THE SLOTTED IRIS STRUCTURE

G. Döme and W. Schminke

ABSTRACT

An 800 MHz cavity working at the 4th harmonic of the main RF frequency had to be designed for the Landau damping system of the SPS. It was decided to use a periodic disk-loaded structure.

Since it is well-known that in such a structure, besides the accelerating \( E_0 \)-like mode, a strongly deflecting \( EH_{11} \) mode is also present, we looked for a possible way of decreasing the coupling impedance of this unwanted mode for the beam. The influence of slots in the disks was investigated by model measurements for various slot lengths, with slots in line or rotated by 90° from cell to cell.

The main result is that for the slots to have a significant influence on the deflecting \( EH_{11} \) mode, they must be resonant in the passband of this mode. When the slots are rotated by 90° from cell to cell, each original deflecting passband is split into two passbands. For one of them, the coupling impedance is strongly reduced with respect to the case without slots; unfortunately, the coupling to the beam remains significant for the other passband. In this "rotated" case, it is shown that the slots should not be made longer than 65°.

In the "inline" case we have a similar situation: the slots remove the degeneracy of the two polarizations in the deflecting passband. For one polarization, the coupling impedance is reduced, whereas the other one is only slightly affected by the slots.

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1. THEORETICAL BACKGROUND OF THE MEASUREMENTS

1.1 The structure

In the present case, the planned use of the cavity in the Landau damping system fixed the following parameters for the accelerating mode

i. travelling-wave structure.

ii. 4th harmonic of the main accelerating frequency (200,222 MHz) \(^1\).  

iii. operating mode at transition energy: \(\pi/2\) phase shift per cell at \(\omega_0\).

iv. maximum \(Re/Q\) value for highest voltage at given power.

v. \(v_g/c\) around 0.04 to 0.06, for limited phase slip, \(\tau = \lambda/v_g (\omega - \omega_0)\), over the frequency range used.

With a nominal \(\gamma_{tr} = 24\), \(\beta_{tr} = 0.999132\), the cell length is given by

\[
L = \frac{\beta_{tr}c}{2\pi f} \cdot \frac{\pi}{2} = \frac{\beta_{tr}}{4f} \cdot \frac{c}{f},
\]

\(L = 93.5\) mm for \(f = 800.888\) MHz.

The influence of different slot lengths is then investigated on a 1:2 scale model, with a cell length of 46.75 mm, and adequate diameters of tube and iris to match the frequency at \(\beta L = \pi/2\). Two slots are cut at 180° from each other, at the radius with highest magnetic field. If one considers that the field distribution is similar to that in a pillbox, the maximum magnetic field \(H_\phi\) will be found when

\[
\frac{d}{d\rho} \left( \frac{j_{11}^1}{D} r \right) = 0
\]

with

\[
\frac{j_{11}^1}{D} r = 1.8412
\]

that is

\[
r = b \cdot \frac{1.8412}{2.4048} = b \cdot 0.7656.
\]
In fact, since the normal electrical field also couples through the
slot and compensates part of the magnetic coupling, another r may give
roughly the same overall coupling; for details see reference (2).
For our measurements, the following dimensions were chosen, given as for a
full-scale cavity (see Fig. 1).

\[
\begin{align*}
  b &= 140 \text{ mm} & t &= 10 \text{ mm} \\
  a &= 45 \text{ mm} & s &= 4 \text{ mm} \\
  r_1 &= 100 \text{ mm} & p &= 10 \text{ mm} \\
  r_2 &= 120 \text{ mm} & L &= 93.5 \text{ mm}
\end{align*}
\]

The angle \( \alpha \) is increased stepwise. For each \( \alpha \) a set of cells is assembled
to form a resonator, with slots in line, or rotated by 90° from cell to cell.
The resonator is closed at both ends with half cells as shown in Fig. 1c.

1.2 Dispersion diagram and R/Q values

1.2.1 The perturbation technique

For describing a periodic structure the dispersion diagram, relating
phase shift per cell and frequency, is of great importance. Using an N cell
resonator, one can find N resonances in one passband, yielding N discrete
frequencies on the dispersion curve. By using some numerical interpolation
technique, the actual dispersion curve can be drawn for continuous phase shift
per cell. Since the end-plates are imaging the resonator in such a way that
an infinitely long periodic structure results, one has to take care where to
put them. A periodic structure with slots rotated by 90° is only simulated
if the mirror lies in the iris (as in Fig. 1). Here the beam holes have been
machined in the half cells. For the sake of simplicity, the half slots have
not been cut into the end-plates. The error introduced by this approximation
should be very small, since the presence of an end-plate requires the electric
field to vanish in those "half" slots.

Generally, the boundary conditions at the end-plates are such that
in a given passband (for some wavetypes), the 0 mode, the \( \pi \) mode or the
two together (e.g. for the slot passband) cannot be excited. This depends
on the specific field distribution of the wave in the cavity. For more
details see reference (3).

