SCALING OF LUMINOSITY DATA BETWEEN $e^+e^-$ STORAGE RINGS

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Abstract

A scaling law for luminosity between different $e^+e^-$ storage rings is derived. It is used to estimate the luminosity in LEP. It is found that the luminosity in LEP should be about equal to the luminosity actually achieved in PETRA.

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1. Introduction

In the design of future $e^+e^-$ storage rings such as LEP the question arises on how to use the experimental data available from existing machines in a rational fashion to make predictions about their performance. This question has been addressed in the past\textsuperscript{1,2}). Here we formulate scaling rules which should permit valid comparisons between the luminosities of different machines or extrapolation from existing machines to machines under design. The scaling rules specify parameter values for the new machine which should be met in order to assure successful operation with the appropriately scaled luminosity.

In Section 2 these rules are expressed in the strict form which would be appropriate for controlled experimental comparison between different machines. Fortunately controlled experimentation in this area is notoriously difficult and rare. But, to a large extent, the parameters of the various machines tend to be related roughly in accordance with the scaling rules given here. That is because single-beam considerations (which have dominated existing designs) lead to conclusions similar to beam-beam considerations (which are the subject of this note).

In Section 3 we make the not-strictly-valid assumption that existing machines satisfy the scaling relations. Making allowances for the crudeness of this assumption there is reasonable agreement amongst SPEAR, CESR and PETRA. Buoyed by this we make projections for LEP which appear, to us, to be fairly reliable.

The main way in which this discussion differs from the ideas on which existing machines have been based is that we permit the maximum tune shift parameters $\xi_x$ and $\xi_y$ to depend on the betatron damping rate. Actually, to assume the contrary as has been beam-beam orthodoxy flies in the face of reason.

In what follows, the asymmetry between horizontal, $x$, and vertical, $y$, is the result of our assumption that the beams are ribbon-shaped, much wider than they are high. This is valid for the machines to be discussed.
2. **Scaling procedure**

The present scaling procedure is designed in such a way that the physics of single and colliding beams scales between the two machines considered. The rules are chosen to be such that the same scaled results are guaranteed to be obtained if both machines are simulated on a computer as has been done recently with considerable success

Specifically, the assumptions are the following:

i) The ratio between the dynamic acceptances \( A_x \) and \( A_y \) and the beam emittances \( e_x \) and \( e_y \) in the horizontal and vertical directions are the same in the two machines. In an existing machine the dynamic acceptance can be obtained by scraper measurements; for the non-existent machine either the physical acceptances, given by the vacuum chamber dimensions, or the stability limits obtained by particle tracking must be used, whichever are the smaller. The beam emittances are taken while the beams are in collision, i.e. they include any beam blow-up which might occur. This rule fixes the beam emittances \( e_{x2} \) and \( e_{y2} \) in the future machine.

\[
e_{x2} = e_{x1}(A_{x2}/A_{x1}) ; \quad e_{y2} = e_{y1}(A_{y2}/A_{y1})
\]

ii) The beam energy \( E_2 \) in the future machine is fixed in such a way that the number of beam-beam collisions in a damping time \( \tau \) is the same in the two machines:

\[
k_2\tau_2 f_2 = k_1\tau_1 f_1
\]

Here \( k \) is the number of bunches in one beam and \( f \) is the revolution frequency. Another way of expressing this condition is that the fractional damping decrement between crossings, \( \delta \), is the same in the two machines. That is because

\[
\delta = 1 - \exp \left( -\frac{1}{k\tau f} \right) = \frac{1}{k\tau f}
\]

The variable \( \delta \) is useful in making comparisons between different machines.
iii) The number $N_2$ of particles in a bunch is chosen to be such that the linear beam-beam tune shifts $\xi_x$ and $\xi_y$ are the same in the two machines. The number $N_2$ follows from the standard formula, using $E_2$ and $e_{x2}$ and $e_{y2}$ as obtained earlier:

$$
N_2 = N_1 \cdot \frac{E_2}{E_1} \min \left[ \frac{e_{x2}}{e_{x1}}, \frac{e_{y2}}{e_{y1}}, \frac{\beta_{x2}}{\beta_{x1}}, \frac{\beta_{y2}}{\beta_{y1}} \right]^{\frac{1}{2}} \left( \frac{\alpha_{x2} \alpha_{y2}}{\alpha_{x1} \alpha_{y1}} \right)^{\frac{1}{2}} \min \left( \frac{\beta_{x2} \beta_{y1}}{\beta_{x1} \beta_{y2}} ; 1 \right)
$$

(4)

Here, the $\beta$'s are taken at the crossing point. To be on the safe side, we use the smaller current from the horizontal and vertical beam-beam limit. The two numbers will be the same when the horizontal and vertical acceptances have the same ratio in the two machines.

