Static behaviour of a 2-metre straw tracker

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Introduction

Conventions and notations

\( x \) : longitudinal coordinate of object, zero half-way
\( L \) : total length of object between fixation points
\( y \) : transverse coordinate, vertical-\textit{upward} for horizontal object
\( T \) : axial tension on object
\( U \) : value of high voltage
\( R_1 \) : radius of anode wire
\( R_2 \) : radius of cathode cylinder (straw)
\( \varepsilon \) : dielectric permittivity of chamber gas; we will take \( \varepsilon \approx \varepsilon_0 = 8.85 \text{pF/m} \)

In many equations, the natural logarithm of the ratio of radii is found. Since this ratio is high, the logarithm is a \textit{mild} function of that ratio:

\[
\ln \frac{R_2}{R_1} = \begin{cases} 
5.78 & \text{for } 2R_2 = 9.75\text{mm }, 2R_1 = 30 \mu\text{m} \\
5.09 & \text{for same } R_2 \text{, but } \text{doubled } R_1
\end{cases}
\]

Anode wire

Electric field gives a destabilizing phenomenon, associated with a negative transverse stiffness, per unit length:

\[
k_{\text{neg}} = -2\pi \varepsilon \left( \frac{U}{R_2 \ln \frac{R_2}{R_1}} \right)^2
\]

The minimal wire traction in order to stabilize equilibrium:

\[
T_{\text{min}} = |k_{\text{neg}}| \left( \frac{L}{\pi} \right)^2
\]
**Horizontal straw : vertical position \(y(x)\) of anode wire in sagging straw \(y_s(x)\)**

Assume \(y_s(x) = Y_0 \cos \frac{\pi x}{L}\)

and let \(w\) be the wire weight per unit length.

**Equilibrium equation :**

\[
T \frac{d^2 y}{dx^2} + k_{\text{neg}} |y - y_s| - w = 0
\]

subject to \(x\)-symmetry \(y(-x) = y(x)\), and to boundary condition : \(y(L/2) = \delta\)

**Note :**

- disturbance = wire weight ;
- imperfection = misfit of wire positioning at end plug.

**Solution ; position of wire relative to straw (sign-borne eccentricity) :**

\[
y - y_s = \tilde{Y} \cos \left( \sqrt{\frac{T_{\text{min}} \pi x}{T L}} \right) + \frac{Y_0}{T_{\text{min}}} \cos \frac{\pi x}{L} + \frac{w}{k_{\text{neg}}} \quad \text{where} \quad \tilde{Y} = \frac{\delta - w}{k_{\text{neg}}} \cos \sqrt{\frac{T_{\text{min}} \pi}{T L 2}}
\]

Below is the graph of \(y(0) - y_s(0)\) as a function of \(T\). Parameters to the curves : \(Y_0\) (0 and -0.2mm) and \(\delta\) (-0.1, 0 and +0.1mm). It shall be noted that the numerical values used for the imperfections are ambitious. Other numerical values used : \(2R_1 = 30\mu\text{m}, 2R_2 = 9.7\text{mm}, U = 2.6\text{kV}, L = 2.15\text{m}\), from which follows : \(T_{\text{min}} = 0.224\text{N}\). With a wire density of 19.3kg/ltr we get \(w = 1.34 \times 10^{-4}\text{ N/m}\).

**Horizontal straw : vertical position, half-way, of anode wire w.r.t. straw**

---

**Axes:**
- Wire tension \([\text{N}]\)
- Relative position of wire \([\text{mm}]\)

**Legend:**
- **Solid lines:** straw perfectly straight
- **Dashed lines:** straw sagging by 0.2mm
- **Fixation of wire at end plugs:**
  - 0.1mm too high
  - Perfect
  - 0.1mm too low

**Critical tension:** 0.224N
Cathode cylinder – axial traction vs. lateral deflections

Destabilizing phenomena:
- electric field, we will neglect it for the time being;
- gas pressure.