Having excited some resonance by small coupling probes at the end-plates,
the phase shift per cell and the R/Q value have to be found.
Here a perturbation technique is used to sample the electric field on the axis of the cavity. This method, described by Slater et al.\(^4,5\) uses the fact that the resonant frequency of a resonator is shifted by \(\Delta \omega\) if a small perturbing object is introduced into the resonator. It can be shown that for a small metallic bead

\[
\frac{\Delta \omega}{\omega_1} = -\frac{\frac{3}{4} v \varepsilon_0 |E_1|^2 - \frac{3}{2} v \mu_0 |H_1|^2}{W_s}
\]

with

- \(v\) volume of the bead
- \(E_1, H_1\) unperturbed electric and magnetic field at the position of the bead
- \(W_s\) total stored energy
- \(\omega_1\) unperturbed resonance frequency
- \(\Delta \omega = \omega - \omega_1\)

For measuring only one electric field component, small needles can be used. Calibration of such cylindrical needles in a test cavity leads to good agreement with formulae given in reference (6), rather than with Slater's formula for ellipsoidal needles.

In fact, the approximate formula for a long thin metallic needle given by Slater leads to frequency shifts which are a factor \(\frac{2}{3}\) too small.

The formula cited in reference (6) can be obtained by first calculating the perturbed field under the assumption that a long ellipsoid is used instead of a cylindrical needle. The polarization of the cylindrical needle is then calculated using the field strength found. This leads to

\[
\frac{\Delta \omega}{\omega} = -\frac{\pi}{16} \frac{\varepsilon^3}{\ln(\frac{2}{\rho}) - 1} \frac{\varepsilon_0 |E_{1p}|^2}{W_s}
\]

with \(E_{1p}\) the unperturbed electric field parallel to the axis of the needle

- \(\ell\) the length of the needle
- \(\rho\) the radius of the needle
As explained in reference (6), the best way of measuring $\Delta \omega$ is to measure the phase shift $\Delta \phi$ of the transmission factor through the resonator at the resonance frequency $\omega$.

Then

$$\Delta \omega = \frac{\omega}{2Q_\perp} \cdot \tan \Delta \phi$$

with $Q_\perp$ the quality factor of the resonator (with probes).

A longitudinal electric field exists on the axis only for waves with rotational symmetry around the structure axis. That is, only waves whose fields behave like $E_{0n}$ waves near the axis can accelerate particles longitudinally. On the other hand, only waves having non-vanishing transverse electromagnetic fields on the structure axis can deflect particles which travel along the axis. Combining both facts, it follows that of the infinite variety of different periodic waves which exist in a waveguide accelerator, only two classes are of interest for the particle beam: for longitudinal effects all $E_{0n}$ types and for deflecting effects all $H_{1n}$- and $E_{1n}$-types. As a basic condition, if the structure is long compared to the axial wavelength of the fields, these effects require synchronism between wave and particle — that is the phase velocity has to equal the particle velocity. In such a case, as shown in reference (7), the deflecting force is proportional to $\nabla \times E_z$ on the axis. Therefore the measurements are performed with a small bead or needle on the axis for longitudinal modes, and with a needle offset from the axis for transverse modes. In both cases, $E_z$ has to be measured, so that the needle is parallel to the axis.

To control the motion of the perturbing object and to record the corresponding phase shift, the set-up shown in Fig. 2 is used.

The measurement procedure is as follows: first a resonant frequency is found by looking for the maximum transmission factor through the cavity. On the polar diagram of the network analyzer a simple resonance produces the well-known circle when the frequency is varied.

Two phase measurements at two frequencies in the vicinity of resonance deliver the $Q_\perp$ value. Then the perturbing object is moved stepwise through the resonator with a phase measurement at each step, yielding, after some calculations, the ratio (electric field)$^2$/[stored energy]. Since the electric
field enters the perturbation formula quadratically, its sign has to guessed once a plot of the field along the axis has been performed.

1.2.2 The Floquet Theorem and space harmonics

Floquet's theorem says that for a travelling wave in a periodic structure, a specific field component at some point \( x,y,z \) differs from that at another point \( x,y,z+L \) one period length \( L \) away only by some complex factor \( \exp(-\gamma L) \), with \( \gamma = \alpha + j\beta \), \( \beta = \beta_0 + 2n\pi/L \).

In fact, the period length \( L \) of interest here is the electrical period length. Depending on the characteristics of the wavetype and the structure, the electrical period length does not always equal the geometrical period length (see reference 9).

In what follows, we assume that the structure is lossless, so that \( \alpha = 0 \) within a passband. Each field component may be expanded in a series of space harmonics. For the longitudinal electrical field on the axis, we may write

\[
E_z^+ = \sum_{n=-\infty}^{+\infty} a_n \exp(z \cdot j\beta_n z),
\]

\( \beta_n = \beta_0 + 2n\pi/L \), \( n = 0, \pm 1, \pm 2 \ldots \), \( a_n \) being the amplitude of the \( n \)th space harmonic\( ^8 \). For a loss-less structure, all the \( a_n \) may be taken as real quantities.

