EFFECTS DUE TO THE DISCONTINUOUS REPLACEMENT OF RADIATED ENERGY
IN AN ELECTRON STORAGE RING

by

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ABSTRACT

The discontinuous replacement of radiated energy is the source of
relevant effects for LEP. These are: splitting of both relative
energy and closed orbits for $e^-$ and $e^+$, difficulty of a perfect over-
lapping at the crossing points, opposite optical aberrations and mis-
crossing due to the combined effect of misalignments and aberrations.

1. INTRODUCTION

Electrons and positrons circulating in a storage ring lose energy due to synchrotron
radiation. The relative energy loss per turn $P_0$ is

$$P_0 = 88.5 \times 10^{-6} \frac{E^3}{\rho \text{m}}$$

For the synchronous particle, this energy loss is compensated by acceleration in an RF
system. As a consequence, the relative energy errors $\Delta E/E = p^z(s)$ for electrons and
positrons vary along the circumference of the storage ring in a sawtooth manner approxi-
mately opposite for $e^+e^-$. If we call $n_{RF}$ the number of equispaced RF stations the maximum
absolute value of the functions $p^z(s)$ is $p_0/n_{RF}$. In the early electron-positron storage
rings, the energy loss $P_0$ per turn was small. In their design, the variation of $p^z(s)$ was
usually neglected, i.e. $p^z(s) = 0$ was taken and it was assumed that the central orbit in
the machine $X_0(s) = 0$ was a true closed orbit (C.O.) both for electrons and positrons. In
the meantime $P_0$ has gone up steadily as shown in Table I, and the variation of the energy
error along the orbit $p^z(s)$ may no longer be neglected. It must be stressed already here
that all the more dangerous phenomena to be discussed below do not arise due to the sawtooth

<table>
<thead>
<tr>
<th>Machine</th>
<th>$E(\text{GeV})$</th>
<th>$\rho(\text{m})$</th>
<th>$P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACO</td>
<td>0.54</td>
<td>1.1</td>
<td>$1.26 \times 10^{-5}$</td>
</tr>
<tr>
<td>ADDONE</td>
<td>1.5</td>
<td>5</td>
<td>$6.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>SPEAR</td>
<td>4.2</td>
<td>12.7</td>
<td>$5.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>PETRA</td>
<td>19</td>
<td>192</td>
<td>$3.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>LEP</td>
<td>62.3</td>
<td>3544</td>
<td>$6.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>LEP</td>
<td>130</td>
<td>3544</td>
<td>$5.5 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

shape of $p^z(s)$ but rather from the fact that $p^+(s)$ and $p^-(s)$ and the resulting C.O.'s,
$X_0^+(s)$ and $X_0^-(s)$ of $e^+e^-$ are opposite signs for electrons and positrons (see section 2).
In the following we will make reference mainly to the LEP-8 structure[1]. The number $n_{RF}$
of RF stations can be 4, 8, 16 (see Fig. 1) corresponding to a periodicity $p_{RF}$ of the RF
system 2, 4, 8. Independently from the $p_{RF}$ value the lattice is always assumed to have a
periodicity 8 in the theoretical considerations. All the crossing points (defined in Fig.
1) and the midpoints of each period are assumed to be symmetry points. Actually, the LEP-8
structure does not have precisely the periodicity 8; the computational results take this
fact into account.

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2. **EQUATIONS TO DETERMINE** $p(s)$ **AND** $X_0(s)$

A better understanding of the functions $p(x)$ and $X_0(s)$ can be obtained by writing two differential equations. For $p(s)$ the equation is

$$p'(s) = -\frac{1.409 \times 10^{-5} E_s^2 (\text{GeV})}{E_0 (\text{GeV})} \left( \rho^2(s) - E_s(s)/E_0 (\text{GeV}) \right)$$

where $\rho$ is expressed in metres and the longitudinal field $E_s$ due to RF in GeV/m.