Stabilizing mechanisms:
- straw's bending stiffness;
- axial, external traction on straw.

Disturbance: straw weight. We will study the most unfavourable orientation: horizontal straw.

See Appendix B for a general discussion on slender cylinders under internal pressure. We will use the same notations hereunder.

Without intermediate supports (“spacers”)

We will take a free length $L=2.15m$. Straw diameter $2R_2=9.7mm$, so $A=7.4 \times 10^{-5} \text{ m}^2$. With $p=1\text{bar}$ we get $Ap=7.4\text{N}$. Below, we will study two variants, one based on Mylar$^1$ and one on CFRC$^2$. The reason of assuming different thicknesses should be obvious; one can easily challenge the sense of realism around the 0.1mm-thin composite assumption!

<table>
<thead>
<tr>
<th></th>
<th>Mylar</th>
<th>CFRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [ GPa]</td>
<td>4$^3$</td>
<td>80$^4$</td>
</tr>
<tr>
<td>density [ kg/ltr]</td>
<td>1.37</td>
<td>1.6</td>
</tr>
<tr>
<td>thickness [ mm]</td>
<td>0.036</td>
<td>0.1</td>
</tr>
<tr>
<td>$I$ [ $10^{-11}$ m$^4$]</td>
<td>1.3</td>
<td>3.6</td>
</tr>
<tr>
<td>$EI$ [ Nm$^2$]</td>
<td>0.0516</td>
<td>2.88</td>
</tr>
<tr>
<td>$w$ [ N/m]</td>
<td>0.015</td>
<td>0.048</td>
</tr>
<tr>
<td>$EI/(ApL^2)$ [-]</td>
<td>1.51 $10^{-3}$</td>
<td>8.4 $10^{-2}$</td>
</tr>
</tbody>
</table>

To obtain central sagging of 0.2mm:

| $T$ [ N] | 50 | 146 |
| $\sigma$ [ MPa] | 45 | 48 |
| $\varepsilon$ [%] | ?! | 0.6 |
| $\Delta L$ [ mm] | ?! | 1.3 |

1 Trade name from DuPont de Nemours for film of biaxially oriented PETP, poly(ethylene terephthalate), with microcrystallites
2 Carbon fibre reinforced composite
3 Initial modulus, NOT TO BE USED FOR LONG-TERM EFFECTS!
4 Assumed effective tensile
Observations:

- The dimension-less factor $EI/(ApL^2)$ is small. **Bending stiffness can be neglected and the cylinder can be approached like a tensioned cable.** See B4.4. This even applies, albeit somewhat approximately, for the CFRC variant, with much higher $EI$. At least if one takes the “hinged” case. Indeed, trying to clamp a composite pipe rigidly in the frame, for whichever marginal benefit it might bring, may not be a good idea.

- Consequently, the simple “cable” relation can be used: 
  \[
  \text{sagging} = \frac{wL^2}{8(T-Ap)}
  \]

- Now, since the weight is the driving force, and the densities do not differ much, the necessary traction scales roughly with pipe thickness, and the resulting pipe wall tensile stress is roughly invariant... for an imposed straw straightness.

- **An ambitious straightness calls for very important tractions.** The resulting material loading is out of safe reach for a thermoplastic like PETP.

- Since straightness can only be obtained through traction, it is important that one can **ascertain** that traction, in spite of uncertainties, and in the long term, over many years!

- Internal pressure is a destabilizing phenomenon, and it is good to understand it properly. However, it has, for lateral effects, merely the effect of decreasing the cable tension.... not by much in our cases!

---

**The usefulness of “spacers”**

Supporting the cylinder with $n$-1 equidistant “spacers” means that the free length is divided $^5$ by $n$, and one could argue that the benefit $^6$ scales with $n^2$ .... or more, since, for shorter free lengths, the bending stiffness does start playing a role.

This important potential of such spacers may counterbalance their disadvantage of adding locally a lot of material.