1.2.3 Calculation of the space harmonics

On the other hand, the standing wave field pattern found in a 2N cell resonator may be expanded as

\[
E_z = \sum_{m=1}^{\infty} 2b_m \cos \frac{m\pi}{NL} z + b_0
\]

\[
= \sum_{m=1}^{\infty} b_m (e^{-j\frac{m\pi}{NL} z} + e^{j\frac{m\pi}{NL} z}) + b_0
\]

\[
= \sum_{m=-\infty}^{\infty} b_m e^{-j\frac{m\pi}{NL} z}, \quad b_m = b_{-m} \text{(boundary conditions)}
\]

\[
b_m = \frac{1}{NL} \int_0^{NL} E_z e^{-j\frac{m\pi}{NL} z} \, dz.
\]
The $b_m$ may be calculated by a complex FFT, after extending the field pattern $E_z$ over $2NL$ by just imaging the pattern at $z = NL$ or at $z = 0$. For resonators which are part of a periodic structure, the field $E_z$ will be the sum of the fields of two travelling waves, propagating in opposite directions

$$E_z = E_z^+ + E_z^-.$$

For both waves, the Floquet theorem yields the expansion

$$E_z^\pm = \sum_{n=-\infty}^{\infty} a_n^\pm e^{\mp j\beta_n z}, \quad \beta_n = \beta_0 + \frac{2n\pi}{L}.$$

It follows

$$E_z = \sum_{n=-\infty}^{\infty} \left( a_n^+ e^{-j\beta_n z} + a_n^- e^{+j\beta_n z} \right), \quad \frac{a_n^-}{a_n^+} = \rho \text{ independent of } n.$$

The boundary conditions at $z = 0$ and $z = NL$ yield respectively $\rho = 1$, $\sin(\beta_0 NL) = 0$. Therefore

$$a_n^+ = a_n^- = a_n \text{ with } \beta_0 = \frac{q\pi}{NL}.$$

The phase shift/cell is thus

$$\beta_0 L = \frac{q\pi}{N} \quad \text{where} \quad q = 0, 1, 2, \ldots, N.$$

Since the Fourier decomposition and the expansion into space harmonics must yield the same field it follows for the amplitudes that

$$a_n^- = a_n^+ = b_{-m} = b_m, \quad \text{with } m = |q + 2nN|.$$

If $q \neq kN$ ($k$ integer), the stored energy $W_{SW}$ in the resonator is twice the energy $W_{TW}$ in each of the travelling waves

$$W_{SW} = 2W_{TW} = 2 \left[ \frac{1}{2} \int_E \rho_0 |E^+|^2 \, dv \right].$$
For $q = kN$ there is no difference between $E_z^+$ and $E_z^-$ and it holds

$$E_z = E_z^+ = E_z^- = \sum_{n=-\infty}^{\infty} a_n e^{-j(2n+k)\frac{\pi z}{L}} = \sum_{n=-\infty}^{\infty} a_n e^{j(2n+k)\frac{\pi z}{L}}$$

changing the index in the second term to $n + n - k$ gives

$$0 = \sum_{n=-\infty}^{\infty} (a_n - a_{n-k}) e^{j(2n+k)\frac{\pi z}{L}}$$

and hence

$$a_n = a_{n-k}.$$ 

In this case, the total stored energy in the resonator is

$$W_{SW} = \frac{1}{2} \int \varepsilon_0 |E^+|^2 \, dv$$

$$= W_{TW}$$

As a consequence of these amplitude relations between space harmonics and Fourier components, it follows that the Fourier components have to vanish unless the condition

$$m = |q + 2nN|, \quad m = 0, 1, 2, \ldots$$

$$n = 0, \pm 1, \pm 2, \ldots$$

holds. Therefore this condition determines $q$ and as a consequence the phase shift per cell

$$\beta_0L = \frac{q \pi}{N}$$

(for the 0th space harmonic).
Writing down the Fourier components in \((2N-1)\) columns leads to:

for \(q = 0\)

<table>
<thead>
<tr>
<th>Column</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>\ldots</th>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(a_1 + a_{-1})</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(a_2 + a_{-2})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

for \(q \neq kN, k = 0, 1, 2 \ldots\)

<table>
<thead>
<tr>
<th>Column</th>
<th>0</th>
<th>1</th>
<th>\ldots</th>
<th>(q)</th>
<th>\ldots</th>
<th>((2N-q))</th>
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<td>(a_0)</td>
<td>(a_{-1})</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>(a_1)</td>
<td>(a_{-2})</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>(a_2)</td>
<td>(a_{-3})</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

for \(q = kN, k = 1, 2, 3, \ldots\)

<table>
<thead>
<tr>
<th>Column</th>
<th>0</th>
<th>1</th>
<th>(N)</th>
<th>\ldots</th>
<th>((2N-1))</th>
</tr>
</thead>
<tbody>
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<td>Fourier-components</td>
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<td>0</td>
<td>(a_0 + a_{-1})</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>(a_1 + a_{-2})</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>(a_2 + a_{-3})</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

the \(a_n\) being the amplitudes of travelling space harmonics.

