ULTIMATE LONGITUDINAL PARTICLE DENSITY

IN HIGH ENERGY PROTON BUNCHES

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Summary

Instantaneous particle density in proton bunches is limited by collective phenomena. Depending on the particular conditions, these can take the form of microwave instabilities or low order coherent bunch mode instabilities. Theories in rather good shape exist now for both phenomena, out of which universal scaling laws can be derived to be used in the design of modern machines.

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1. INTRODUCTION

Both pp and ep colliders ask for short, dense bunches of protons. One main reason is that in order not to lose luminosity, the bunch length must not exceed the betatron amplitude function $\beta^*$ at the interaction point. In modern projects $\beta^*$ is some tens of centimetres. Another important point to bear in mind is that the bunch length must be relatively small compared to the bucket length, otherwise diffusion due to RF noise poses unsolvable problems. On the other hand, the line density of bunches will ultimately be limited by collective phenomena. Much work on this subject has been going on in different places for a while, out of which a detailed model of the beam-surrounding interaction has emerged, and is currently used in electron machine design. Observations and measurements made recently on the CERN SPS pp collider can be used to further refine and extend the current understanding of the problem.

2. THE COUPLING IMPEDANCE

We restrict ourselves to the problem of dense bunches spaced sufficiently apart in the machine so that the inter-bunch coupling can be neglected, or does not play a prominent role.

It is believed that the various boxes, flanges, vacuum chamber cross-section variations encountered by the beam along its path add up randomly to create the equivalent of a broad-band coupling impedance rising up to a maximum around the vacuum pipe cut-off frequency (1.3 GHz) and decaying slowly at higher frequencies (Fig. 1).

It is sometimes approximated by the impedance of a low Q resonator, when this can help analytical or computer calculations. For our purpose here, the exact form is of no real importance. We shall just keep in mind that:

- the real part (resistance) of the coupling impedance reaches large values at frequencies high compared to the bunch spectrum. Up to these frequencies, the value $\frac{Z}{n} (n = f/\nu)$ is about constant.
- At frequencies corresponding to the bunch spectrum (0 to 1 GHz) the beam couples to a large inductive impedance: the "inductive wall".
In the CERN ISR, an inductive \( Z/n \) of 20–30 \( \Omega \) has been measured\(^6\). In the SPS, extensive data on bunch stability can be fitted very well with an inductive \( Z/n \) of 30 \( \Omega \). We suppose that the resistive part has about the same amplitude.

3. THE MICROWAVE INSTABILITY

We apply the Boussard criterion\(^7\), which is an extension to bunched beams of the Keil-Schnell criterion. It says that in order for the bunch to remain stable, the local intensity \( I \) and momentum spread \( \Delta p/p \) along the bunch must fulfill the relation:

\[
\frac{Z}{n} < F' \frac{E_0}{e} \frac{\gamma}{Y} \left( \frac{\Delta p}{p} \right)^2
\]

(1)

\( E_0 = \) rest mass of the particle of charge \( e \)
\( \gamma = \) Lorentz factor
\( \eta = \frac{1}{\gamma^2} - \frac{1}{\gamma^2} \)

\( F' \) is a form factor which can be taken equal to one for our purposes.

Let us express this relation in terms of bunch parameters. If \( h \) is the harmonic number, \( F \) the frequency and \( V \) the amplitude of the accelerating voltage, the maximum momentum spread in the bunch is related to the bunch length \( T \) in metres by:

\[
\left( \frac{\Delta p}{p} \right)^2 = \frac{\pi}{2} \frac{e V}{c^2 E_0 \gamma h \eta} F^2 T^2
\]

(2)

with \( c = \) the velocity of the light.

Writing the peak intensity

\[
I = N e \frac{F}{h} \frac{2 \pi R}{T}
\]

\( N = \) number of particles in the bunch, \( R = \) machine radius, the criterion for stability can be put into the form:

\[
\frac{Z}{n} < \frac{1}{4e c^2} \frac{V F}{N R} T^3
\]
\[
\tau_m^3 > 5.8 \times 10^{-4} \frac{N}{V \text{MV}} \left( \frac{F \text{MHz}}{10^3} \right) R Z n \quad (3)
\]

where \( N \) is expressed in units of \( 10^{10} \) particles, \( V \) in megavolts and \( F \) in megahertz.

Remark: the relation (2) is valid only for particles with small synchrotron amplitudes. Therefore when bunches not much shorter than the bucket are considered, the criterion (3) is an approximation suitable only for scaling purposes.

4. THE COHERENT MODES INSTABILITY

Sacherer\(^8\)) gives the criterion for stability of coherent bunch modes

\[
\frac{S}{\Delta \Omega_{sc}} > k_m
\]

where \( S = \Delta \Omega \) is the frequency spread in the bunch, \( \Delta \Omega_{sc} \) the synchrotron frequency shift due to the interaction, and \( k_m \) a coefficient depending on the mode number \( m \).

For a purely local interaction like the inductive wall, and a parabolic bunch:

\[
\Delta \Omega_{sc} = \frac{3}{2\pi^2} \frac{I}{h V} \left( \frac{2\pi R}{T} \right)^3 \frac{Z}{n}
\]

where \( I \) is the d.c. current.