And this would make the discussion straightforward, if the goal of these spacers were to “stiffen” in the traditional engineering context, i.e. preventing a collapse, for example.

But it is not.

What matters is the eccentricity of the anode wire with respect to its accompanying cathode cylinder.

**With spacers, one tries to achieve local precision with very remote means.**

---

$^5$ To keep the discussion simple, we neglect the hinged/clamped notion.

$^6$ ... for the sum $T-Ap$. The needed traction would become smaller, and thus, the relative importance of pressure would become greater.
The sketch below depicts the chain of uncertainties and tolerances. Those of the wire are not included. In other words, a perfectly straight wire, held at the straw ends without misfit, is assumed.

1. Positioning of straw ends (end plugs) in the “piano frame”.
2. Precision of frame.
3. Deformations of frame (various types of loading).
4. Precision of hang-down of “line B”.
5. Line B leads an other life than line A, especially if they are from different materials: temperature, humidity. If line B is made from polymer, additional trouble may come from relaxation of internal stresses, which may be due to the fabrication, some fraction of it may be released in weeks, another in years. Remember that 0.1mm over 1m is a strain of $10^{-4}$ only. This is a small strain, especially for polymers.
6. Precision of the hang-down assembly: line B and spacer ring.
7. Positioning of straw cylinder in spacer ring.
**How to apply traction to straws**

The “best” way would be by a mechanism that would ascertain the magnitude and constancy of force. For physics detector construction, this usually boils down to adding an external, metallic spring to the element to be put under traction. The higher the compliance of that spring compared to the one of the detector element, the better one approximates the ideal of “a given force”.

Realizing such construction in a straw chamber is not trivial, mainly because of the constraint of gas tightness.

On the contrary, “potting” the straw ends in the “piano frame” seems an elegant way of making that frame work as the gas barrier.

However, in such case, the traction is induced by a position-defined mechanism; the accompanying issues should not be taken lightly:

- The imposed element elongation should be of reasonable order of magnitude: 1cm is, 1mm is not.

- The piano frame must be so stiff that its deformations do not alter the magnitude of the induced elongation by too much. If not, then complicated pre-loading schemes must be found. They may turn out to be prohibitive. Especially when the final loading, to cause frame deformations, does not come from the “elements” alone. In our case, the gas pressure shall not be forgotten!

- Possibly the most serious issue is that the element_to_be_strained lives its own life. Absorbed humidity may evolve between assembly and operation. More important is the “relaxation”: the essential time-dependence, in case of polymers, of elastic modulus. One can “loose” a significant portion of the initial traction over the years, depending upon the location of the (primary) glass transition on the time axis. This phenomenon is linear, occurring even for the smallest strains. Evidently, for higher loads, possible material non-linearities (creep) would add to the worries.

---

7 At operational temperature
Appendix A : Static (in)stability, negative stiffness, amplification of parasitics

\[ E_{\text{pot}} = k \frac{x^2}{2} + mgy = |k - |k_{\text{neg}}|\frac{x^2}{2} \quad , \quad k_{\text{neg}} = -2mgA \]

Destabilizing effects can be associated with \textit{negative stiffness} (not always constant : not always a linear effect).

Stable for \( k > |k_{\text{neg}}| \) ...

.... but \textit{effective} stiffness is only \( k - |k_{\text{neg}}| \) !

- “Disturbance” : horizontal force \( Q \) on ball will result in \( x \)-deflection : \( \frac{Q}{k - |k_{\text{neg}}|} \)

- “Imperfect geometry” : offset \( x_{\text{off}} \) of spring will result in \( x \)-offset of ball : \( \frac{1}{1 - \frac{|k_{\text{neg}}|}{k}} x_{\text{off}} \)

A destabilizing mechanism amplifies :

- the effect of a disturbance ;
- an inherent imperfection.
To buckle or not to buckle?