Keeping in mind that in the special cases with \(q = 0\)

\[
a_n = a_{-n},
\]

and \(q = kN, k = 1, 2, 3 \ldots\)

\[
a_n = a_{-n-1}
\]

we realize that the calculated Fourier components are the sum of two equal space harmonic amplitudes. The maximum amplitude yields the mode number \(n\) of the leading space harmonic.
1.2.4 The R/Q value

For all space harmonics R/Q values can be calculated. These values apply for particles whose velocity equals the phase velocity of the corresponding space harmonic. The R/Q value is defined as

$$\left[ \frac{R}{Q} \right]_n = \frac{|E_{z,n}|^2 \cdot \omega}{\omega \cdot W_{TW}}$$

with $E_{z,n}$ the field strength seen by a particle synchronous with the $n$th space harmonic of a travelling wave, $l = NL$ the waveguide length, $\omega$ the operating frequency $\times 2\pi$, $W_{TW}$ the total stored energy in the travelling wave.

The field strength $E_{z,n}$ equals the amplitude of the synchronous space harmonic of the travelling wave $E_z^+$, that is

$$E_{z,n} = a_n \cdot$$

On the other hand, the perturbation formula yields for a purely electric perturbation by a metallic bead of volume $V$:

$$\Delta \omega = -\frac{\gamma}{4} \cdot V \cdot \omega \cdot \varepsilon_0 \cdot \frac{|E_z|^2}{W_{SW}}$$

and, if we apply travelling waves in opposite directions

$$\Delta \omega = -\frac{\gamma}{4} \cdot V \cdot \omega \cdot \varepsilon_0 \cdot \frac{|E_z^+ + E_z^-|^2}{2W_{TW} (1 + \delta_q kN)}$$

Hence it follows

$$E_{z^+} + E_{z^-} = \pm \sqrt{-\frac{8 \Delta \omega W_{TW}}{3\gamma \cdot \omega \cdot \varepsilon_0} \cdot (1 + \delta_q kN)}$$

The proper sign has to be guessed at to conform to the field pattern.

The decomposition into Fourier components

$$F_m = \frac{1}{NL} \int_0^L \sqrt{\frac{\Delta \omega (z)}{\omega}} e^{j \frac{m \pi}{NL} z} dz$$
leads to
\[ b_m = \sqrt{\frac{8}{3} \frac{W_{TW}}{\nu \varepsilon_0} (1 + \delta_{q,kN})} \cdot F_m \]

The relations between Fourier components and the space harmonics amplitudes lead to

\[ a_n (1 + \delta_{q,kN}) = b_m, \quad m = |q + 2nN| \]

and hence

\[ a_n = \sqrt{\frac{8}{3} \frac{W_{TW}}{\nu \varepsilon_0} \frac{1}{1 + \delta_{q,kN}}} \cdot F_m \]

Putting everything together, it follows (for accelerating waves)

\[
\begin{bmatrix}
R \\
Q
\end{bmatrix}
= \frac{8}{3} \cdot \frac{NL \cdot F_m^2}{(1 + \delta_{q,kN}) \varepsilon_0 \omega \nu} \cdot 
\begin{bmatrix}
\Omega \\
\frac{-m}{\hbar_n}
\end{bmatrix}
\]

in MKSA units, with \( m = |q + 2nN| \).

For deflecting waves the transverse \( R/Q \) is defined as in reference (6)

\[
\begin{bmatrix}
R \\
Q
\end{bmatrix}_{\perp,n} = \frac{|\text{grad}_z \frac{E}{\beta^2 n \omega \cdot W_{TW}}|}{\frac{E_z(r = r_0)}{r_0}} \cdot 
\begin{bmatrix}
\Omega \\
\frac{-m}{\hbar_n}
\end{bmatrix}
\]

Near the axis we have approximately

\[ \text{grad}_z E_z = \frac{dE_z}{dr} \approx \frac{\Delta E_z}{\Delta r} = \frac{E_z(r = r_0)}{r_0} \]

with \( r_0 \) the off-axis distance. Then we get

\[
\begin{bmatrix}
R \\
Q
\end{bmatrix}_{\perp,n} \approx \frac{|E_{z,r N}(r_0)|^2}{\beta^2 r_0^2 \omega \cdot W_{TW}} \cdot 
\begin{bmatrix}
\Omega \\
\frac{-m}{\hbar_n}
\end{bmatrix}
\]

in MKSA units.
1.2.5 Examples of the relations between Fourier components and space harmonics

As an example of the above relations, two measurements of modes in the accelerating passband are given, together with field plots and Fourier components. Fig. 3a shows the longitudinal standing wave $E_z$ field on the axis, measured with a small needle at 512 equidistant points for a resonator with $N = 8$ cells. Obviously, it is a $0$-mode. For the FFT, the measured data have first to be extended over one full period, that is twice the resonator length. Because of the boundary conditions, data are simply imaged point by point at the right endplate. The FFT on this 1024-point set of data is then performed. The result is as follows:

