HOM issues in 704.4 MHz and 1.3 GHz superconducting cavities

M. Schuh
Beams Department

Keywords: SPL, ESS, superconducting cavities, HOM, beam dynamics, ILC

Summary
The basic Higher Order Modes (HOM) properties of different elliptical superconducting cavities operating at 704.4 MHz and 1.3 GHz are compared for the operation in a high power proton linac.

1 Introduction

In the early design stages of high-power proton linacs, the question is often raised if one can use the ILC [1] RF frequency of 1.3 GHz for operation, as this may enable the use of already existing ILC hardware. This question was studied in detail for the Superconducting Proton Linac (SPL) at CERN in [2] with the conclusion that lower frequencies, such as 704 MHz for the SPL, are preferred for the operation of $\beta < 1$ linacs. When it comes to the impact of Higher Order Modes (HOMs) on beam stability of linacs at both frequencies, a generic analytical scaling was developed in [2] with the conclusion, that the influence of HOMs at lower frequencies is less severe than at higher frequencies (and at higher cell numbers).

In this note beam dynamics simulations with the code Simulation of higher order Mode Dynamics (SMD) are presented to make a numeric comparison for the HOM impact on low and high-frequency proton linacs [3–5]. As an example we chose the current design of the European Spallation Source (ESS) linac [6–8] and an artificially created 1.3 GHz linac with the same operational parameters in terms of current, pulse structure, beam energy, and beam power. The results of the comparison are perfectly representative for the SPL design and were therefore not repeated with the specific SPL layout.

A set of two cavities was designed for both frequencies [9] along with a non-optimised basic linac layout for each frequency [10]. SMD uses field maps of the HOMs in these cavities, and performs bunch tracking through cavities with HOMs.

First the two investigated linac layout options and the used cavity designs are introduced. Then different beam dynamics scenarios are studied, where the effect of monopole modes is investigated in detail. Dipole modes are not discussed in this survey.

This is an internal CERN publication and does not necessarily reflect the views of the CERN management.
2 Linac layout options

For this survey two linac layout options, illustrated in Figure 1, are used [8, 10]. In both cases a normal conducting front end is used up to 50 MeV followed by a section of superconducting (SC) spoke cavities. Afterwards two families of SC elliptical cavities accelerate the beam up to 2.5 GeV. All relevant layout parameters are summarised in Table 1. The main difference of the two options is the operation frequency in the elliptical cavities and the number of cells per cavity, which are the focus of this study. HOM effects are only studied in the SC elliptical cavity sections between 380 MeV and 2.5 GeV.

![Figure 1: Schematic layout of the linac options used in this study. The 1.3 GHz option is about 70 m longer, due to the smaller energy acceptance range of nine cell cavities. This is compensated by a longer SC Spoke cavity section.](image)

![Table 1: Section parameters of the studied linac options.](image)

<table>
<thead>
<tr>
<th>Option</th>
<th>Section</th>
<th>$\beta_g$</th>
<th>cells</th>
<th>$f$ [MHz]</th>
<th>$E_{in}$ [MeV]</th>
<th>length [m]</th>
<th>cavities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spoke</td>
<td>0.63</td>
<td>5</td>
<td>352.2</td>
<td>50</td>
<td>57</td>
<td>42</td>
</tr>
<tr>
<td>704 MHz</td>
<td>Medium $\beta$</td>
<td>0.63</td>
<td>5</td>
<td>704.4</td>
<td>225</td>
<td>52</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>High $\beta$</td>
<td>0.74</td>
<td>5</td>
<td>704.4</td>
<td>500</td>
<td>237</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>Spoke</td>
<td>0.74</td>
<td>9</td>
<td>325</td>
<td>50</td>
<td>117</td>
<td>81</td>
</tr>
<tr>
<td>1.3 GHz</td>
<td>Medium $\beta$</td>
<td>0.74</td>
<td>9</td>
<td>1300</td>
<td>380</td>
<td>59</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>High $\beta$</td>
<td>0.84</td>
<td>9</td>
<td>1300</td>
<td>660</td>
<td>240</td>
<td>160</td>
</tr>
</tbody>
</table>