"perfect" with "disturbance" imperfect geometry

Spring is ALWAYS drawn in its REST position, and "follows" to stay horizontal

\[ \theta \quad \text{[radians]} \]
\[ \frac{N}{kL} \quad \text{[\text{\text{-}}]} \]

\[ \theta_0 \quad \text{[radians]} : \]
\[ 0 \quad -0.001 \quad -0.003 \quad -0.01 \]

\[ \frac{Q}{kL} \quad \text{[\text{-}}] \]
\[ 0 \quad 0.001 \quad 0.003 \quad 0.01 \]
Buckling: only well-defined in the “perfect” case. Even “far away” from “buckling”: destabilizing mechanism may give significant trouble.

Moment equilibrium about hinge, taken along disturbance and offset:

\[ NL \sin \theta + QL \cos \theta = kL (\sin \theta - \sin \theta_0) L \cos \theta \]

Simplifies to, in 1st-order approximation:

\[ Q + kL \theta_0 = -N \theta + kL \theta \]

Beam compression gives negative lateral stiffness.

\[ \theta = \frac{1}{1 - \frac{N}{kL}} \left( \frac{Q}{kL} + \theta_0 \right) \]

Output = (amplification multiplier) x (normal result of stimuli)

Often, for conception of physics detectors:
- magnitude of destabilizing mechanism (\( N \)) is given;
- intensity of stabilizing mechanism (\( k \)) must be negotiated, but comes at a cost.

Example: anode wire in (perfectly) cylindrical cathode, horizontal, in gravity field.

- \( |k_{\text{neg}}| \approx \left(\frac{\text{voltage}}{\text{diameter}}\right)^2 \) (approximately)
- disturbance: e.g. wire weight
- imperfection: e.g. wire positioning at ends
- result: e.g. mid-way sagging of wire
- stabilizing mechanism: wire tension (no “e.g.”, well-defined)
Appendix B: Destabilizing effects on hollow beams (slender pipes) subjected to inner pressure

The subject remains utterly confusing. Below, an attempt is made to confront the reasoning outlined in [1] with that found in [2], and to supply unambiguous recipes for several geometry cases.

B0. Conventions, assumptions and notations

Let \( x \) be the beam's longitudinal coordinate, and \( y(x) \) its transverse deflection. Section forces: normal force \( N \), positive in compression, transverse force \( Q \) and bending moment \( M \). As in the classical column buckling theory, we will assume linearity, so linear elastic material and small deflections, and we will neglect influence of gravity.

Let \( EI \) be the beam's bending stiffness, \( A_{beam} \) the cross-sectional area of the beam's flesh (pipe wall), and \( A \) that of the enclosed fluid at pressure \( p \).

B1. Discussion of reference [1]

The reasoning is done on a pipe closed with an ideal piston (see the sketch of case I below). The piston is part of the system: pipe plus fluid. The force \( N_{ext} \) on the piston to compress the fluid is external. This external force is taken into account, in the classic way, to compute the section forces at an imaginary cut of the beam system which includes the fluid. It is then, correctly, stated that transverse force and bending moment have to be taken up by the wall. However, the discussion takes an unfortunate short cut, “ignoring the shear effect”\(^8\) and ending up with a second-order differential equation governing \( y(x) \). It happens to yield the correct solutions (only) for a distinct set of boundary conditions. A more rigorous treatise, generally valid, would yield:

\[
EI \frac{d^4 y}{dx^4} + N_{ext} \frac{d^2 y}{dx^2} = 0
\]

Substitute the beam compressive force by the piston force in the classic Euler equation.\(^9\)

In the case studies below, we will learn how to use this recipe more generally, by inserting an imaginary piston and redefining system boundaries.

---

\(^8\) Neither the assumption that one neglects the shear component in the deflection, nor the constancy of the shear force, mean that this shear force may always be omitted from the equation.

\(^9\) Obviously, in case of an additional mechanical external force on the pipe, this shall be appended to \( N_{ext} \).

This article gives a different, complementary view by *separating* the bare pipe and the contained fluid.