<table>
<thead>
<tr>
<th>FOURIER TRANSFORM CCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>---</td>
</tr>
<tr>
<td>49.7941</td>
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<tr>
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<tr>
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</tr>
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<td>-0.0745</td>
</tr>
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</tr>
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<td>8</td>
</tr>
<tr>
<td>-0.1251</td>
</tr>
<tr>
<td>-0.1242</td>
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<tr>
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</tr>
<tr>
<td>-0.0617</td>
</tr>
<tr>
<td>-0.0445</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MODE/PI</th>
<th>FREQ/MHZ-NORM</th>
<th>R/Q(OHM/METER)</th>
<th>R/Q-NORM</th>
<th>AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/8</td>
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<td>5050.806</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>

The Fourier components are written down element by element on the first line of the first block, then continuing on the first line of the second block (because 2N columns are needed), and so on with the 2nd, 3rd lines etc.
Thus, all non-zero space harmonics have to appear in two columns corresponding to

\[ c_1 = q \quad c_2 = 2N - q \]

In our case, for \( N = 8 \) with \( q = 0 \) we have \( c_1 = 0, c_2 = 16 \), which means: the same column. The same is true for \( q = N = 8 \), with \( c_1 = c_2 = 8 \).

In fact, all other amplitudes which should ideally be zero are finite, although very small. This is due to imperfections in the model or in the measurement itself, and sometimes due to mode mixing because of two resonances very close together. Application of our above formulae leads directly to the \( R/Q \) value for the 0 and 1 space harmonic, printed in the two last lines.

The \( R/Q_{\text{norm}} \) value applies to the full-scale cavity.

Fig. 3b) shows the standing-wave pattern of the \( \pi/2 \) mode, the operating mode in the planned system. Zeros with a change of sign are assumed at points 61, 191, 321 and 451. As before, the field has been imaged at the right hand endplate in order to extend the data to 1024 points. Because \( q = 4 \), columns 4 and 12 should be non-zero. The amplitudes found by the FFT are:

<table>
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<th>2</th>
<th>3</th>
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<td>-0.6771</td>
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<td>0.0347</td>
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<tr>
<td></td>
<td>0.0916</td>
<td>-0.0826</td>
<td>0.1582</td>
<td>0.1242</td>
<td>-0.1790</td>
<td>0.0298</td>
<td>-0.0402</td>
<td>0.0819</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MODE/PI</th>
<th>FREQ/MEZ-NORM</th>
<th>R/Q(Ohm/Meter)</th>
<th>R/Q-NORM AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/8</td>
<td>793.704</td>
<td>2737.054</td>
<td>1368.527 25.378</td>
</tr>
<tr>
<td>12/8</td>
<td>793.704</td>
<td>97.527</td>
<td>48.764 4.790</td>
</tr>
</tbody>
</table>

As before, some of the other amplitudes are still quite strong. The -1. space harmonic is of more importance than in the first example.

The \( R/Q_{\text{norm}} \) value of about 1369 \( \Omega/m \) is the one which applies for operation in the planned full-scale system.
1.3 The effect of the slots

1.3.1 Coupling and different polarizations for the slots (in-line)

First of all the slots provide a supplementary coupling. For a \(\phi\)-dependent wave this coupling depends on the polarization relative to the slots. In the \(E_{ln}, H_{ln}\) case perfectly conducting plates can be introduced in the symmetry planes of the structure along \(\phi\) (without disturbing the fields). For the two different polarizations these planes are orthogonal. In the circularly symmetric waveguide without slots, no difference exists for the waveguide sectors defined by both planes, that is, the two-polarizations are degenerate. They have the same resonant frequency. Once the slots are introduced and aligned, the symmetry planes define two different waveguide sectors, see Fig. 4.

Different resonant frequencies will be found for the two cases. In case (1) the length of the slot is half the length of case (2). To excite the two polarizations we used probes placed as shown in Fig. 4. They have to be very small in order not to perturb the field distribution.
1.3.2 Higher symmetries for the rotated slots

For slots rotated by 90° from one cell to the next, the above-mentioned perfectly conducting planes will cut the slots in every second cell only. The two polarizations mentioned in the previous chapter 1.3.1, will now degenerate (as in a structure without slots). In fact, the electrical period of the structure for a \( \sin n\phi \) or \( \cos n\phi \) mode with \( n \) odd is now \( 2L \), and hence equals the geometrical period length. Figure 5 shows the different dispersion diagrams of e.g. an \( E_{11} \)-like wave in a disk-loaded waveguide without slots (Fig. 5a), with vanishing slots rotated by 90° from cell to cell (Fig. 5b) and with quite long slots, also rotated (Fig. 5c).

Now the stop-bands for a \( \pi \) phase-shift per electrical period appear at \( \beta_0 2L = \pi \), or \( \beta_0 L = \pi/2 \). Comparison with the slots in line shows that the \( \pi/2 \) standing wave modes (with slots in line) are equivalent to the \( \pi \) modes (with slots rotated) since in both cases every second slot pair is not excited. Since different polarizations in the in-line case yield different \( \pi/2 \) mode frequencies, we have to expect different \( \pi \) mode frequencies in the case of rotated slots for the two different probe positions.