In Figure 2 the effective accelerating voltage\(^1\) $V_{acc}$ and synchronous phase $\phi_s$ in all superconducting structures are shown. The power per cavity is limited to 0.9 MW, which leads to a maximum effective accelerating voltage of about 12 MV at a beam current of 75 mA. $\phi_s$ is

\[^1\] $V_{acc} = E_0 T(\beta) t_{active} \cos(\phi_s)$
reduced for longitudinal beam matching after the SC Spoke cavity section due to the frequency jump. In the 1.3 GHz option this is very important because the natural bucket size decreases by a factor of four in phase due to the four time higher operation frequency.

(a) Effective accelerating voltage

(b) Synchronous phase

![Graphs showing effective accelerating voltage and synchronous phase along the linac with a power limitation of 0.9 MW per cavity for both options. The phase is reduced at the frequency jump to longitudinally match the beam.]

**Figure 2:** Effective accelerating voltage (a) and synchronous phase (b) along the linac with a power limitation of 0.9 MW per cavity for both options. The phase is reduced at the frequency jump to longitudinally match the beam.

### 3 Elliptical cavities

For this survey four elliptical cavities were designed by R. Calaga [11, 9] and analysed with SUPERFISH [12]. Some of the cavity parameters are listed in Table 2 and the monopole spectrum is shown in Figure 3. In all cavities there are 3 bands of interest each containing 5 or 9 modes ($TM_{010}$, $TM_{011}$, $TM_{020}$), respectively, below the beam pipe cutoff frequency, which is a function of the beam pipe radius. All HOMs are far away from machine lines and resonant excitation should not be an issue as long as no pulse substructure (e.g. beam chopping) is introduced. In case of the 1.3 GHz cavities, there are more modes with high ($> 1$) $(R/Q)(\beta)$ values and the values itself are, as expected from the frequency scaling, about a factor two higher than in the 704 MHz cavities.

<table>
<thead>
<tr>
<th></th>
<th>704.4 MHz</th>
<th>1.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_g$</td>
<td>0.63</td>
<td>0.74</td>
</tr>
<tr>
<td>Cells</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$L_{(active)}$ [m]</td>
<td>0.99 (0.67)</td>
<td>1.11 (0.79)</td>
</tr>
<tr>
<td>$R_{iris}$ [cm]</td>
<td>5.5</td>
<td>6.2</td>
</tr>
<tr>
<td>$(R/Q)(\beta_g)$ [Ω]^†</td>
<td>238</td>
<td>307</td>
</tr>
<tr>
<td>$E_0T(\beta_g)$ [MV/m]</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>$f_{cutoff}$ [GHz]</td>
<td>2.09</td>
<td>1.85</td>
</tr>
</tbody>
</table>

^† linac definition

Machine lines are integer multiples of the bunch frequency.
Figure 3: Monopole spectrum and maximum \((R/Q)(\beta)\) value in the used energy range of all four cavities. The beam pipe cutoff frequencies are marked with dashed lines and are the same for both 1.3 GHz cavities, due to the same beam pipe radius.