If $s$ is assumed to be the azimuthal coordinate and $p(s)$ to have the nominal value $p$ of the curvature radius, then $p(s)$ can be evaluated without taking into account the C.O. $X_0(s)$. Minor effects can be taken into consideration in computer programs such as PETROS. These include radiation inside quadrupoles and sextupoles, local lengthening or shortening of the orbit and the variation of the curvature with energy. The second differential equation relative to $X_0(s)$ can be obtained from the classical dispersion equation $D''(s) + K(s).D'(s) = 1/\rho(s)$ by replacing the $1$ on the right hand side by $p(s)$. This gives

$$X_0'' + KX_0 = p(s)/\rho(s) - X_0'.E_s(s)/E_0$$

The second term on the right hand side is added to take into account the slope variation due to the azimuthal electric field (damping term). Synchronization with the RF imposes the constraint $\int X_0 ds/\rho(s) = 0$. If equations (2.1) and (2.2) hold for electrons travelling clockwise, the equations for positrons can be obtained by changing the sign on the right hand side of equation (2.1). This can be interpreted that a positron 'coming backwards' captures energy in the bending field and gives energy to the RF cavities. From this consideration on the sign an approximate property of the $p^+(s)$, $p^-(s)$, $X_0^-(s)$ and $X_0^+(s)$ functions may be deduced. In fact, if the above-mentioned minor effects of radiation are neglected then $p^+(s) = -p^-(s)$ and from equation (2.2) $X_0^+(s) = -X_0^-(s)$.

For a real ring, equations (2.1) and (2.2) can only be solved by using a computer. However, it is useful to examine a simplified model.

Assuming uniform focussing and only 1 RF cavity at $s = 0$ the solution of (2.1) and (2.2) is

$$p(s) = (p_0/2).\left(1-s/(nR)\right) \quad X_0^\pm = \frac{p(s).D + \left[p_0 R \sin Q(s/R-\pi)\right]/(2Q^2 \sin^3 Q)}{}$$

The following characteristics of the solution should be noted.

(i) It has the periodicity of the RF system.
(ii) At $s=0$ and $s=nR$, we find $X_0^+ = X_0^- = 0$. 
(iii) The solution is made up of the function \( p(s)D(s) \) plus a betatron oscillation. In general this is not true though \( p_{\text{max}}D_{\text{max}} \) gives a good approximation for \( X_{\text{omax}} \) (see eq. 3.2 of section 3).

From previous considerations and from the simplified model, it follows that an overlapping between the \( e^+ \) and \( e^- \) bunches at a given crossing point (C.P.) is no longer guaranteed. However this can still be achieved by imposing symmetry and periodicity conditions on the machine design. Point (ii) gives such an example. Two methods frequently proposed \( 3, 4, 5, 6 \) will be resumed in the following paragraphs.

3. OVERLAPPING METHODS

Let us consider the 1st layout of Figure 1. It can be seen that the symmetrical arrangement of the 4 RF stations about points 1, 3, 5, 7, which are also symmetry points of the lattice, assure the overlap at these points independently of the local dispersion \( 3,6 \).

It follows from the above that for LEP, 8 RF stations (2nd layout of Fig. 1) would guarantee overlap at 8 C.P.'s. It is however shown in the following that 16 RF stations, (3rd layout of Fig. 1) leading to a smoother \( p(s) \) function, are still necessary to reduce the value of \( X_0 \) and, more fundamentally, to reduce the imperfect overlap resulting from alignment errors.

Passing to the second method we observe that, even if the 4 RF stations guarantee 8 crossings in time of 4+4 bunches, the 1st layout of Fig. 1 does not ensure overlap at all the 8 C.P.'s. For this to happen the dispersion and its derivative must vanish at the C.P.'s. The proof of this will be somewhat different with respect to previous analysis which is based on matrix formalism \( 3,5 \) or on a Fourier analysis \( 6 \).

In the C.O. equation with radiation

\[
X_0'' + K'X_0 = p(s)/c(s) \tag{3.1}
\]

it is a good approximation to assume that the driving function on the right hand side vanishes outside the bending magnets and \( p'(s) \) is periodic with the lattice periodicity.

The function \( p(s) \) can then be separated into two parts:

1. \( p_C(s) \), constant between two successive C.P.'s, and equal to the average of \( p(s) \) over the bending magnets in the period.

2. \( p_D(s) \), periodic, with zero average and given by the difference \( p(s) - p_C(s) \).