Figure 6 shows the cross-sections of the structure with rotated slots as a resonator in the plane of exciting probe and perturbing needle. One realizes that moving the probe and needle through \( \phi = 90° \) on the endplate is equivalent to a lateral shift of the structure by \( z = L \), keeping the probe and needle as before. This lateral shift means a shift of the shorting end-plane by \( L \), half an electrical period. For \( L = \pi/2\beta_0 = \lambda_g/4 \) this is equivalent to a replacement of the end condition "short" by "open"; for more information see Reference (3).
1.3.3 Space harmonics for different excitations

Let us assume that with different excitations we excite the two degenerate polarizations composed by almost the same $E^+_{z}$ travelling waves: when counted from $z = 0$, $E^+_{z_{2b}}$ is the same as $E^+_{z_{2a}}$ except for a constant factor (see Fig. 6). The decomposition into space harmonics for the configuration of Fig. 6a gives, for an $N$ double-cell resonator:

$$E^+_{z_{2a}} + E^-_{z_{2a}} = \sum_{-\infty}^{\infty} a^+_n e^{-j\beta_n z} + a^-_n e^{+j\beta_n z}, \quad \beta_n = \frac{q + 2nN}{2NL} \pi$$

with $a^+_n = a^-_n = a_n$ real, because of the short circuit at $z = 0$. In Fig. 6b the short circuit is at $z = -L$, and

$$E^+_{z_{2b}} + E^-_{z_{2b}} = \sum_{-\infty}^{\infty} b^+_n e^{-j\beta_n z + jL} + b^-_n e^{+j\beta_n z + jL}, \quad \beta_n = \frac{q + 2nN}{2NL} \pi$$

with $b^+_n = b^-_n = b_n$ real

$$= \sum_{-\infty}^{\infty} b_n e^{-j\beta_n z - j\frac{q}{2N} + (n+1)\pi} + b_n e^{+j\beta_n z + j\frac{q}{2N} + (n+1)\pi}$$

$$= e^{-j\frac{q}{2N} \pi} \sum_{-\infty}^{\infty} b_n (-1)^n e^{-j\beta_n z} + e^{j\frac{q}{2N} \pi} \sum_{-\infty}^{\infty} b_n (-1)^n e^{+j\beta_n z}$$

In both cases the space harmonic amplitudes are the same, apart from a general phase factor $e^{\pm j\frac{q\pi}{2N}}$ and a sign change for all harmonics of odd number $n$. For $E^+_{z_{2b}} = E^+_{z_{2a}}$, it follows that $b_n = (-1)^n a_n$. Measurements of deflecting modes with different excitations confirm the above results, see the following examples for the $\pi/4$ mode of the first deflecting, $E_{11}$-like higher passband and the $\pi/4$ mode of the second deflecting $H_{11}$-like lower passband. Fig. 7a shows the standing wave longitudinal $E_z$-field $10$ mm off the axis of an eight-cell resonator with $65^\circ$ long, rotated slots, when the input and output probes are placed as in Fig. 6a. For the probes placed as in Fig. 6b the field pattern of Fig. 7b is found. Frequencies and the first four space harmonics for both cases are as follows:
<table>
<thead>
<tr>
<th>Fig. 7a</th>
<th>Fig. 7b</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>f/MHz</strong></td>
<td>2564.486</td>
</tr>
<tr>
<td><strong>Q</strong></td>
<td>8340.649</td>
</tr>
<tr>
<td><strong>mode</strong></td>
<td>π/4</td>
</tr>
<tr>
<td><strong>E_{11}</strong></td>
<td>on 2L</td>
</tr>
<tr>
<td>a₀</td>
<td>-15.49</td>
</tr>
<tr>
<td>a₋₁</td>
<td>-1.16</td>
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<tr>
<td>a₁</td>
<td>0.71</td>
</tr>
<tr>
<td>a₋₂</td>
<td>1.18</td>
</tr>
</tbody>
</table>

The parity relations $b_n = (-1)^n a_n$ expected from our simple assumption fit. The amplitudes of the 0th space harmonic are almost the same for both excitations. Differences in the frequencies and higher-order amplitudes may be due to model imperfections, especially the missing half slots in the endplates. Fig. 7c and Fig. 7d show the $E_z$-field plot for the $H_{11}$-like $7\pi/4$ mode. The plot is quite noisy, due to the low coupling of the measuring probes.