Increasing the frequency leads to a decrease of the cavity cell length. In order to keep the ratio between the effective acceleration length and the total length small, the number of cells have to be increased. In general \((R/Q)/L\) scales linear with the frequency. So a higher frequency and number of cells lead to a higher acceleration efficiency at the cavity geometrical beta \(\beta_g\). The drawback is, that the transit time factor \(T(\beta)\) drops faster for a higher number of cells, if the particle \(\beta\) differs from the geometrical \(\beta\) and so also \((R/Q)(\beta)\) drops. This significantly reduces the energy range, where the cavity efficiently accelerates the beam. This is illustrated in Figure 4, where \((R/Q)(\beta)\) dependency of the \(\beta_g = 0.75\) cavities is shown. The used velocity range, indicated by the vertical dashed lines, is smaller in the 1.3 GHz cavity. Also the number of HOMs and their \((R/Q)(\beta)\) values increases with the number of cells, which is clearly shown in Figure 3, where \((R/Q)(\beta)_{\text{max}}\) values of the HOMs in the 1.3 GHz cavities are at least a factor two higher than in the 704.4 MHz cavities. The \((R/Q)(\beta)\) of TM\(_{010}, 4/5\pi\) mode in the 704.4 MHz cavity and the TM\(_{010}, 8/9\pi\) mode in the 1.3 GHz cavity have a pole at the \(\beta_g\) and rise fast for higher and lower \(\beta\) values and even exceed the \((R/Q)(\beta)\) values of the accelerating mode for certain \(\beta\) values. In the nine cell cavity the \((R/Q)(\beta)\) of the TM\(_{010}, 8/9\pi\) mode rises faster than the TM\(_{010}, 4/5\pi\) mode in the five cell cavity. In general at \(\beta_g\) all non TM\(_{010}, \pi\) modes have a very low \((R/Q)(\beta)\) compared to the accelerating mode, hence these modes are less critical in electron machines, where the electrons always travels with \(\beta_g\). Further frequency scaling laws, also in the context of HOMs are discussed in [2].

Looking at the \((R/Q)(\beta)\)-map of the modes with the highest \((R/Q)\) values in all cavities within their energy range (see Figure 5 and Figure 6) one observes, that in both high \(\beta\) cavities the fundamental passband mode next to the accelerating mode has a higher \((R/Q)\) value than the accelerating mode towards the end of the linac. Hence, it has a high potential to drive instabilities and beam losses. A special focus is set to this mode in the beam dynamics study. All other modes have at least a factor two smaller \((R/Q)(\beta)\) values.
(a) \((R/Q)(\beta)\)-map 704.4 MHz \(\beta_g=0.74\) cavity

(b) \((R/Q)(\beta)\)-map 1.3 GHz \(\beta_g=0.74\) cavity

Figure 4: \((R/Q)(\beta)\)-maps of the first 3 monopole bands in both \(\beta_g = 0.74\) cavities. The solid curves are the TM\(_{010}\) modes. The dashed and dotted curves are the TM\(_{011}\) and TM\(_{020}\) modes. The vertical dashed lines mark the beta range where the cavity is used. There are more modes in the 9 cell 1.3 GHz cavity and the maximum \((R/Q)(\beta)\) of the TM\(_{010}\), \(\pi\) mode (purple) is higher, but drops faster beside the \(\beta_g\) as in the 5 cell 704.4 MHz cavity (violet) with the same \(\beta_g\). Hence, the beta range in which the 704.4 MHz cavity can be used efficiently is much larger.
4 Beam dynamics simulations

In order to directly compare both linac options all simulations start at 380 MeV and go up to 2.5 GeV. All relevant beam parameters are listed in Table 3. The injection noise is extrapolated from Linac 4 [13, 14]. The two times higher injection phase jitter in the 1.3 GHz option is caused by the higher operation frequency and the 2 times higher frequency jump. It is assumed, that the front-end delivers the same beam quality in both options.

4.1 Reference simulations without HOMs

Before looking at the impact of HOMs, simulations without HOMs are carried out in order to check the beam noise parameters at injection. The resulting phase space distributions
Table 3: Simulation parameters used in the beam dynamics simulations. Errors are estimated from Linac 4 simulation data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>704.4 MHz Mean</th>
<th>704.4 MHz Variance</th>
<th>1.3 GHz Mean</th>
<th>1.3 GHz Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch frequency</td>
<td>[MHz]</td>
<td>352.2</td>
<td>325</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulse length</td>
<td>[ms]</td>
<td>∼ 2†</td>
<td></td>
<td>∼ 2†</td>
<td></td>
</tr>
<tr>
<td>Period length</td>
<td>[ms]</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam current</td>
<td>[mA]</td>
<td>50...400</td>
<td>1%</td>
<td>50...400</td>
<td>1%</td>
</tr>
<tr>
<td>Injection energy</td>
<td>[MeV]</td>
<td>380</td>
<td>0.2</td>
<td>380</td>
<td>0.2</td>
</tr>
<tr>
<td>Injection phase</td>
<td>[deg]</td>
<td>-16</td>
<td>0.2</td>
<td>-60</td>
<td>0.4</td>
</tr>
</tbody>
</table>

