Effective field theory – concepts and applications

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Abstract

Effective field theory provides the modern perspective on renormalization theory and explains why we can make meaningful and precise predictions without knowing the Theory of Everything. By separating physical effects on different distance scales it is also an efficient tool to deal with the strong interaction in various regimes and to sum large logarithms in perturbation theory. The two lectures given at the school provide an introduction and cover mostly textbook material. The write-up below is therefore only meant as a brief reminder of the topics discussed, and provides references to textbooks or other lecture notes for further reading. Further details can also be found in the slides available on-line.

1 Concept of effective field theory

1.1 Basics

Stated simply, effective field theories are just general quantum field theories based on the principles of relativity and quantum mechanics but without the restriction of renormalizability. Any term allowed by the assumed symmetries of the theory is included in the action. The characteristic feature of an effective field theory is that it can be valid only up to a certain energy scale $\Lambda$, which must be above the energy scale directly accessible by experiment for the theory to be useful. In contrast, fundamental theories are candidates for theories that describe Nature at any scale. It is worth noting that we will never know whether a given candidate for a fundamental theory is the theory that describes Nature at all energies. Whether such a theory is adopted as the ‘Standard Model’ then depends on aesthetic criteria such as naturalness and theoretical simplicity. Effective theories are more adapted to describe what we can know.

Philosophy aside, a perturbative power-counting analysis shows that fundamental theories must not contain operators with mass dimension larger than four in their Lagrangian, while effective theories contain all possible higher-dimensional operators. Since the effect of short-distance physics looks local to an observer performing experiments at energies $E \ll \Lambda$, the ignorance of this physics can be parametrized by the values of the coupling constants of all possible local operators. Effective theories are predictive, since an operator of dimension $d$ contributes at most of order $(E/\Lambda)^{d-4} \ll 1$ to a dimensionless observable, hence only a finite number of unknown couplings are relevant to achieve a set precision. While so-called non-renormalizable interactions ($d > 4$) thus appear naturally in effective field theories, super-renormalizable interactions ($d < 4$, such as mass terms) are problematic, since their couplings are generically large, of order $\Lambda^{4-d}$. Mass terms of order $\Lambda^2$, however, describe particles that cannot be part of the effective theory. Thus super-renormalizable interactions should be protected by symmetries (chiral, gauge, etc.) to avoid a hierarchy problem.

The previous paragraph summarizes the modern view of renormalization as compared to the prevailing picture form the early days of QED until the late 1970s. The simple picture described above can be modified at strong coupling. On the one hand, theories that appear as candidates for a fundamental theory may not be fundamental after all, because some of the couplings develop a ‘Landau pole’ at a large but finite energy. The Standard Model itself is almost certainly of this kind. On the other hand,
theories that appear non-renormalizable by power counting may possess what is called a non-trivial ultraviolet fixed point, in which case the infinitely many couplings of the higher-dimensional operators are all related to a few. Einstein’s theory of gravitation may be of this kind, in which case there is no need for other theories of quantum gravity.

This is the very general ‘bottom-up’ usage of effective field theory in situations where the fundamental theory is unknown. There is a ‘top-down’ application where the fundamental theory (or another effective theory valid at higher energy scales) is known, and effective theory is used to simplify calculations or make them possible in the first place. A prime example is QCD, a perfect fundamental theory for the strong interaction. The point is that QCD is strongly coupled in certain regimes and weakly in others. Effective field theory allows us to separate the two regimes, to calculate what can be calculated, and to parametrize the remainder by ‘hadronic matrix elements’. In addition, large logarithms of different energy scales can be summed; this makes effective field theories useful even in a purely weak-coupling context. We shall discuss an example in the following subsection. Thus there are many effective field theories of QCD, each appropriate for a specific situation: chiral effective theory for the Goldstone bosons of spontaneous chiral symmetry breaking, nucleon effective theory, heavy quark effective theory, non-relativistic effective theory, soft-collinear effective theory, high-temperature effective field theory, etc.

