across the gap, we have a gap voltage $V_0 = \int_0^l E_p \, dz$

where $E_p$ is the field at a distance $z$ from one end.

The value $E_0^2$ in expression (2) is in fact $\frac{1}{\varepsilon_0} \int_0^l E_p^2 \, dz$.

It is convenient to refer $R/Q$ to the peak value (in space and time) of $E_p$, and combine variation in space and time in the transit-time factor. In the case where $E_p$ varies over a half-sinusoid, $R/Q$ referred to peak field is twice that given by expression (2). This $R/Q$ value can be compared with that obtained using a short wire and calculated from expression (1) above.
The variation in relative field across the gap can be checked by moving a dielectric bead across the gap. If there is no magnetic field in this part of the cavity, a metal bead may be used. In either case the frequency shift due to the bead is proportional to the square of the field $E_p$.

As an example of measurement of R/Q on an actual vacuum chamber, we consider the "PC pump" unit (BEIC note 166). This consists of a chamber of roughly elliptical section 177 mm high and 300 mm wide, in fact formed of straight lines and semi-circles. It has a length of 252 mm, and joins to the standard 4161 mm beam pipe at each end. The chamber carries a sublimation filament assembly, mounted on a flange on one end, with the filaments parallel to the beam axis. RF measurements were made on the chamber with and without the filament assembly.

The lowest TM mode in the empty chamber, $eTM_{010}$ in elliptical waveguide terminology, was found with the arrangement of Fig. 4 at 1038 MHz. The Q value measured was 2300. A copper wire, $\phi 2 \text{ mm} \times 17 \text{ mm}$ long mounted in the end of a styrofoam rod was moved into the chamber. With the wire parallel to the chamber axis, a maximum frequency shift of $-0.25 \text{ MHz}$ was observed. This was at the centre of chamber cross-section; on the beam axis the shift was $-0.2 \text{ MHz}$. Substitution of these figures in expression (1) gives values $R/Q = 248$, 299 ohms respectively.

The field was also perturbed by placing a teflon rod, $\phi 8.9 \text{ mm}$, along the beam axis. This gave a frequency shift of $-1.85 \text{ MHz}$. Substitution in expression (2) then gives $R/Q = 237$ ohms, integrated right along the cavity.

Calculated values for $R/Q$ in a closed cavity of the same frequency were:

- Rectangular section ($TM_{110}$ mode) $352$ ohms
- Circular section ($TM_{010}$ mode) $417$ ohms

Since in this mode there is no variation of field in the axial direction (for a closed cavity), the calculated values can be compared directly with that measured in the centre of the chamber. It is seen that the calculated values are all somewhat higher.

Measurements were also made in the lowest TE mode in the empty chamber, the $eTE_{111}$ mode. This mode had a frequency of 792 MHz with a Q value of 3170. In this case the electric field lines are vertical, so the perturbing wire was fitted across the end of the styrofoam rod and the field variations explored. A maximum shift of $-0.3 \text{ MHz}$ was measured in the centre of the chamber, and $-0.25 \text{ MHz}$ on the beam axis. Substitution in expression (1) gives values
R/Q = 511 and 426 ohms respectively.

For an equivalent rectangular section (TE₀₁₁₁) closed cavity, the calculated R/Q ratio, considering only electric field, was 671 ohms. These measured and calculated figures all refer to the field halfway through the cavity - the electric field in this mode varies over a half-sinusoid along the axis. In this mode there is magnetic field as well as electric on the beam path, and this should strictly be taken into account in determining the R/Q value. However, consideration of electric field only will give a pessimistic result which should be adequate for the present purpose.

The transit-time factor must now be used to take account of the variation of the electric field as the beam passes down the chamber. In the TM₀₁₀ modes there is only the variation with time to consider, so the (voltage) transit-time factor will be given (for a relativistic beam) by

$$T = |\sin \frac{\pi l}{\lambda}| / \frac{\pi l}{\lambda}$$

where $l$ is the cavity length.

For the eTM₀₁₀ mode this gives $T = 0.14$, so the expression $R/Q T^2$ will equal 199 $(.14)^2 = 3.9$ ohms on the beam path. This will give a coupling impedance ratio $Z/n = 2.8$ ohms for the empty chamber, for a Q of 2300 and a revolution frequency of 318 kHz.

