NONLINEAR DYNAMICS INDUCED BY 1-D MODEL OF PINCHED ELECTRON CLOUD

G. Franchetti¹, F. Zimmermann

Abstract

The presence of an electron cloud in an accelerator generates a number of interesting phenomena. In addition to electron-driven beam instabilities, the electron “pinch” occurring during a beam-bunch passage gives rise to a highly nonlinear force experienced by individual beam particles. A simple 1-dimensional model for the effect of the electron pinch on the beam reveals a surprisingly rich dynamics. We present the model and discuss simulation results.

1 GSI, Darmstadt, Germany

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INTRODUCTION

The interaction of the electron cloud in a synchrotron with the passing beam creates a rich jungle of phenomena. The electron-driven instabilities have received full attention as the time scale of their appearance is fast. It has, however, been recently recognized that even the stable dynamics of the coupled system bunched beam / electron cloud, can originate a complex beam dynamics leading to phenomena of periodic resonance crossing [1]. The time scale of these phenomena is much longer. Reference [2] has studied in detail the “pinch” dynamics of the electrons during the passage of a bunched proton beam. Under the action of the proton Coulomb field, the electrons undergo a pinch which creates a complex structure of “rings”. This structure weakly feeds back onto the tunes of the proton beam according to the actual location of these particles in the bunch frame. The global effect is an induced periodic tune modulation leading to possible resonance crossing. As shown in Ref. [2], this phenomenon closely resembles that encountered in the case of a stored high intensity bunched beam, where the periodic tune modulation is due to space charge [3]. The challenge in predicting the evolution of the proton bunch lies in the slowness of the full phenomena. The periodic crossing of a resonance creates a very slow beam diffusion which can be detected only on a long time scale (hundreds of synchrotron oscillations). This diffusion-generating mechanism is highly relevant for a collider like the LHC, but also in the case of space-charge driven periodic resonance crossing, as found for SIS100 [4]. The understanding of this coupled beam dynamics is essential for very long term prediction. A suitable approach uses a frozen model. Here the effect of the pinch is modeled with a simplified EC structure, where the structure of the pinch is kept the same from turn to turn while allowing the proton bunch dynamics to evolve. The main approximation made in Ref. [2] was to consider the EC pinch in an axisymmetric approximation. Although the approximation allowed an estimate for accelerators like LHC, RHIC, and SIS100 (for the latter with space charge), a discussion of the underlying nonlinear dynamics was left unexplored. In this paper we extend a simplified model, earlier presented in Ref. [5], and discuss its dynamical properties, gaining some insight into the complex dynamics governing a “resonance scattering” regime induced by the electron cloud.

THE ELECTRON CLOUD STRIPE MAP

We here consider the beam dynamics induced by the pinched electron cloud for protons moving only in one plane, so that the system becomes 1-dimensional. We also assume that the field created by the pinched electrons can, locally in z, be approximated by the one of an ideal planar structure consisting of 2 infinitely wide parallel planes of negligible thickness, and located at a vertical distance \( y_{EC} = \pm R_1 \) from the axis. The Coulomb field produced by this system is \( E = 0 \) between the two planes, and \( E = \pm \text{const.} \) outside them (Fig. 1). The quantity \( R_1 \) mod-

![Figure 1: Schematic of the 1-dimensional model of the electron cloud pinch.](Image 364x450 to 491x544)
as $1/y$, but similarly to the space charge the force is antisymmetric with regard to the exchange $y \leftrightarrow -y$. The repeated application of the map (2) creates, for $Q_y$ close some structure resonance $Q_r = m/n$, a set of $n$ fixed points according to the distance $\delta$ from the resonance. The phase advance is $\omega = 2\pi m/n + \delta$. For $m = 1$ one of these $n$ fixed points, we call it $y_0^{(n)}$, has the location:

$$y_0^{(n)} = e^{i[-\pi/2 - \omega/2]} 2\tan(n\delta/4)\sin(\omega/2).$$

The other $(n-1)$ fixed points are found by repeatedly mapping $y_0^{(n)}$ with (2). In Fig. 2 we show the phase space plot for $Q_y$ close to the resonances at $Q_r = 1/4$ (left), and $Q_r = 1/6$ (right). In both pictures $\delta = -0.0003$ and $R_1 = 0$.

Figure 2: Phase space structure for $Q_r = 1/4$ (left) and $Q_r = 1/6$ (right). In both pictures $\delta = -0.0003$ and $R_1 = 0$.