† 700,000 bunches
‡ at the operation frequency

of the nominal injection beam jitter, which will be used for comparison in the following HOM simulations, is shown in Figure 7, where the phase error in the 1.3 GHz linac is normalised to the unit of the 704 MHz linac. No filamentation is observed at the end of both linacs. The phase spread is less in the 1.3 GHz linac due to the higher operation frequency.

(a) 704 MHz linac

(b) 1.3 GHz linac

Figure 7: Phase space distribution of one pulse at the end of both linacs with the nominal injection energy and phase jitter. In the 1.3 GHz linac the phase error is normalised to 704 MHz for comparison reasons. The phase error is smaller in the 1.3 GHz linac due to the higher operation frequency.

Increasing the injection energy and phase jitter by a factor two leads to a phase space increase as shown in Figure 8, where a starting filamentation is observed in both linac options. The resulting output energy and phase errors are as expected about twice the values of the simulation with nominal injection jitter.

As a measure of the influence of various effects such as injection noise, RF errors or HOMs the effective phase space area of a pulse is calculated via

$$\epsilon = \pi \sqrt{\langle dE^2 \rangle \langle d\phi^2 \rangle - \langle dEd\phi \rangle^2}$$

and compared afterwards with the nominal simulation. The phase space area increases by a factor 4.65 in the 704 MHz linac and 4.27 in the 1.3 GHz linac in the simulations with increased injection noise.
Figure 8: Phase space distribution of one pulse at the end of both linacs with a two times higher injection energy and phase jitter. In the 1.3 GHz linac the phase error is normalised to 704 MHz for comparison reasons. The phase space distribution shows a starting filamentation.

The effect of RF-errors originating from the RF system is analysed for both linacs assuming an error of 0.5 degrees in phase and 0.5 % in amplitude. Both errors have a uniform distribution along the linac. The phase space distribution resulting from 1000 different error is shown in Figure 9. An increase in the occupied phase space area is observed due to the RF errors. The energy error is about the same in both options, while the phase error is almost twice as large in the 704 MHz linac compared to the 1.3 GHz linac. As a consequence the calculated phase space area is 4.76 times higher in the 704 MHz linac but only 3.18 times higher in the 1.3 GHz linac. The impact of RF errors seems to be less significant in the 1.3 GHz linac.

Figure 9: Phase space distribution of one pulse at the end of 1000 linacs with an uniform distributed RF-error (0.5% in amplitude and 0.5 degree in phase) for both options. In the 1.3 GHz linac the phase error is normalised to 704 MHz for comparison reasons. An increase in the phase space, especially in the energy error is observed in both options.

The effect of RF errors will be later on compared to the effect of HOMs to judge their influence on the beam. The phase space increase due to RF errors can be used as upper tolerable limit for the phase space increase caused by HOMs.
4.2 HOMs with high \((R/Q)(\beta)\) values

The HOM with the highest \((R/Q)(\beta)\) value is chosen individually in each cavity for the corresponding \(\beta\), see Table 4. A HOM frequency spread of 1 MHz in the 704 MHz option and 2 MHz in the 1.3 GHz option is used. In order to avoid pulse to pulse coupling the HOM damping in terms of \(Q_{\text{ex}}\) is set to \(10^8\) in all cavities for both options. The beam current is varied between the nominal 50 mA and 400 mA to explore the safety margin.

Table 4: HOMs used in the simulations. The \((R/Q)(\beta)\) map of the modes is shown in Fig. 5 and Fig. 6.