The solution of 3.1 can then be written as:

\[
X_0(s) = p_C(s)D(s) + X_D(s) \tag{3.2}
\]

Since \( D(s) \) vanishes at the C.P. and \( X_D(s) \) is periodic and since the optics seen by \( e^+ \) and \( e^- \) rotating in opposite directions are the same, the values of \( X_0^+(s) \) and \( X_0^-(s) \) at the C.P.'s must be the same, giving the required overlap (the slopes are nevertheless opposite).

This conclusion has been checked by means of the PETERS program for the LEP-8 structure at an energy of 62.3 GeV with 4 RF stations. The result is surprisingly good, in spite of the approximations made. For all C.P.'s one has

\[
|\Delta x/\Delta x_{\text{C.P.}}| < 4 \times 10^{-2} \tag{3.3}
\]

while along the regular arc one has

\[
|\Delta x/\Delta x_{\text{max}}| = 6 \tag{3.4}
\]

This result agrees with the above model so well because all the 2nd order dipole kicks (neglected in the model and due to quadrupoles, sextupoles and bending magnets) are locally equal for \( e^+ e^- \) for parity reasons.
4. DISTORTION OF FUNCTION $\mathcal{B}(s)$ AND $\mu(s)$ IN THE PRESENCE OF RADIATION

So far only the influence of radiation on the closed orbit has been examined. We now examine the optics around the two closed orbits $X_0^-(s)$ and $X_0^+(s)$. The main change is the appearance of a gradient distortion function (GDF), opposite for $e^-e^+$ which has 2 components:

(i) GDF at the quadrupoles : $\Delta K_Q = P(s)K_Q(s)$
(ii) GDF at the sextupoles, due to the C.O. : $\Delta K_S = X_0(s)K_S(s)$.

$\Delta K_Q$ and $\Delta K_S$ lead to distortions of $\mathcal{B}(s)$ and $\mu(s)$, $\delta \mathcal{B}(s)$ and $\delta \mu(s)$. The result of PETROS calculations at 62.3 GeV with or without radiation were used to find the maximum optical distortions. They are shown in Table 2.

<table>
<thead>
<tr>
<th>$n_{r-f}$</th>
<th>$\mathcal{B}/\mathcal{B}(X)$</th>
<th>$\omega_x$(rad)</th>
<th>$((\mathcal{B}/\mathcal{B})<em>{P</em>{max}})_X$</th>
<th>$\mathcal{B}/\mathcal{B}(Z)$</th>
<th>$\omega_z$(rad)</th>
<th>$((\mathcal{B}/\mathcal{B})<em>{P</em>{max}})_Z$</th>
<th>$P_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>.067</td>
<td>.102</td>
<td>44.4</td>
<td>.105</td>
<td>.182</td>
<td>69.9</td>
<td>1.5 x 10^{-3}</td>
</tr>
<tr>
<td>8</td>
<td>.040</td>
<td>.0464</td>
<td>52.45</td>
<td>.042</td>
<td>.062</td>
<td>56.0</td>
<td>7.55 x 10^{-4}</td>
</tr>
<tr>
<td>16</td>
<td>.007</td>
<td>.0065</td>
<td>18.04</td>
<td>.016</td>
<td>.013</td>
<td>43.5</td>
<td>3.77 x 10^{-4}</td>
</tr>
</tbody>
</table>

The ratio $(\Delta \mathcal{B}/\mathcal{B})/P_{max}$ lies between 18 and 70. This is about the same ratio as obtained in the absence of radiation losses for the usual chromatic gradient distortion function, where we have $p = const.$ and $X_0(s) = pB(s)$. However, it must be noted that, as in the case of C.O.'s, $\delta \mathcal{B}$ and $\delta \mu$ have now locally opposite values for $e^-e^+$. This, together with alignment errors, is the source of the undesired miscrossing.