Bibliography


1.2 Integrating out a heavy quark

Suppose there is nothing else in the world than gluons and six flavours of quarks one of which, the top quark, is much heavier than the others. The fundamental theory is

\[
\mathcal{L} = -\frac{1}{4} G^{A}_{\mu\nu} G^{A\mu\nu} + \sum_{f=1}^{5} \bar{\psi}_f (i \not{D} - m_f) \psi_f + \bar{Q} (i \not{D} - m_Q) Q.
\]  

At energies \( E \ll m_Q \) much below the top quark mass, physics is described by an effective field theory without the top quark field \( Q \) but containing all higher-dimensional operators built from \( \psi_f \) and the gluon field constrained only by the SU(3) gauge symmetry. Since the fundamental theory is known and weakly coupled at the scale \( m_Q \), the effective field theory Lagrangian can be calculated order by order by a procedure called ‘matching’. The simplest diagram containing the top quark is the gluon vacuum polarization at one loop. The QED analog of this calculation can be found in any quantum field theory textbook. Expand the result in external momentum \( q^2 \ll m_Q^2 \). The result can be reproduced in an effective theory without the top quark with Lagrangian

\[
\mathcal{L} = -\frac{1}{4} \left[ 1 - \frac{\alpha_s T_F}{3\pi} \ln \frac{m_Q^2}{\mu^2} \right] G^{A}_{\mu\nu} G^{A\mu\nu} + \sum_{f=1}^{5} \bar{\psi}_f (i \not{D} - m_f) \psi_f + \frac{\alpha_s T_F}{60\pi m_Q^2} G^{A}_{\mu\nu} D^2 G^{A\mu\nu} + \ldots
\]  

This simple example illustrates a few important points:
– The couplings of the dimension 4 terms have been modified, here the gluon kinetic energy term,
and a series of higher-dimensional operators has been generated. Matching the gluon three-point
function would generate another dimension 6 operator \( f^{ABC} G^A_{\mu\nu} G^{B\mu\nu} G^C_{\alpha\beta\gamma} \), and so on.

– To express the low-energy Lagrangian in terms of canonically normalized fields, we must rescale
the gluon field: \( \hat{A}_\mu = (1 - \frac{a_s T_F}{m^2} \ln(m^2_Q/\mu^2)) A^A_\mu \). Then the effective quark–gluon coupling is
\[
g_s \bar{\psi} f \hat{A} \psi_f = \frac{g_s}{1 - \frac{a_s T_F}{m^2} \ln(m^2_Q/\mu^2)} \bar{\psi} f \hat{A} \psi_f \equiv \hat{g}_s \bar{\psi} f \hat{A} \psi_f. \tag{3}
\]

The strong coupling in the effective theory is not the same as in the fundamental theory. However,
\( \alpha_s(m_Q) = \hat{\alpha}_s(m_Q) \) in the present approximation.

– If \( \mu^2 \frac{d\hat{\alpha}}{d\mu^2} = -\beta_0^{(6)} \hat{\alpha}_s^2/(4\pi) \) with \( \beta_0^{(6)} = 11 - \frac{2}{3} \cdot 6 \), then \( \mu^2 \frac{d\hat{\alpha}}{d\mu^2} = -\beta_0^{(5)} \hat{\alpha}_s^2/(4\pi) \) with \( \beta_0^{(5)} = 11 - \frac{2}{3} \cdot 5 \) just as expected. At scales \( \mu \ll m_Q \) one must use the effective five-flavour Lagrangian to sum logarithms \( \alpha_s \ln(m^2_Q/\mu^2) \) to all orders in perturbation theory, which otherwise ruin perturbation theory even when \( \alpha_s \) is small.

If we did not know the fundamental theory, we could not tell the existence of the top quark from the
dimension 4 terms, since their effect is hidden in the value of the low-energy strong coupling \( \hat{\alpha}_s \) that is
determined experimentally. However, as experiments approach the top quark scale, neglecting the
dimension 6 operator becomes a poorer and poorer approximation, and from the failure to describe
experimental data accurately we could infer the existence of a scale \( \Lambda \approx m_Q \), where the effective theory
breaks down. We might imagine measuring the couplings of the dimension 6 operators and from their
value derive the hypothesis that there must be a yet undiscovered heavy quark species.