<table>
<thead>
<tr>
<th>Section</th>
<th>704 MHz</th>
<th>1.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_n)</td>
<td>(\sigma_{f_n})</td>
</tr>
<tr>
<td>Medium (\beta)</td>
<td>1817.37</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1835.18</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1849.61</td>
<td>1</td>
</tr>
<tr>
<td>High (\beta)</td>
<td>1623.77</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1623.84</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(\dagger\) linac definition

First, it is validated, that at \(Q_{\text{ex}} = 10^8\) no pulse to pulse coupling occurs by simulating 100 consecutive pulses with a beam current of 400 mA. In Figure 10 the maximum observed cavity HOM voltage in the linac is plotted at the end of each pulse for both options. It shows a completely random distribution and no correlation between the pulses is observed. The induced voltages in the 1.3 GHz case are significantly higher than for the 704 MHz case. At weaker damping (higher \(Q_{\text{ex}}\)) an increase of cavity HOM voltage over the pulses is observed due to the pulse to pulse coupling.

![Figure 10](image-url)

**Figure 10:** Maximum HOM voltage in the linac after each pulse for 100 consecutive pulses at 400 mA, with \(Q_{\text{ex}} = 10^8\) and for the HOMs listed in Tab 4. No systematic pattern is observable in both linacs. The voltage in the 1.3 GHz option is significant higher.

Based on this result, only one pulse is simulated with no HOM voltage present in each
cavity at the start of the simulation. 100 different error sets for the HOM frequency distribution along the linac are simulated for different beam currents. The average HOM voltage, present at the end of the pulse, in each cavity is shown in Figure 11 for both linacs. In addition the maximum observed HOM voltages are plotted for the 400 mA case.

![Figure 11](image_url)

**Figure 11:** Average and maximum HOM voltage after one pulse for 100 linacs (different HOM frequency patterns) and different beam currents where one HOM per cavity is present. The maximum observed HOM voltage at 400 mA is about 10 kV in the 704 MHz linac and exceeds 100 kV in some cavities in the 1.3 GHz linac.

The HOM voltage scales linearly with the beam current and the variations along the linac are due to the \((R/Q)(\beta)\) change. In the 1.3 GHz option the voltages are on average about one order of magnitude higher than in the 704 MHz option and the observed peak voltages at 400 mA overshoot 100 kV, while in the 704 MHz linac the maximum is about 10 kV. This result can be explained by the higher HOM frequencies and \((R/Q)(\beta)\) values in the 1.3 GHz linac. Both lead to a higher induced HOM voltage by each bunch. At nominal current the HOM voltage is at a very moderate level in both options and causes no significant growth in the energy and phase spread at the end of the linac compared to RF-errors.

In Table 5 the average change in the energy and phase error as well as the phase space increase is shown which is caused by HOMs. Additionally the data from the initial simulations without HOMs are added for a better comparison. Even at 400 mA the influence of HOMs in both options is minor compared to the impact of RF errors. Comparing the two linacs one finds that at 400 mA the influence of HOMs is higher in the 1.3 GHz linac.

In the next scenario, the same HOM configuration as before is used and a sweep of the HOM damping \((Q_{ex})\) is done for different beam currents. Two pulses are simulated and the phase space area increase of the second pulse is compared with the case where no HOMs are present (Figure 7). The results for both linacs are shown in Figure 12. Here only one HOM frequency pattern is simulated to get a qualitative overview, where operational boundaries are located. Further simulations with different HOM frequency pattern should be carried out to quantify these results. At 400 mA, the HOM influence starts to rise above \(Q_{ex} = 10^6\) and reaches a plateau around \(Q_{ex} = 10^9\). If \(Q_{ex}\) increases further, a steep increase in the phase space area is observed due to the pulse to pulse coupling. The plateau is shifted to a higher \(Q_{ex}\) in the 1.3 GHz linac because of the shorter decay time \(T_d\) at higher frequencies \((T_d \propto 1/\omega_n)\). In general, the phase space increase is about a factor ten higher in the 1.3 GHz linac as in the 704 MHz linac caused by the higher HOM voltages and shorter fill time \((\propto \omega_n)\). At nominal current no influence is observed below \(Q_{ex} = 10^8\) in both linacs.
Table 5: Average energy and phase error deviation as well as the phase space increase at the end of the linacs for injection beam jitter, RF-errors and HOMs.