Isolating a *fluid* element, resolving force equilibrium perpendicular to its axis:

\[ q = Ap \frac{d\theta}{ds} \]

**The contained fluid exerts a lateral force on a deflected pipe.** Its magnitude, per unit length, is the product of pressure, cross-section, and curvature. It acts towards the outside of the curve.

Isolating now an element of *bare pipe*, one can derive the column buckling equation in the traditional way. We transmit the lateral reaction of the fluid element; because of small deflections, we have dx=ds and \(\cos\theta=1\).

\[ \text{Moment equilibrium :} \]
\[ \frac{dM}{dx} + N \frac{dy}{dx} - Q = 0 \]

\[ \text{Transverse force equilibrium :} \]
\[ q + \frac{dQ}{dx} = 0 \]

\[ \text{Bending stiffness relation :} \]
\[ M = EI \frac{d^2 y}{dx^2} \]

And we have from above:
\[ q = Ap \frac{d^2 y}{dx^2} \]  

(1)
Differentiating the moment equation, we get, after substituting $M$ and $Q$, $q$:

$$EI \frac{d^4 y}{dx^4} + N_{\text{eff}} \frac{d^2 y}{dx^2} = 0$$

where:

$$N_{\text{eff}} = N + Ap$$

and where:

$$N = - \int_{A_{\text{beam}}} \sigma \, dA \quad (\sigma = \text{tensile stress})$$

is the compressive normal force in the pipe wall.
**B3. Various geometry cases**

We will determine \( N_{ext} \) and \( N_{eff} \), in the spirit of [1] and [2], respectively. We will see that \( N_{ext} \) always equals \( N_{eff} \).

The reaction force from the lower support always equals \( N_{ext} \).

**Case I: simple piston as in [1].**

\[
N_{ext} = Ap \quad N = 0 \quad N_{eff} = Ap
\]

**Case II: pipe “plunged” in pressure vessel.**

\[
N_{ext} = Ap \quad N = 0 \quad N_{eff} = Ap
\]
Case III: connection with vessel through reduced section $A_{red}$.

$$N_{ext} = A_{red} p \quad N = (A - A_{red}) p \quad N_{eff} = A p + N = A_{red} p$$

Case IV: pipe-internal reduction. It should be obvious that a plug with orifice, *held by the pipe*, makes no difference to case II.
Case V: including axial mechanical load on the pipe. Neglect clamping effects of the external tensioning action.

\[ N_{\text{ext}} = Ap \cdot T \quad N = -T \quad N_{\text{eff}} = Ap - T \]

At least for transverse effects, pressure has the effect of decreasing the pipe traction.

Case VI: sideways connection.

\[ N_{\text{ext}} = 0 \quad N = -Ap \quad N_{\text{eff}} = 0 \]
**B4. Stable deflections in case of a known transverse load**

We will assume the transverse load \( w \) per unit length to be constant.

\[
\begin{align*}
C_{\text{hinged}} &
\end{align*}
\]

\[
\begin{align*}
C_{\text{clamped}} &
\end{align*}
\]

\[
\begin{align*}
L &
\end{align*}
\]

### 4.1. Including bending stiffness

The critical \( N_{\text{eff}} = Ap - T \) is the one corresponding to the first buckling mode:

\[
N_{\text{eff, crit}} = f \frac{\pi^2 EI}{L^2}
\]

where \( f = 1 \) for the *hinged* case, and \( f = 4 \) for the *clamped* case.

The solutions to the differential equations below are always subject to symmetry, \( y(x) = y(-x) \), and to boundary condition \( y(L/2) = 0 \).

For the *hinged* case, the additional boundary condition \( \frac{d^2 y}{dx^2}(L/2) = 0 \) applies.