In the TE₁₁₁₁ modes the field varies in both space and time. In the closed-cavity case used for calculation the transit-time factor is

$$T = \frac{2n}{n} \cos \frac{\pi l}{\lambda} \left( \frac{2\pi}{\lambda} \right)^2 - n^2$$

for odd longitudinal order $n$.

For the eTE₁₁₁₁ mode this gives $T = 0.41$, so the measured figure $R/Q = 426$ ohms on the axis gives $R/Q T^2 = 72$ ohms and $Z/n = 91$ ohms. As already mentioned, these figures take into account only the electric field.

**Q measurement**

The Q value is determined from the frequency difference $\Delta f$ between half-power points, i.e. half-voltage points for a square-law detector, using the sweeper arrangement shown in Fig. 4.
\[ Q_L = \frac{f}{\Delta f} \]

\( Q_L \) is the loaded \( Q \) value, and includes the loading effect on the cavity of the loops. With small loops, however, it is sufficiently close to the unloaded value. This can be checked by measuring again with another size of loops.

**Damping resistors**

The ratio \( R/Q \) of a structure is a function of the physical form of the structure. If this cannot be modified to reduce \( R/Q \), the coupling impedance can still be reduced by increasing the RF losses. Various forms of resistor have been tried for this purpose. The most effective of those tried were ceramic cylinders of diameter about one fifth of beam pipe diameter, coated on the outside surface with an evaporated layer of nichrome. Surface resistances in the range 50 to 200 ohms per square were successfully employed. The ceramic tubes of standard production resistors were found to be too porous for use in the ultra-high vacuum system of the ISR, so pure alumina tubes were bought from industry and sent to a resistor manufacturer for coating.

The resistors may be placed at the bottom of the vacuum chamber, fixed by wires or clips if necessary, or hung on bars at the top or sides. Where the chamber changes from circular to elliptical section, resistors are fixed on the bottom of the circular-section so that waves reflecting from the change of section are damped.

The wall thickness (about 2.7 mm) of these ceramic-tube resistors is sometimes undesirable due to the shadow caused to nuclear-physics measuring equipment. In such cases tubes with thinner walls, but smaller diameter, have been used.

In Fig. 5 power loss in a fairly typical chamber, of volume 42 litres, is plotted against gross resistor volume. It was found empirically for the two diameters of resistor used that the volume coordinate \( \pi/4 \, D^2 \) gave better plots than surface area or \( D^3 \). This graph shows that the loss is greater when the resistors project somewhat away from the wall, through either larger diameter or stacking. A scale of \( Z/\pi \) calculated from the dimensions and \( Q \) value for the cavity is shown on the right of the graph. By extrapolating these results it is seen that even for the worst case the cavity should be sufficiently well damped when the total resistor volume is between 0.5 and 1.5% of cavity volume. This has been confirmed by measurements on various other cavities formed by enlargement of beam pipe diameter.
Thin ceramic plates coated with nichrome have also been used as damping resistors where it was necessary to keep material thickness to a minimum. These have been mounted in both inverted-vee and cross form. In terms of gross occupied volume they are less effective than cylinders, but in many cases adequate cavity damping has been provided.

Plates inside vacuum chambers used as clearing electrodes for trapped electrons can be damped externally, using the feedthrough insulator by which the high voltage is applied. The inner conductor of the feedthrough is taken through a high resistance to the high-voltage supply, and through a capacitor to a coaxial connector. A 50 ohm termination is screwed on to the connector. The RF circuit through the feedthrough and capacitor is good only up to about 400 MHz. However, this is sufficient for the lower line modes on clearing plates. Higher modes are taken care of by the cavity damping resistors.

Lossy ferrites

As an alternative to resistors, lossy ferrite material has been tried. This should have an advantage where the damping material has to be kept close to the chamber wall, since this is usually a region of strong RF magnetic field, but very little electric field parallel to the wall.

Measurements have been made (ref. 16) on a typical intersection chamber containing "Ecosorb ZN" lossy ferrite tiles. The tiles were found to be effective in damping various modes, particularly if they were attached at two or more points around the periphery. Cleaning and baking were found to be necessary (ref. 17) before the tiles were acceptable for the ISR vacuum system. Damping properties were not permanently changed by baking to 500°C.

During ISR construction, ferrite rings were fitted for damping purposes in the beam-dump kicker magnets. They were placed round barium-titanate rods used as RF bypass capacitors on the pulsed conductors.

Acknowledgements

Most of the measurement methods and damping arrangements described in this report result from the work of many people in CERN, members of BEIC or Vacuum or RF groups and others.
References


APPENDIX

Transit-time factor

This factor relates the actual integrated effect of the electric field to the peak value in space and time determined by measurement or calculation.