1.3 Effective weak interactions

From the 1930s to the late 1960s the weak interactions were described by a set of dimension 6 four-
fermion operators. The non-renormalizability of these interactions provided strong motivation for searching
for a fundamental theory of the weak interactions which resulted in the discovery of the SU(2) \( \times \) U(1) gauge structure now included in the Standard Model. The value of the Fermi constant \( G_F \) together with
naturalness arguments points towards the 100 GeV scale as the limiting scale of Fermi’s theory of the
weak interaction; indeed the W and Z bosons have masses of this order.

Now that the gauge sector of the Standard Model is established, we use the reverse procedure.
Since all flavour-changing processes observed up to now occur in processes where the external scales are
at or below the bottom quark mass \( m_b \ll M_W, M_Z, m_t \), we can use an effective field theory approximation
to the Standard Model where the W and Z boson, and the top quark have been integrated out. The result
is five-flavour QCD discussed above and an extended version of Fermi’s four-fermion interactions for the
weak force:
\[
\mathcal{L}_{SM} \rightarrow \mathcal{L}_{QCD,2+QED} + \mathcal{L}_{\text{eff,weak}}. \tag{4}
\]

It is worth noting that the leading term in the effective weak interactions \( \mathcal{L}_{\text{eff,weak}} \) is a dimension 6 interaction, which explains why the weak interactions are weak at low energies \( E \ll M_W \) — the
effective coupling is \( g_s^2 (E/M_W)^2 \) — but stronger than the electromagnetic interactions at the Fermi scale
100 GeV. In the lecture I discussed the structure of \( \mathcal{L}_{\text{eff,weak}} \) in some more detail and explained how
short-distance QCD effects lead to additional operators and an important change of the values of the coefficients of the four-quark operators responsible for quark flavour-changing interactions. See the bibliography below for details. The effective Lagrangian above is itself the starting point for deriving
further effective theories of great use in flavour physics, such as heavy quark effective theory. For practical
purposes there is no loss in precision in using the effective weak interactions for flavour physics rather than the full Standard Model, and it is computationally simpler since the Fermi scale does not appear explicitly. It has been dealt with once and forever (barring New Physics) in deriving the effective Lagrangian.
2 Application: heavy quark effective theory

Heavy quark effective theory (HQET) is an approximation to QCD valid for observables in which a heavy quark participates (so it cannot be integrated out), but the essential dynamics involves momentum scales $\Lambda_{\text{QCD}} \ll m_Q$. This applies to the spectrum of heavy hadrons, and to their decay, provided the light degrees of freedom in the decay have momenta much smaller than $m_Q$.

I have chosen HQET as my example application, since it is a relatively simple case of an effective theory of QCD that illustrates the concepts of perturbative matching, higher-dimensional operators, emergent symmetries and parametrization of hadronic matrix elements. The following topics have been covered in the lecture:

- Derivation of the effective Lagrangian at leading order in the heavy quark expansion in $1/m_Q$ from the full QCD heavy quark propagator and quark–gluon vertex.
- Spin–flavour symmetry of the leading-order effective Lagrangian. Note that these are emergent symmetries at low energies. They are not symmetries of the full QCD Lagrangian and appear only after integrating out the fluctuations at the heavy-quark scale.
- Derivation of the $1/m_Q$ terms in the HQET Lagrangian, which break the spin–flavour symmetry.
- Trace formalism to parametrize hadronic matrix elements and derive spin–symmetry relations between them.
- Heavy meson mass formulae.
- Expression of $B \rightarrow D^{(*)}$ form factors in terms of a single HQET form factor (the Isgur–Wise form factor) and its normalization at $w = v \cdot v' = 1$.
- Determination of $|V_{cb}|$ from the $B \rightarrow D^{(*)} \ell \nu$ decay spectrum at $w = 1$.
- Matching of $b \rightarrow c$ currents at one loop.

Since these are textbook topics, I refer to the bibliography for further details.

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