For this example the data are:

<table>
<thead>
<tr>
<th>Fig. 7c</th>
<th>Fig. 7d</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>f/MHz</strong></td>
<td>2798.415</td>
</tr>
<tr>
<td><strong>Q</strong></td>
<td>3917.273</td>
</tr>
<tr>
<td><strong>mode</strong></td>
<td>7π/4</td>
</tr>
<tr>
<td><strong>H_{11}</strong></td>
<td>on 2L</td>
</tr>
<tr>
<td>a₀</td>
<td>0.749</td>
</tr>
<tr>
<td>a₋₁</td>
<td>-8.466</td>
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<tr>
<td>a₁</td>
<td>5.63</td>
</tr>
<tr>
<td>a₋₂</td>
<td>-1.55</td>
</tr>
</tbody>
</table>

As before, the parity relations fit. Furthermore, frequencies and all given space harmonics agree quite well for both excitations. Evidently the $H_{11}$-like mode is less affected by imperfections, that is the two polarizations are more similar. In fact, the parity relation can be used to determine whether a deflecting mode has to be considered. If this relation is not verified, the $\phi$-dependent component of the mode is evidently not of great importance.

2. **THE RESULTS**

2.1 **Dispersion curves**

2.1.1 **Model without slots**

Fig. 8 shows the dispersion diagram for the disk-loaded, slotless model. The thick lines are the dispersion curves found by resonator measurements for accelerating and deflecting modes. The thin lines represent the dispersion curves for the limiting case with vanishing periodic loading of the structure,
that is, for a circular waveguide without disks. These curves are labelled with the wavetype names of the unloaded circular waveguide. Comparison with the loaded structure sometimes shows good agreement for the 0-mode. We may therefore consider the respective periodic wave as an E or H-like wave. Thus we may classify as follows:

accelerating wave-types

1. $E_{01}$-like
2. $E_{02}$-like coupled to $E_{01}$, lower part
3. as before, upper part,
4. $E_{03}$-like, coupled to $E_{02}$
5. $E_{03}$-like

deflecting wave-types

1. $E_{11}$-like, hybrid
2. $E_{12}$-like, coupled to $E_{11}$, hybrid, lower part
3. as before, upper part

Coupling between different wave-types is possible, if the circumferential \( \phi \) dependence is the same, and the branches of the hollow waveguide dispersion curves cross each other. Hybrid waves are six component waves (with both $E_z$ and $H_z$) in our case all $\phi$-dependent waves are hybrid. In Fig. 8 the line $v = c$ is also introduced. Crossing points with the dispersion curves yield the frequencies and modes for synchronism with particles of velocity $\approx c$. The first back-folded part of the $v = c$ line corresponds to $\pi < \beta_0 < 2\pi$, that is to synchronism with the -1 space harmonic. For the 1st accelerating and 1st deflecting passband $R/Q$ values are given for modes near synchronism for the fullscale structure. It turns out that $R/Q_{||} = 1345 \ \Omega/m$ for the accelerating $\pi/2$ mode. This value can be maintained also for cut slots, whilst the $R/Q_{\perp} = 794 \ \Omega/m$ can be decreased.

2.1.2 Model with $65^0$ slots, inline and rotated from cell to cell

Increasing the slot length step by step from $15^0$ upwards finally led to the $65^0$ slot model. Comparison of the lower passbands in Fig. 9 for the slots inline with those in Fig. 8 gives:
1. the first accelerating forward wave of the disk-loaded structure has been changed into a backward wave.

2. the two different polarizations of the first deflecting wave yield different passbands. Here the first polarization is excited with a probe as in Fig. 4b, the second as in Fig. 4a.

3. the second accelerating passband turns out to be strongly dependent on the slot length. We may call this wave the "slot wave", because it corresponds to a slot resonance.

4. because of the slots a new second deflecting passband has appeared. It can be considered as \( H_{11} \)-like. The first polarization is strongly affected by the slot length. The second polarization is practically unaffected by the slots; as in the case of no slots, this passband is so high in frequency that it is outside the range of our plot.

Changing over to the model with the slots rotated by \( 90^\circ \) from cell to cell leads to Fig. 10. The most important differences with the inline case are the new stopbands at \( \beta_0 L = \pi/2 \) for the deflecting waves (as \( \cos \phi \) or \( \sin \phi \)). As in Fig. 8 the \( v = c \) line is drawn, yielding the appropriate \( R/Q \) values for the full-scale structure at synchronism. Fig. 11 shows an enlarged plot of the region of synchronism for frequencies between 2 and 3 GHz. Evidently the increased reduction of \( R/Q_{1} \) has, as a counterpart, the appearance of two synchronous waves.

2.2 Influence of the slot length on different passbands

2.2.1 The first accelerating passband

Stepwise lengthening of the slots gives rise to a magnetic coupling which opposes the electric coupling through the irises for the first accelerating wave. This leads first to a decrease of overall coupling, and for slots with \( \alpha > 45^\circ \) to a dominant magnetic coupling and hence backward wave character.