(i) Constant field along the beam path, sinusoidal variation in time (e.g. zero-mode TM₀ cavity).

In the most favourable case where particles are midway across the gap at peak RF voltage,

\[ E_{\text{eff}} = \frac{i}{L} \int_{-\frac{\pi L}{\lambda}}^{\frac{\pi L}{\lambda}} E_0 \cos \omega t \, dz = E_0 \frac{\lambda}{\pi L} \sin \frac{\pi L}{\lambda} \]

where

- \( E_{\text{eff}} \) is the effective field seen by the beam
- \( E_0 \) is the peak value of the field
- \( L \) is the gap length
- \( \lambda \) is the RF wavelength

The transit-time factor \( T = E_{\text{eff}} / E = \frac{\lambda}{\pi L} |\sin \frac{\pi L}{\lambda}| \) and is close to unity for a short gap. In the case where particles cross the gap in half an RF period, \( T = 2/\pi \), i.e. the R/Q figure is multiplied by \( T^2 = 0.41 \).

(ii) Sinusoidal variation of field in both space and time (e.g. TM₀₁₁ cavity).

In this case, for relativistic particles crossing at the most favourable time,

\[ E_{\text{eff}} = \frac{i}{L} \int_{-\frac{\pi L}{\lambda}}^{\frac{\pi L}{\lambda}} E_0 \sin \frac{2\pi z}{\lambda} \sin \frac{\pi uz}{L} \, dz \]

for \( n \) odd, where \( n \) is the axial mode number,

- \( z \) is the distance across the gap.

In the case of even axial modes the sine terms are replaced by cosines.

For TM modes in a cylindrical cavity, this gives

\[ T = \frac{1}{\pi} \frac{L}{\lambda} \left( \frac{\lambda c}{L} \right)^2 |\cos \frac{\pi z}{\lambda}| \quad \text{(or Sin)} \]

where \( \lambda c \) is the cutoff wavelength.
For TM_{01} modes we can substitute \( \lambda_c = 2.61a \), where \( a \) is the cavity radius, giving

\[
T = \frac{0.54 \, \frac{\lambda}{\lambda}}{(\frac{\lambda}{2a})^2} \left| \cos \frac{\pi \lambda}{\lambda} \right| \quad \text{(or Sin)}
\]

\( \frac{\lambda}{\lambda} \) is given by the expression \( \left[ \left( \frac{\lambda}{2.61a} \right)^2 + \left( \frac{n}{2} \right)^2 \right]^{\frac{1}{2}} \).

If we take the most favourable case (for the particles) of \( \frac{\lambda}{\lambda} = 1 \), we have

\[
T = 0.54 \left( \frac{2a}{L} \right)^2 = \frac{a}{3\pi} \quad \text{when} \ n = 1.
\]

So that calculated or measured R/Q figures can be multiplied by \( T^2 = 0.18 \).

This figure is strictly applicable only for the case \( \frac{\lambda}{2a} = 1.13 \), but will always be less for other dimension ratios, so \( T^2 = 0.18 \) represents the most pessimistic case.
FIG. 1. RESONATOR SHAPES.
FIG. 2. \( \frac{Z}{Q} \) from BEIC/42 and BEIC/33.

\[
\frac{Z}{Q} = \frac{R}{Q} \times \text{transit-time factor}
\]

\( \text{ohms} \)

60
50
40
30
20
10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
\( \frac{b}{d} \)

TMot mode

Transit-time factor 0.9
FIG. 3. NORMALIZED SHUNT IMPEDANCE OF e^{TM_{010}} MODE IN ELLIPTICAL SECTION RESONATOR.

\[ R \frac{L}{L} (1 + \frac{a}{L}) \]

\[ \begin{array}{c}
  200 \\
  180 \\
  160 \\
  140 \\
  120 \\
\end{array} \]

\[ \frac{b}{a} \]

0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

FIG. 4. RF equipment for perturbation and \( Q \) measurement.
FIG. 5. LOSS vs. RESISTOR VOLUME.

TM_{00} and TE_{31} modes in "Half-bicone".

- $\theta \leq 0.30$ mm resistors
- $\theta \times 0.16$

Stacked TM_{00} mode
Resistors axial

Stacked TE_{31} mode

Flat

Total Resistor Volume / Cavity Volume, %

0.1 0.2 0.3 0.4 0.5