The distance of the EC planes, $R_1$, is 1/8 the island radius in Fig. 2, and $\delta = -0.0003$

**RESONANCE CROSSING**

At the start of the pinch no electrons are accumulated and the strength of the electric field is $F(t) = 0$. During the build up of the pinch $F(t)$ grows along the bunch, from zero up to a maximum value, and then it remains constant (in this modeling). Let us assume that $F(t) = \tilde{F} \delta/dN = \text{const.}$, with $N$ denoting the turn number. Due to the increase of $\tilde{F}$ the fixed points created by a structure resonance $Q_r = m/n$ migrate outwards according to

$$\frac{dS_0^{(n)}}{dN} = \frac{\tilde{F}}{2\tan(n\delta/4)\sin(\omega/2)}.$$

Here $\tilde{y} = \tilde{y} + iy' = y\tilde{F}$. The moving islands crossing a particle orbit will trap the latter if: The speed of rotation of the particle in the islands is faster than the speed of escape of the fixed points. The maximum speed of a particle in the islands is

$$v_p = |\omega_{sec}|y_0^{(n)} = \frac{\delta}{2\tan(n\delta/4)}.$$

Detrapping [6] occurs if the opposite condition is fulfilled, i.e. if $v_p < |dS_0^{(n)}|/dN$. The transition can be characterized by a dimensionless parameter $T$ defined as the ratio of these two speeds

$$T = \frac{1}{\omega_{sec}} \frac{dS_0^{(n)}}{dN}.$$

The condition of detrapping reads $T > 1$. $T$ can be also interpreted as an adiabaticity parameter: for $T \ll 1$ the motion of the fixed points is adiabatic with respect to the motion in the islands. The condition $T = 1$ corresponds to an EC strength of $\tilde{F}_t = \tilde{F} \delta/d[\sin(\omega/2)]$. With $\tilde{F} = \tilde{F} \delta/dN = \text{const.}$ the previous formula yields the number of turns $N$ at which the transition takes place: $\Delta N_t = 1/|\delta|\sin(\omega/2)]$. At this transition instant the fixed points are located at $|\tilde{y}_{e}^{(n)}| = \tilde{F} \delta/2[\tan(n\delta/4)\sin(\omega/2)]$. If beam particles have amplitudes larger than $|\tilde{y}_{e}^{(n)}|$ they will be trapped, otherwise scattered [7, 8]. However, when $\tilde{y}_{e}^{(n)}$ is at $T = 1$, the part of the island closer to the origin...
can still trap beam particles. Therefore, trapping does never occur only for those particles with $|\hat{y}| < |\hat{y}_{\Delta t}|$, where

$$|\hat{y}_{\Delta t}| = |\hat{y}_{0}^{(n)}| |1 - \sin(\omega/2)|.$$  

For those trapped their maximum amplitude is $|\hat{y}_{\max}| = |\hat{F}_{\max} \hat{y}_{0}^{(n)}|$. In Fig. 4 we show the benchmarking of (8) with the map (1).

$$\text{SCATTERING REGIME}$$

From (8) trapped particles should have an amplitude larger than $|\hat{y}_{\Delta t}| \propto \hat{F}_{\delta}$. $\hat{F}_{\delta}$ depends on the synchrotron tune, on the longitudinal particle amplitude and on the extent of the pinch along the longitudinal axis ($z$). As the latter is much shorter than the bunch length it is likely that the scattering regime is dominating the EC periodic crossing dynamics. We here introduce an analysis of the scattering regime. The beam dynamics in the scattering regime ($T > 1$) is characterized by the absence of trapping [9, 10, 11]. This means that when the resonant orbit crosses the particle orbit, the CS invariant of the particle is subjected to a random kick of maximum amplitude equal to the island size. If we characterize the particle via its radius $r_0$ (proportional to the root square of the action), we can represent the scattering process via the map

$$r_1 = r_0 + \xi \frac{\pi}{n} r_0,$$  

where $r_1$ is the new particle radius after one crossing. In (9) the quantity $\pi/nr_0$ is the radius of an island, and $\xi$ is a random variable of standard deviation $\sigma_\xi$, the value of which incorporates the effectiveness of the island to scatter the particle radius (invariant) at $r_0$. For $T > 1$ the islands cross the particle orbit very fast producing a small scattering, hence $\sigma_\xi \rightarrow 0$. The general dependence is a complex function of $T$, like $\xi = \xi(1/T)$.

$$\text{REFERENCES}$$