<table>
<thead>
<tr>
<th></th>
<th>704 MHz</th>
<th>1.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_E$</td>
<td>$\sigma_\phi$</td>
</tr>
<tr>
<td>nominal</td>
<td>0.113</td>
<td>0.370</td>
</tr>
<tr>
<td>more inj. jitter$^2$</td>
<td>0.214</td>
<td>0.740</td>
</tr>
<tr>
<td>RF errors$^3$</td>
<td>0.498</td>
<td>0.523</td>
</tr>
<tr>
<td>HOM$^4$ 50 mA</td>
<td>0.113</td>
<td>0.415</td>
</tr>
<tr>
<td>HOM$^4$ 400 mA</td>
<td>0.114</td>
<td>0.370</td>
</tr>
</tbody>
</table>

$^1$ degree at 704 MHz; all values are normalised to this unit

$^2$ Reference simulation with nominal injection noise, see Fig. 7.
$^3$ doubled injection energy jitter and doubled injection phase jitter in the 704 MHz linac, see Fig. 8.
$^4$ Average of 1,000 simulations, see Fig. 9.
$^5$ Average of 100 simulations, $Q_{ex} = 10^8$, see Tab. 4.

Figure 12: Phase space area increase of a pulse in both linacs under the influence of HOMs for different beam currents and damping values. At nominal current no effect is observed below $Q_{ex} = 10^8$. Different scales for the y-axis are used, because the phase space increase is about one order of magnitude higher in the 1.3 GHz linac due to the higher HOM voltages, as illustrated in Fig. 11.
4.3 Machine lines

The HOM frequencies found by SUPERFISH are all far from machine lines. However, the effect is analysed, if a HOM with a moderate \( R/Q \) value (1 \( \Omega \)) falls in one cavity on a machine line. The 5\(^{th} \) fundamental machine line (1.761 GHz) is used for the 704 MHz linac and the 9\(^{th} \) (2.925 GHz) for the 1.3 GHz. These are the machine lines, which are closest to any HOMs in the cavities. For this kind of simulation the same HOM settings as before and the nominal current are used, but one cavity is modified. In all simulations the HOM voltage in the complete linac after one pulse is shown in Figure 13–15, and the HOM voltage development during the pulse in the modified cavity is shown in Figure 16–18.

![Figure 13: HOM voltage distribution along the linacs at 50 mA, where a HOM with \( R/Q = 1 \) in cavity 100 falls direct on a machine line. All HOMs are only weakly damped (\( Q_{ex} = 10^8 \)). The HOM voltage in the resonantly excited cavity is three orders of magnitude higher than in the other cavities.](image1)

![Figure 14: Same as Fig. 13, but moderate damping (\( Q_{ex} = 10^6 \)).](image2)

First the HOM meets a resonance and is only weakly damped (\( Q_{ex} = 10^8 \)), see Figure 13 and Figure 16. The HOM voltage increases linearly over the pulse, because of the weak damping and reaches almost 1 MV at the end of the pulse in the 1.3 GHz linac. The voltage in the 704 MHz linac is only half due to the lower frequency. This high voltage disturbs the beam and causes a significant load for the cryogenic systems due to the power dissipation in the cavity walls and interconnections. The peak power dissipation (CW) in the cavity walls for 1 MV would be 100 W assuming \( Q_0 = 10^{10} \). In the second simulation the HOM is still in resonance,
Figure 15: Same as Fig. 13, but the HOM frequency is detuned by 10 kHz in the 704 MHz cavity and 20 kHz in the 1.3 GHz cavity.

Figure 16: Time development of the cavity HOM voltage during a pulse in the cavity with a HOM \((R/Q = 1)\) directly at a machine line and a weak HOM damping \((Q_{ex} = 10^8)\) at beam current of 50 mA. The HOM voltage rises linear in time until the end of the pulse and it increases in the 1.3 GHz cavity about a factor two faster.