At this point all ingredients for luminosity scaling are available. The result is:

$$
L_2 = L_1 \left( \frac{E_2}{E_1} \right)^2 \frac{e_{x2} e_{y2}}{e_{x1} e_{y1}} \left( \frac{\beta_{x2} \beta_{y1}}{\beta_{x1} \beta_{y2}} \right)^{\frac{1}{2}} \min \left( \frac{\beta_{x2} \beta_{y1}}{\beta_{x1} \beta_{y2}} ; 1 \right)
$$

(5)

This equation includes all transverse beam-beam phenomena, but not yet longitudinal phenomena, coupling between transverse and longitudinal effect and errors. Hence the enumeration is continued:

iv) The betatron amplitude functions at the crossing points and the fractional parts of the phases between the crossing points are the same.

v) The unavoidable errors in the machine lattice which may be partially responsible for the drop in the maximum beam-beam tune shift in going from ADONE, SPEAR and VEPP-2M to CESR, PEP and PETRA have an equivalent effect. This statement is easy to quantify for any particular error included in a simulation. Since at best only some statistical properties of the errors in existing machines are known, this rule must be applied in a statistical sense to a future machine. If it were possible to measure all errors accurately enough it would presumably also be possible to correct them.
vi) The longitudinal parameters bunch length, energy spread and synchrotron tune must be identical in the two machines.

The rules given are formulated here in their strictest form. It is likely that some can be relaxed. Furthermore, if it can be shown that varying a parameter in one machine does not change the luminosity then this also holds in the other machine. It is easy to convince oneself that in suitably scaled units where lengths are measured in beam sizes, kicks in beam divergences etc. the beam-beam collisions and the transformations through the machine arcs can be described by the same equations in the two machines, when the above six rules are applied.

3. Application to Existing Machines and Extrapolation to LEP

The main result we wish to use is formula (5). For small, first-generation, storage rings the various factors vary somewhat erratically. But for SPEAR and later larger machines, cost optimization has tended to force a conformity on the designs which facilitates comparisons. We confine our attention to those machines except to comment that the scaling rules described here tend to be progressively harder to achieve as the energy is increased and that may account partially for the higher tune shifts obtained with early machines such as ADONE and VEPP-2M.

Steffen has given formulae for the scaling with energy of "cost-optimized" $e^+e^-$ storage rings. While existing machines do not precisely satisfy his formulae the trends with energy of the important quantities from linear lattice theory and the single beam distribution functions are roughly followed. We will use his formulae for initial discussion. More quantitative results can be obtained using the data of Table I.

Machine radii scale roughly as the squares of the nominal energies. This causes the factors $(E_2/E_1)^2$ and $f_2/f_1$ in (5) to tend to cancel each other. The single beam emittances $e_x$ and $e_y$ at the nominal energy are approximately constant. It has been natural to design the apertures $A_x$ and $A_y$ with reference to $e_x$ and $e_y$ this makes $A_x$ and $A_y$ about the same for different machines. If in (5) we also assume identical intersection region optics (so that the $\beta$-factors cancel) we get the simple relation
\[ L_2 = \frac{k_2}{k_1} \]

The approximations leading to (6) have been crude enough that they should not be used for quantitative purposes. But what it is intended to show is that there are no important, inexorable, "kinematic" factors either helping or hurting at high energy. As an aside we can observe that (6) seems to favour making \( k_2 \) large (assuming single beam considerations do not prevent that). But it must be remembered that changing \( k_2 \) changes the reference energy according to (2). This reduces but does not eliminate the advantage of making \( k_2 \) large.