The spread \( S \) is very well approximated for a bunch of length \( T \) by

\[
S = \frac{\tau^2}{64} \left( \frac{2\pi R}{c F} \right)^2
\]

and the coefficients \( k_m \) can be found in Besnier's thesis\(^9\)). Table 1 gives their values for lower order modes.
Table 1: Stability Coefficients

<table>
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<tr>
<th>m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_m$</td>
<td>3.4</td>
<td>1.6</td>
<td>.9</td>
<td>.65</td>
</tr>
</tbody>
</table>

So the criterion can be written:

$$T^5 > k_m \frac{Z}{n} \frac{48e c^4}{\pi^3} \frac{N R}{VF^3}$$

or with the same units as in the preceding chapter:

$$T^5_{tm} > 20 k_m \frac{Z}{n} \frac{N^{1.10} R}{V_{MV}{F^3_{MHz}}}$$ (4)

APPLICATION

When storing dense bunches of protons in the SPS, the lower order dipole and quadrupole modes are damped by feedback systems. Then the next troublesome modes are the sextupole or the first non-rigid dipole, both with $k_m \sim .9$.

So we can evaluate the lower limit of the bunch length in our case with $N = 10$, $V = 4.8$, $F = 200$, $R = 1100$, and an inductive impedance $Z/n$ evaluated to 30 $\Omega$:

$$T_c > .7 \text{ m}$$

The microwave instability criterion gives in the same situation

$$T_{mw} > .58 \text{ m}$$

So in the SPS, even coherent modes higher than quadrupole dominate the microwave instability, for the kind of impedance considered here.

This is well verified experimentally.

5. HOW TO USE THESE CRITERIA

The microwave instability is a phenomenon which develops typically in a fraction of a synchrotron oscillation period: it is a fast instability.
The coherent modes instability develops typically in several periods of synchrotron oscillation. This is because the strong local inductive wall interaction (or the space-charge interaction at low energy) suppresses Landau-damping but does not induce any growth of the modes amplitude: growth is due to interaction with the resistive part of the impedance. Whereas multibunch instabilities can be rather fast when strong, narrow-band coupling impedances are present, instabilities of single bunches are usually very slow, and it is even hard to find mechanisms explaining their growth\(^{11}\).

So the situation can be very different from one machine configuration to another:

- Whenever the microwave instability can develop, it supersedes the coherent modes and determines eventually the bunch length. In proton machines overshoot phenomena are important in this respect.
- The absence of the microwave instability does not ensure stability. One must also enquire whether the lower coherent modes are stable, and if not, whether they have sufficient time to grow. Overshoot will again determine bunch length.

Let us consider some examples:

1. **The SPS in storage mode at 270 GeV/c**

   With the beam parameters (intensity, bunch emittance) which have been available up to now during storage tests with protons, no microwave instability is predicted by criterion (3) with a coupling impedance \( \frac{Z}{n} = 30 \Omega \). We have been able to study bunches of \( N = 10^{11} \) p with initial length larger than .6 m, or bunches of length .45 m but with intensities lower than \( 3 \times 10^{10} \).

   Actually, no microwave instability has ever been detected.

   On the other hand, criterion (4) tells us that our bunches should be unstable on dipole, quadrupole and sextupole modes. This is well verified: if not damped by active feedback, the quadrupole mode, which develops over minutes (> \( 10^4 \) synchrotron oscillations) eventually blows up the bunch emittance by a large factor, due to overshooting\(^1\). The sextupole mode, which starts closer to its threshold, is subject to less overshooting and produces only a moderate emittance growth.
2. **The SPS during injection of 6 proton and 6 antiproton bunches at 26 GeV/c**

RF voltage at injection, as determined by matching considerations, is much lower than at high energy. Therefore criterion (3) tells us that the low emittance bunches the CPS is able to provide will be subject to the microwave instability. To avoid this, it is foreseen to blow up their emittance in the CPS by a factor $3^{12}$This should also ensure stability of the coherent modes.

3. **Electron storage rings**

All machines show some kind of bunch-lengthening, now currently ascribed to a microwave instability, or "turbulence". In SPEAR, large amplitude coherent modes are also observed, whereas in PETRA bunch-lengthening is not accompanied by such macroscopic motion.

The simple criteria developed in this report cannot be applied to electron machines. The reason is that the form of coupling impedance assumed is not appropriate for the very short bunches of electrons. Bunches with characteristic lengths $\sigma = 2$ cm (SPEAR) or 1 cm (PETRA) have wide spectra which couple to frequencies much higher than the vacuum pipe cut-off.

It has been shown $^4$ that the SPEAR bunch-lengthening can still be explained by a more elaborate application of the Boussard criterion for microwave instability. Whether this is equally valid in the extreme case of PETRA remains to be seen.

We also know that the coherent modes in SPEAR show up when Landau-damping is lost due to a real frequency shift coming from the interaction of the bunch with a reactive coupling impedance. But this effective coupling impedance is about 10 times smaller than the low frequency inductive wall $^{13,14}$ because the short bunch couples essentially to high frequencies (see Fig. 1).

The absence of strong coherent modes in PETRA could be explained in two ways: either the effective reactive coupling impedance is so low that the modes are Landau-damped, or they are not Landau-damped but their growth-rate is smaller than the radiation damping rate.
7. CONCLUSION

The higher possible density of non-interacting proton bunches in an accelerator or storage ring depends only, once the radius of the machine and the voltage and frequency of its accelerating field have been chosen, on the coupling impedance (i.e. the smoothness) of the vacuum chamber.

The lower possible limit of bunch length, as determined by bunch stability, is given by two very simple and universal formulae which are presented in this report, and can be used to optimize a machine design.

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Fig. 1: Real and Imaginary part of coupling Impedance (ref. 14)