5. MISCROSSING DUE TO RADIATION LOSSES AND ALIGNMENT ERRORS

A miscrossing at the C.P.'s must be expected when the combined effect of the opposite aberrations and of alignment errors is considered. In fact, the closed orbit caused by the random transverse kicks $\delta_i$, and including optical aberrations (see section 4), can be written

$$X_0^\pm = \sum_i \sqrt{(\delta_i \pm \delta_i)} \cos(\theta - m_i \pm \delta_i)/(2 \sin Q)$$

By differentiation we obtain for the miscrossing

$$\Delta X = X^+ - X^- = \sum_i \delta_i (\delta_i^+ \pm \delta_i^-) \frac{1}{2} \left(\frac{1}{2} (\partial X/\partial \theta) \cos(\theta - m_i \pm \delta_i) + \delta_i \sin(\theta - m_i \pm \delta_i) / \sin \theta\right)$$

Numerical results for the r.m.s. value of $\Delta X$, $\Delta Z$ can be obtained by PETROS. The assumed structure is LEP-8 with $E = 62.3$ GeV (stage 1/3), $Q_X = 70.3$, $Q_Z = 74.54$ and for the beam dimensions at C.P.'s and for the design coupling $\sigma_X = .260$ mm at $\theta_X = 1.6$ m, .367 mm at $\theta_X = 3.2$ m, $\sigma_Z = .016$ mm at $\theta_Z = .1$ m, .0227 mm at $\theta_Z = .2$ m.

The assumed r.m.s. value for all the position errors is .1 mm. For completeness .24 mm for the tilts and 5.1 x 10^{-4} for the relative strength errors have also been included. The results of Table 3 were obtained by the following procedure:

i) assume a certain set of random distortions
ii) run PETROS for $e^-$ and $e^+$ with radiation and distortion on, calculate $\Delta X$, $\Delta Z$ at different C.P.'s,
iii) repeat i) and ii) ten times,
iv) compute the r.m.s. value of $\Delta X/\sigma_X$, $\Delta Z/\sigma_Z$ over the optically equivalent C.P.'s (o.e.e.c.p.) and the 10 machines.
Table 3

<table>
<thead>
<tr>
<th>n_{rf}</th>
<th>O.E.C.P.</th>
<th>(ΔX/σ_X)</th>
<th>(ΔZ/σ_Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1, 5</td>
<td>.47</td>
<td>2.93</td>
</tr>
<tr>
<td></td>
<td>2, 4, 6, 8</td>
<td>.24</td>
<td>.866</td>
</tr>
<tr>
<td></td>
<td>3, 7</td>
<td>.095</td>
<td>.788</td>
</tr>
<tr>
<td></td>
<td>1, 2,...,7, 8</td>
<td>.3</td>
<td>1.75</td>
</tr>
<tr>
<td>8</td>
<td>1, 3, 5, 7</td>
<td>.265</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>2, 4, 6, 8</td>
<td>.162</td>
<td>.796</td>
</tr>
<tr>
<td></td>
<td>1, 2,...,7, 8</td>
<td>.219</td>
<td>.975</td>
</tr>
<tr>
<td>16</td>
<td>1, 3, 5, 7</td>
<td>.041</td>
<td>.257</td>
</tr>
<tr>
<td></td>
<td>2, 4, 6, 8</td>
<td>.022</td>
<td>.234</td>
</tr>
<tr>
<td></td>
<td>1, 2,...,7, 8</td>
<td>.033</td>
<td>.246</td>
</tr>
</tbody>
</table>

From Table 3 it may be seen that:

i) the effects are not negligible,

ii) the largest mis crossings appear for C.P.'s near the RF stations where the slope of p(δ)
    function is the biggest; the mis crossing decreases by increasing n_{rf} as expected.

The worst value of ΔZ/σ_Z for n_{rf} = 4 is 2.83 with uncorrected orbits due to error.