Figure 17: Same as Fig. 16, but moderate damping \((Q_{ex} = 10^6)\). The HOM voltage saturates at the same value in both cavities, but in the 1.3 GHz cavity the rise time is about a factor two faster.
Figure 18: Same as Fig. 16, but the HOM frequency is detuned by 10 kHz in the 704 MHz cavity and 20 kHz in the 1.3 GHz cavity. The HOM voltage oscillates in both cavities and the amplitude is several orders of magnitude lower as in 16. In the 1.3 GHz cavity the amplitude and the oscillation frequency are about a factor two higher due to the higher frequency.

but moderately damped \( (Q_{ex} = 10^6) \), see Figure 14 and Figure 17. There the HOM voltage saturates at a level of 50 kV in both linacs, which is not a problem from the beam dynamics point of view and also not for the cryogenic system. In the 1.3 GHz linac the saturation value is reached faster due to the higher frequency. Finally the weakly damped HOM is detuned by 10 kHz in the 704 MHz linac and by 20 kHz in the 1.3 GHz linac, see Figure 14(a) and Figure 17(a). In this case the the HOM voltage oscillates with a frequency of 10 kHz or 20 kHz, which correspond to 35,200 Bunches in the 704 MHz linac and 16,250 Bunches in the 1.3 GHz linac. The maximum amplitude is about 7 kV in the 1.3 GHz linac and 9 kV in the 704 MHz linac.

Only in the case, where the HOM falls directly on the machine line and is weekly damped a significant influence on the beam is observable. This is illustrated in Figure 19, where the energy error evolution of one pulse along the linacs is shown. The HOM lead to deceleration kick, which is observable as sudden increase in the energy error.

Figure 19: Energy error evolution of one pulse along the linac, where in cavity 100 a HOM with \( (R/Q) = 1 \Omega \) falls directly on a machine line and is only weakly damped \( (Q_{ex} = 10^8) \). The beam gets a significant kick from the HOM, which leads to sudden increase in the energy error.
4.4 Fundamental passband modes

The \( TM_{010,4/5\pi} \) mode in the 704 MHz cavities and the \( TM_{010,8/9\pi} \) mode in the 1.3 GHz cavities have a significant \((R/Q)(\beta)\), see Table 6, which is even higher than the accelerating mode at certain \( \beta \) values.

**Table 6**: Fundamental passband modes used in the simulations. The \((R/Q)(\beta)\) map of the modes was shown in Fig. 5 and Fig. 6.

<table>
<thead>
<tr>
<th>Section</th>
<th>( f_n ) [MHz]</th>
<th>( \sigma_{f_n} ) [kHz]</th>
<th>( (R/Q)(\beta)_{\text{max}} ) [( \Omega )]</th>
<th>( f_n ) [MHz]</th>
<th>( \sigma_{f_n} ) [kHz]</th>
<th>( (R/Q)(\beta)_{\text{max}} ) [( \Omega )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium  ( \beta )</td>
<td>702.98</td>
<td>10</td>
<td>155.1</td>
<td>1298.91</td>
<td>20</td>
<td>230.30</td>
</tr>
<tr>
<td>High  ( \beta )</td>
<td>702.63</td>
<td>10</td>
<td>314.76</td>
<td>1299.27</td>
<td>20</td>
<td>545.35</td>
</tr>
</tbody>
</table>

† linac definition

Beam dynamics simulations are executed with these particular modes using a frequency spread of 10 kHz in the 704.4 MHz linac and 20 kHz in the 1.3 GHz linac. The fundamental power coupler provides a certain damping to these modes which correspond to a \( Q_{\text{ex}} = 10^6 \). This is about a factor ten higher than the \( Q_{\text{ex}} \) of the accelerating mode. 100 linacs are simulated with different beam currents. The average and maximum HOM voltage in all cavities is shown in Figure 20.

![Figure 20](image)

**Figure 20**: Average and maximum cavity HOM voltage in the \( TM_{010} \)-mode closest to the accelerating mode, see Tab. 6, after one pulse for 100 linacs with different beam currents. Instabilities occur at the end of the 1.3 GHz linac operating with 400 mA, where HOM voltages in the same order as the accelerating voltage are present.

