RADIAL AND PHASE BEAM POSITIONING DEVICE
FOR 70 GeV SYNCHROTRON

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When adjusting the accelerator upon start-up and also under special accelerating conditions, it is necessary to control the radial position of the centre of gravity of the beam and the phase of the beam (in relation to the accelerating voltage). In particular, it is necessary to shift the beam in a radial direction when it is aimed at a target, and also to shift the reference phase in the phase detector of the RF beam control system in proportion to the variation of the rise-time of the magnetic field towards the flat top.

The methods previously used in Russian proton synchrotrons (for instance, in the 7 GeV accelerator) for shifting the radial beam position by adding auxiliary voltage to the output voltage of the radial position detector entering the feedback circuit of the RF beam control system do not give great accuracy because of the variability of the transconductance of the detector, which may be greater than ±20%. The possibility of using desymmetrization of the differential pick-up electrodes by varying their capacitance is also limited by the radiation instability of the controlled semi-conductor capacitances used for this purpose. ...*) in 1952 by Feinberg et al.¹, who formulated equations of motion of oscillating and equilibrium particles:

\[
\frac{dp}{dt} = e \left[ E_m \cos \phi + \frac{\rho M}{\varepsilon_0} (x - x_0) \right],
\]

\[
\frac{dp_s}{dt} = e \left[ E_m \cos \phi_s + \frac{\rho M}{\varepsilon_0} (z - z_0) \right],
\]

and by subtracting them obtained the phase equation. Let us put this equation in relativistic form²,³)

\[
\frac{(1-\beta^2)^{3/2}}{\beta} \frac{d}{dt} \left\{ \frac{1}{(1-\beta^2)^{3/2}} \frac{d}{dt} \beta (y - y_s) \right\} + \Omega_0^2 \left[ \frac{\cos \psi_s - \cos \psi}{\sin \psi_s} - S (y - y_s) \right] = 0. \quad (2)
\]

*) The beginning of this sentence appears to have been omitted from the Russian text.
Here \( \Omega^2 = 2\pi \alpha m_0 (1 - \beta^2)^{3/2} \), \( \beta = v/c \); \( e, m_0, v, p, z, \phi \) are the charge, rest mass, velocity, momentum, longitudinal coordinate and phase of the particle; \( z_0 \) is the coordinate of the centre of the bunch; \( c \) is the speed of light; \( t \) is time; \( E_m \) is the amplitude of the accelerating wave; \( \lambda \) is the wavelength; \( M_z \) is a coefficient depending on the shape of the bunch; \( S = \rho \beta \lambda M_z / 2 \pi \alpha m_0 \sin \phi_s \) is a parameter proportional to the charge density \( \rho \). The derivative of the potential function \( U(\phi) \) is in square brackets. With good accuracy

\[
U'(\phi) = \frac{(\phi - \phi_s)^2}{2 \gamma_s} + (\phi - \phi_s)(1-S), \quad U(\phi) = \frac{(\phi - \phi_s)^3}{6 \gamma_s} + \frac{(\phi - \phi_s)^2}{2}(1-S). \tag{3}
\]

In Ref. 1 a qualitative deduction was also made of the reduction of the phase stability region (bucket) under the action of the bunches' own charge.

In 1960 we determined the dimensions of the bucket and calculated the critical current \(^{2,3}\). The potential function \( U(\phi) \) has a minimum when \( \phi = \phi_s \), a maximum when \( \phi = \phi_s - 2\phi_s (1 - S) \), and a value equal to the maximum when \( \phi = \phi_s (1 - S) \). Therefore the length of the bucket and the longitudinal axis of the bunch are equal to

\[
\Delta \phi = 3 \gamma_s (1-S), \quad 2 \beta = 3 \gamma_s (1-S) \frac{\beta \lambda}{2 \pi} \tau, \tag{4}
\]

diminishing proportionally to \( 1 - S \) and vanishing when \( S = 1 \). Here \( \tau \) is the utilization factor of the bucket (in Refs. 2 and 3, as a precaution \( \tau \) was taken as equal to 0.75). The particle current is

\[
I = \frac{c}{\lambda} \frac{L}{3} \pi \alpha_x \alpha_y \beta \gamma = \frac{2 \pi \tau \alpha_x \alpha_y E_m \gamma_s \sin \gamma_s}{\sqrt{\epsilon_0}} \frac{S(1-S)}{M_z}. \tag{5}
\]

With quadrupole focusing, the perpendicular semi-axes of the bunch \( a_x, a_y \) vary periodically, and \( a_{x,\text{max}} / a_{x,\text{min}} = \kappa > 1 \). However, \( a_x, a_y, b, \rho \) remain constant throughout the period. The current (5) is proportional
to the product of the charge density $\rho \sim S/M_2$ and the length of the bunch $2b \sim 1 - S$. The current vanishes both when $S = 0$ ($\rho = 0$), and when $S = 1$ ($b = 0$), reaching the maximum possible value

$$I_m = \frac{2\pi \tau R^2 E_m y_S \sin y_S}{\sqrt{\mu_0/\varepsilon_0} \kappa \lambda} \left[ \frac{S(1-S)}{M_2} \right]_{\text{max}}$$

(6)

for some intermediate value of $S = S_M$. Here $\sqrt{\mu_0/\varepsilon_0} = 120\pi$ cm, $R = \sqrt{a_x a_y} = a_{x,\text{max}}$ is the radius of the part of the aperture actually used.

Generally $S_M = 0.3-0.4$. The details of the calculation of $S_M$ and $M_2(S_M)$, depending only on the value ..., in Fig. 4, is a follow-up system, comprising an accurate gauge of the phase difference of the signals at the input and output of the device (phase detector and wide-band amplifiers with automatic control), d.c. amplifier and fast phase regulator. The equipment is designed so that the phase of the input signal can be varied between 30 and 150° in the frequency range 2.5-6.5 Hz with an accuracy of ±5°.

The slope of the regulating characteristic in relation to the control voltage is 1.5 rad/V. The maximum time of the phase shift through 1 rad with intermittent variation of the control voltage does not exceed 100 μsec.

In conclusion it should be pointed out that the work of directing the beam on to the target using the radial shift device was carried out in collaboration with a group of colleagues from the High Energy Physics Institute, directed by K.P. Myznikov. The author wishes to express his thanks to A.A. Kuzmin for organizing the work and for valuable discussions.

Note

The List of References has been omitted from the Russian text.

*) The end of one sentence and the beginning of the next have been omitted in the Russian text.
Fig. 1 Block diagram of radial beam shifting device.
Fig. 2 Oscillogram of control signal (upper ray) and radial beam shift (lower ray).
Fig. 3 Oscillogram of intensity (upper ray) and radial beam shift (lower ray).
Fig. 4 Block diagram of electronic phase shifter.

U_{in} = U_m \sin(\omega t)

U_{output} = U_m \sin(\omega t + \phi)

Phase regulator

Wide-band amplifier

d.c. amplifier

Phase detector

Wide-band amplifier

U control