Corrected orbits were computed for LEP-8 by two methods: the program "ALIGN" which
improves the orbit by a factor 16 (Table 4.4 of ref. 1) and the "beam bump method of
PETROS" (Table 4.2 of ref. 1) which improves the orbit by a factor 2.5. Making the
hypothesis that the mis crossing decreases like the r.m.s. value of the C.O., one gets .18
by the first method and .13 by the second one instead of 2.83. However, the hypothesis of
a mis crossing decreasing like the r.m.s. C.O. is over optimistic; if the correctors are not
very numerous and not uniformly distributed we must expect a less efficient correction of
the mis crossing. Preliminary results on this point in a single case confirms this. The
corrected ratio ΔX/(σ_Xσ_0) and ΔZ/(σ_Zσ_0) are respectively 1.4 and 2.2 times the uncorrected
values, even if the corrected ΔX and ΔZ are reduced, and the worst case 2.83 can only be
reduced to .4 instead of .18. Better statistics are still needed but it already seems clear
that the LEP-8 structure cannot work with only 4 RF stations at 62.3 GeV. Even
with 16 RF stations we obtain for the worst ΔZ/σ_Z, .07 at 86.11 GeV and .16 at 130 GeV with
constant coupling (the effect goes like E^2), it can be up to 2.5 times worse if the
coupling is reduced to the minimum 13. The conclusion is that a specific correction is
needed for the mis crossing effect.

6. THEORETICAL CONSIDERATIONS

According to (5.2) the mis crossing depends also on ΔB/B and Δμ apart from Δp.

ΔB/B is defined by the equation:

\[ \frac{\Delta B}{B} = 4 \beta^2 \frac{\Delta \phi}{\beta} + 2 \sigma_B \frac{\Delta B}{B} \int \frac{\Delta k}{\delta} \, d\phi + \text{const.} \]  \hspace{1cm} (6.1)

Another equation for Δμ can be easily obtained:

\[ \Delta \mu = -Q \int \frac{\Delta k}{\delta} \, d\phi + \text{const.} \]  \hspace{1cm} (6.2)

where \( \phi \) is the unperturbed normalized phase advance and \( \Delta k \) is the G.D.F. defined in
Section 4. Differences (ii) among different optical C.P.'s are related to the constant in
6.2 which must be determined to cancel Δμ at any chosen C.P. considered in eq. 5.2. As far
as the influence of the Q values on the mis crossing is concerned, we can observe:

The single kicks of eq. 5.1 are resonant at the integer values of Q. In addition, the
G.D.F. (see Section 4), which appears like a driving function on the right hand side of
eq. 6.1 and 6.2, has the same periodicity p_{rf} of the R.F. system, while ΔB/B and Δμ are
oscillating at frequency \( Q \). We therefore have to satisfy for both radial and vertical plane the 2 conditions:

\[
Q \neq m \quad (6.3) \\
2Q \neq n\,p_{rf} \quad (6.4)
\]

The best way to satisfy (6.3) and (6.4) is to set \( Q \) between two successive resonances, namely:

\[
Q = \frac{m}{1/2} \quad (6.5) \\
2Q = n\,p_{rf} \pm p_{rf}/2 \quad (6.6)
\]

Eq. (6.5) is well satisfied by both 70.304 and 74.541. With regard to eq. (6.6) we have

\[
2Q_x = 140.608^{+2}.70+608^{+4}.35+608^{+8}.18-3.39=8.17+4.61 \\
2Q_z = 149.08^{+2}.74+1.08^{+4}.37+1.08^{+8}.19-2.92=8.18+5.08
\]

Eq. (6.6) is well satisfied for periodicity "2" and "8" but not for "4". Thus in Table 2 the case of 8 RF stations is not much better than 4. At maximum periodicity 8 we cannot expect, as far as the miscrossing is concerned, better values than those of Table 3 by changing \( Q_x, Q_z \). We must find other ways of correction. The optical aberrations \( \Delta Q, \Delta y \) are obviously linked to the Fourier coefficients \(^7\) of the right hand side of 6.1. The sextupole arrangement can be changed to reduce these coefficients; this will be attempted but may conflict with the standard chromaticity corrections. The RF arrangement can also be modified to do that, for instance 8 RF stations at mid period points (see if of section 5). This possibility (which has the advantage of no interference with normal chromaticity correction) demands supplementary dispersion suppressors; it is very expensive and it is very different from the actual structure. Finally, we can imagine a specific measurement and correction of the miscrossing effect but it is still to be demonstrated that this is a real possibility.

ACKNOWLEDGEMENTS

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REFERENCES

5. M. Bassetti, Is it possible to cross the beams in all symmetry points with one RF only in presence of radiation losses? Idem.