Significant HOM voltages are observed for 400 mA in both linacs. A strong self amplifying effect is observed at the end of the linacs, especially in the 1.3 GHz option. In this situation a HOM in one cavity drives the HOMs in the further downstream cavities in a constructive manner. This is possible, because the cavity to cavity HOM frequency spread is sufficiently small. HOM voltages at that level can cause significant beam losses, which are a major concern in the operation of such a linac. Even at nominal current the HOM voltages are about 0.1% of the accelerating voltage in the 1.3 GHz option and start to be comparable to the RF errors which are typically about 0.5% of the accelerating voltage.
For these modes a sweep of $Q_{ex}$ and $f_b$ is done and the phase space increase is illustrated in Figure 21 for both linacs. One simulation is executed for each combination and simulations are stopped for higher $Q_{ex}$ values in case the resulting phase space area is a factor 100 higher than the reference area.

(a) 704 MHz linac  
(b) 1.3 GHz linac

Figure 21: Phase space area increase of one pulse under the influence of non accelerating $TM_{010}$-modes for different beam currents and damping. The phase phase increase due to RF errors is indicated with the dashed line. No significant influence is observed at nominal current. Increasing the current lead to a strong phase space increase. At 400 mA the phase space more than a factor 100 larger for $Q_{ex} > 10^6$ and in the 1.3 GHz linac even at 200 mA for $Q_{ex} > 10^7$.

At nominal beam current the effect is still minor, but increases significantly above 100 mA in the 1.3 GHz linac and 200 mA in the 704 MHz linac. The fundamental power coupler should provide a damping in the order of some $10^5$. At that damping level, the phase space increase in the 704 MHz linac is tolerable for all simulated currents, while in the 1.3 GHz linac the phase space increase exceeds the value which is caused by RF errors in case of 400 mA. The safety margin is about a factor two higher in the 704 MHz linac option. In general the $\beta_g$ should be reconsidered to reduce the $(R/Q)(\beta)$ values of the fundamental passband modes beside the accelerating mode.

5 Conclusions

Comparing directly the cavities properties one finds that the $(R/Q)(\beta)$ increases with the number of cells for all modes while the energy acceptance decreases with the number of cells. Hence, the high $(R/Q)(\beta_g)$ of the accelerating mode drops fast for velocities different from $\beta_g$. From this point of view a lower number of cells is preferable for $\beta < 1$ particles, which implies the use of a lower operation frequency in order to keep a high real estate gradient. In the investigated case the $(R/Q)(\beta)$ are about a factor two higher in the 1.3 GHz cavities compared to the 704 MHz cavities.

The phase space increase due to HOMs at nominal current is negligible compared to the effect of RF errors, if they are not excited resonantly. Based on the discussed scenarios a weak HOM damping in the order of $Q_{ex} \sim 10^8$ could be fine in both linacs, if there is no HOM with high $(R/Q)(\beta)$ close to fundamental machine lines. The fundamental passband modes are critical in both options, because of the high $(R/Q)(\beta)$ at the end of both linacs. This can cause a phase space increase especially in the 1.3 GHz linac for high currents. For a final
conclusion on HOM damping requirements a more detailed study has to be performed with the final cavity design and linac layout. Furthermore dipole modes should be considered, which are not discussed here.

Both designs are not yet optimised and need further improvements. Especially the choice of the geometrical $\beta$ in the high $\beta$ sections should be revised in order to avoid the unfavourable case, where other $TM_{010}$ modes have higher $(R/Q)(\beta)$ values than the accelerating mode.

6 Acknowledgements

I would like to thank Rama Calaga and Mamad Eshraqi for providing me the necessary input data used in this study. Furthermore I want to thank Frank Gerigk for the fruitful discussions and suggestions during the preparation of this note.

References

[1] International Linear Collider (ILC), Homepage, http://www.linearcollider.org/


[10] M. Eshraqi, private communication