We now proceed more quantitatively, for actual application to existing machines. Consider an aperture factor such as \( A_{x2} \). To the extent the scaling relationships are valid \( A_{x2} \) influences the luminosity multiplicatively as in (5). But suppose that \( A_{x2} \) is larger than is required by scaling. This would lead to a more subtle change in luminosity as \( \xi_x \) and/or \( \xi_y \) could presumably be made slightly higher in machine 2 than in machine 1. Such an effect can be studied (with difficulty) in a numerical simulation but is specifically beyond the power of our scaling arguments. On the other hand the dependence of \( \xi_x \) or \( \xi_y \) on \( A_{x2} \) is presumably rather weak. Hence we expect the relations between \( \xi_{x2} \) and \( \xi_{x1} \) and between \( \xi_{y2} \) and \( \xi_{y1} \) to be approximately the scaling relationship (namely equality). To summarize: comparison of tune shifts \( \xi_x \) and \( \xi_y \) is less sensitive to uncertainties about apertures than would be comparisons of luminosity. This observation is certainly not original. It is the main virtue of the tune shift parameters.

Data on \( \xi_y \) are shown in Fig. 1 for the various machines under consideration. When plotted against the damping decrement \( \delta \), data from all machines should fall on a universal curve (assuming, as we now are, that the scaling relationships are satisfied). Since the best available data is from SPEAR we use it as the reference (though for extrapolating to LEP it will be more reliable ultimately to refer to PETRA). Fig. 1, taken from Wiedemann\(^9\), originally contained just the SPEAR points. The scale of decrements \( \delta \) has been added. Rather than referring to this scale it is convenient to make an energy scale appropriate to each
accelerator. This has been done for PETRA and LEP using the energy ratios \(E_2/E_1\) worked out with (2) from the damping times. All these items are listed in Table I.

Fig. 1 shows that the limit for SPEAR is systematically higher than for the later machines, (though \(\xi_y\) is rarely as high as the infamous 0.06 which for years was taken as the standard of excellence because it was said to be achievable at SPEAR). We blame the disagreement not on the scaling formulae but on the rather loose assumptions which have been made in this Section 3. As explained in footnote 8) there is an economic temptation to skimp on aperture and that may account for the somewhat poorer performance in PETRA and CESR than in SPEAR. For example, referring to Table I, it can be seen that the horizontal aperture scaling requirement is not respected in going from SPEAR to CESR to PETRA to LEP. On the other hand the vertical aperture scaling is roughly followed.

We regard the agreement in Fig. 1 as good enough to justify extrapolation to LEP and that is the purpose of the smooth curve. This amounts to nothing more than the assumption that at corresponding energies the performance of LEP and PETRA will be equal. This is somewhat optimistic for LEP as, referring again to Table I, \(A_x\) appears to be somewhat lower for LEP than for PETRA. Of course, one should use the magnetic aperture, but that may also be worse for LEP relative to PETRA. At this level more quantitative work remains to be done.

Of greater importance than \(\xi_y\) is the luminosity and that is shown in Fig. 2. The luminosities are taken from lab reports\(^{10}\),\(^{11}\). There are enough uncertain factors in formula (5) that we will not attempt to check it. The certain factor \(k\epsilon^2\) is surprisingly important as can be seen from the last entry of Table I. This presumably accounts for the PETRA luminosity being higher than SPEAR even though the tune shift parameter is less. For extrapolation to LEP this factor is about the same as for PETRA. As a result we will assume the LEP luminosity is equal to the PETRA luminosity at the corresponding energy. This is a bit optimistic since, as mentioned previously, the acceptance factor for LEP may be somewhat inferior to that of PETRA. With that reservation and assuming the same intersection region optics at LEP and PETRA, the smooth curve in Fig. 2 is an estimate of the LEP luminosity. The very good luminosity at PETRA at low energy (1.8\(\times\)10\(^{30}\) cm\(^{-2}\)s\(^{-1}\) at 7 GeV) is
apparently due partially to artificial increase of the horizontal emittance\textsuperscript{10}. If the scaling arguments presented in this paper are valid the same strategy should work at low energy at LEP. Hence one expects $1.8 \times 10^3$ cm$^{-2}$ s$^{-1}$ at about 18 GeV.