For the *clamped* case, the additional boundary condition \( \frac{dy}{dx}(L/2) = 0 \) applies.
4.1.1. Low (or even negative) traction: $Ap - N_{eff,crit} < T < Ap$

Equation:
\[ \frac{d^4 y}{dx^4} + \mu^2 \frac{d^2 y}{dx^2} + \frac{w}{EI} = 0 \quad , \quad \mu^2 = \frac{Ap - T}{EI} \]

Solution:
\[ y(x) = \frac{wL^2}{8(Ap - T)} \left[ 1 - \left( \frac{2x}{L} \right)^2 + C \left( 1 - \frac{\cos \mu x}{\cos (\mu L/2)} \right) \right] \]

\[ C_{hinged} = \frac{8}{(\mu L)^2} \quad , \quad C_{clamped} = C_{hinged} \frac{\mu L/2}{\tan (\mu L/2)} \]

4.1.2. Tuned traction: $T = Ap$

This specific case reduces down to the traditional beam bending problem.

Equation:
\[ \frac{d^4 y}{dx^4} + \frac{w}{EI} = 0 \]

Solution:
\[ y_{hinged}(x) = \frac{wL^4}{384EI} \left[ -\left( \frac{2x}{L} \right)^4 + 6\left( \frac{2x}{L} \right)^2 - 5 \right] \]

\[ y_{clamped}(x) = \frac{wL^4}{384EI} \left[ -\left( \frac{2x}{L} \right)^4 + 2\left( \frac{2x}{L} \right)^2 - 1 \right] \]

4.1.3. High traction: $T > Ap$

Equation:
\[ \frac{d^4 y}{dx^4} - \kappa^2 \frac{d^2 y}{dx^2} + \frac{w}{EI} = 0 \quad , \quad \kappa^2 = \frac{T - Ap}{EI} \]

Solution:
\[ y(x) = \frac{wL^2}{8(T - Ap)} \left[ \left( \frac{2x}{L} \right)^2 - 1 + C \left( 1 - \frac{\cosh \kappa x}{\cosh (\kappa L/2)} \right) \right] \]

\[ C_{hinged} = \frac{8}{(\kappa L)^2} \quad , \quad C_{clamped} = C_{hinged} \frac{\kappa L/2}{\tanh (\kappa L/2)} \]
4.2. Without bending stiffness , \( T > A p \)

The power of the more general formulation of \([2]\) enables us to use it even entirely outside the column buckling concept. Indeed, in absence of, or neglecting, bending stiffness, we get, with the aid of equation (1), the “piano wire equation”:

\[
(T - Ap) \frac{d^2 y}{dx^2} - w = 0
\]

with the familiar solution\(^{10}\):

\[
y(x) = \frac{wL^2}{8(T - Ap)} \left( \frac{2x}{L} \right)^2 - 1
\]

(3)

At least for transverse effects, **pressure has the effect of decreasing the cable tension.**

The sagging of the “cable”:

\[
S_{cable} = |y_{eq,(3)}(0)| = \frac{wL^2}{8(T - Ap)}
\]

4.3. Reduction of deflection due to bending stiffness , \( T > A p \)

In presence of bending stiffness, recalling section 4.1.3 , the beam sagging becomes, according to boundary case:

\[
S_{hinged} = |y_{eq,(2),hinged}(0)| = r_{hinged}(u) \cdot S_{cable}
\]

\[
S_{clamped} = |y_{eq,(2),clamped}(0)| = r_{clamped}(u) \cdot S_{cable}
\]

where \( u = \frac{\kappa L}{2} \). See graph below.

---

\( ^{10} \) It can be shown that, for both hinged and clamped cases, (2) limits to (3) for vanishing \( EI \).
4.4. Over-all graph of sagging

Below is a set of curves of the normalized sagging \( S \cdot \frac{Ap}{wl^2} \), against the normalized traction \( \frac{T}{Ap} \). Parameter to the curves is the normalized bending stiffness \( \frac{EI}{ApL^2} \).
**B5. Acknowledgement**

Many thanks to A. Catinaccio for discussions and supply of references.

**B6. References**