4. References and notes


6. It is not really necessary to specify whether the horizontal emittance $\varepsilon_x$ is for a single beam or with the beams in collision as experimentally these differ only negligibly. On the other hand the vertical emittance $\varepsilon_y$ is known to be greatly affected by the beam-beam interaction. But if the scaling relations described in this paper are satisfied then the blow-up is by the same factor in the two machines, and again we need not specify whether $\varepsilon_y$ is the single-beam or double-beam quantity.

Unfortunately this is a bit too glib as one controversial feature remains. It is possible, at least in principle, to make the single beam value of $\varepsilon_y$ almost negligibly small. In that case the beam-beam interaction blows up $\varepsilon_y$ by a big factor which would even be infinite if $\varepsilon_y$ were mathematically taken as zero. For present purposes we could exclude this case but instead we regard $\varepsilon_y$ as including any beam-beam blow-up which occurs.

This discussion has not been sheer pedantry as there is some evidence that the luminosity is maximized by minimizing the single beam vertical emittance. But the maximum is probably not very extreme and in any case the scaling relations given here are still valid.


8. As the energy increases $\beta$ increases and this forces the chamber dimensions to increase even though $A_x$ and $A_y$ are constant. Presumably for economic reasons, this has led, in existing machines to a systematic reduction in $A_x$ and $A_y$ at high energy. To illustrate this, vacuum chamber dimensions are listed in Table I. These should be taken only as suggestive as the true apertures may well be magnetic or set elsewhere.


11. CESR. J. Seeman, CBN 81-9, 26th February 1981.

**TABLE I**

<table>
<thead>
<tr>
<th></th>
<th>Symbol</th>
<th>Units</th>
<th>SPEAR II</th>
<th>CESR</th>
<th>PETRA</th>
<th>LEP</th>
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<tr>
<td>Circumference</td>
<td>$2\pi R$</td>
<td>m</td>
<td>234</td>
<td>768</td>
<td>$2.30 \times 10^3$</td>
<td>$2.66 \times 10^4$</td>
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<td>Tune</td>
<td>$Q$</td>
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<td>5</td>
<td>9.2</td>
<td>22</td>
<td>92</td>
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<td>Revolution frequency</td>
<td>$f$</td>
<td>Hz</td>
<td>$1.28 \times 10^6$</td>
<td>$0.39 \times 10^6$</td>
<td>$1.30 \times 10^5$</td>
<td>$1.13 \times 10^4$</td>
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<tr>
<td>Number of bunches</td>
<td>$k$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
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<tr>
<td>Damping time</td>
<td>$\tau(E_T)$</td>
<td>s</td>
<td>$4.4 \times 10^{-3}$</td>
<td>$2.52 \times 10^{-3}$</td>
<td>$1.00 \times 10^{-2}$</td>
<td>$1.015 \times 10^{-2}$</td>
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<td>$E_T$</td>
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<td>3.71</td>
<td>5.5</td>
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<td>50</td>
</tr>
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<td>$\bar{B}$</td>
<td>$R/Q$</td>
<td>m</td>
<td>7.5</td>
<td>13.3</td>
<td>16.7</td>
<td>46.0</td>
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<td>Vacuum chamber width</td>
<td>$W$</td>
<td>m</td>
<td>0.12</td>
<td>0.09</td>
<td>0.114</td>
<td>0.126</td>
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<td>$A_x^1$</td>
<td>$W/\sqrt{\bar{B}}$</td>
<td>m$^{-1}$</td>
<td>0.0438</td>
<td>0.0246</td>
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<td>Vacuum chamber height</td>
<td>$H$</td>
<td>m</td>
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<td>$A_y^1$</td>
<td>$H/\sqrt{\bar{B}}$</td>
<td>m$^{-1}$</td>
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<td>0.0106</td>
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<tr>
<td>Luminosity ratio</td>
<td>$k_2f_2E_2/k_1f_1E_1^2$</td>
<td></td>
<td>1.0</td>
<td>0.96</td>
<td>7.84</td>
<td>8.80</td>
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</table>

* These entries are for illustration only. The true acceptances should be used but they are hard to obtain.
Fig. 1. Dependence of maximum vertical tune shift \( \xi_y \) on damping decrement \( \delta \) for various machines. Energy scales specific to the different machines are given to bypass calculating \( \delta \). The smooth curve gives the projected LEP behaviour.
Fig. 2. Dependence of maximum luminosity of damping decrement $\delta$. The smooth curve is the projected LEP behaviour.