Figs. 12 and 13 show the entire bands for different \( \alpha \), for the inline and the rotated slot case. For \( \alpha = 45^\circ \) in the rotated slot model, some resonances are strongly mixed with others, because of the nearly vanishing overall coupling. Therefore, only part of the passband could be plotted. Fig. 14 gives evidence of the quite insignificant influence of \( \alpha \) on the \( R/Q_{\parallel} \) values. For the \( \pi/2 \) mode especially, variations of \( R/Q_{\parallel} \) are rather small and may be due more to measurement errors or structure imperfections than to true slot dependence.
2.2.2 Slot passbands

With increasing slot angle $\alpha$ a new accelerating passband is found, with overall decreasing frequency (Fig. 15). Comparison of these frequencies with the resonances in a line short-circuited at both ends, of length $\ell_{\text{eff}}$ leads to the conclusion that this passband is due to a wave with resonant slots, that is $k \ell_{\text{eff}} \approx \pi$, see Fig. 16. Further increase of length would give rise to higher slot waves with

$$k \ell_{\text{eff}} \approx n\pi, \; n \text{ integer}.$$  

It should be noticed that all slot waves with $n$ odd are accelerating, whilst those with even $n$ are deflecting. This is easily shown by applying symmetry relations. Obviously the $\pi/2$ mode of the inline and the rotated slot case have the same frequency, see Fig. 15. This is generally true for any wave, since $\pi/2$ phase shift per cell means that in a standing wave only every second cell is excited, and hence "seen" by the wave.

2.2.3 The first deflecting passbands

As already mentioned in chapters 1.3.1 and 2.1.2 for the "inline" case, the slots have important influence only for the first polarization (Fig. 4b) on a deflecting wave. For simplicity sake, Fig. 17 is limited to the first polarization of the inline case and to the lower passband of the rotated slot case. As stated already, the $\beta_0L = \pi/2$ - frequencies of both cases are the same. Because of the double electrical period in the rotated slot case, this frequency is the lower end of a stop-band whose upper end is given by the $\pi/2$ frequency of the upper passband (see Fig. 5). Obviously the coupling through rotated slots is slightly lower than through inline slots.

The transverse $R/Q_\perp$ values for some modes in the full-scale structure are plotted in Fig. 18 and Fig. 19. In the "inline" case a significant dependence on the slot length appears only for the 1st polarization. Most interesting is the result in Fig. 19. In the rotated slot case the $R/Q_\perp$ in the lower passband is obviously decreasing with increasing $\alpha$, while the $R/Q_\perp$ in the upper passband is increasing.
As apparent from Fig. 11, for $\alpha = 65^0$ the synchronous modes of the lower and upper $E_{11}$-like passband are $\beta_0 L \approx 6\pi/8$ and $\beta_0 L \approx 7\pi/8$, respectively. The behaviour of the curves 2, 3 and 4, 5 in Fig. 19 as a function of $\alpha$ shows that the decrease of $R/Q_L$ up to $\alpha = 60^0$ in the lower $E_{11}$ passband is counter-balanced by a rapid increase of the $R/Q_L$ in the upper passband.

Fig. 20 shows the $R/Q_\parallel$ and $R/Q_L$-values of the two other synchronous modes (Fig. 11), the accelerating slot mode and the deflecting $H_{11}$-like mode (both of which only appear due to the slots). The $R/Q$ values of both modes increase rapidly for $\alpha > 60^0$.

From both Figs. 19 and 20 it seems that there is no advantage in increasing $\alpha$ beyond $65^0$.

The $R/Q_L$-value at synchronism ($E_{11}$-mode) can at best be decreased by a factor 2, with the disadvantage that two more synchronous modes with comparable $R/Q$-values are created.

Since the fields in the slots give rise to additional losses, the quality factor $Q$ is strongly affected by increasing $\alpha$, as shown in Fig. 21. A low $Q$ and a low $R/Q$-value both help in decreasing the deflecting influence on the particles.
REFERENCES


Fig. 1 - The slotted iris structure

a) the iris and the cut slots
b) cross-section of a cell
c) cross-section of an end half cell
d) details of the iris, inner hole
Fig. 2: The set-up for measuring the transmission factor (phase) through a resonator model. To stabilize the frequency, a special phase lock system is used. The measurement is controlled by a desk-top computer. A link to a somewhat faster computer is used for such time-consuming calculations as the Fast Fourier Transformation.
Fig. 3: The field plots of the longitudinal electric field $E_z$ on the axis of an 8-cell resonator (slotted iris structure as in Fig. 1 with $\alpha = 65^\circ$, rotated by $90^\circ$)

a) for $f = 1655.84$ MHz, 0-mode

b) for $f = 1587.407$ MHz, $\pi/2$ mode

(the appropriate zeros have to be chosen first)
**Fig. 4**: Symmetry planes in the slotted iris structure.

- P position of the exciting and receiving probe.
  
  a) symmetry plane on the $y$-axis, cutting the slots in half.
  
  b) symmetry plane on the $x$-axis.
Fig. 5: The dispersion diagram of an $E_{11}$-like wave.

a) in a disk-loaded waveguide without slots, geometrical and electrical period length $L$

b) in a disk-loaded waveguide with vanishing slots rotated by $90^\circ$ from cell to cell, electrical and geometrical period length $2L$.

c) as b) with longer slots.
Fig. 6: The cross-section of the resonator structure, with rotated slots, in the plane of exciting probe and perturbing needle.

a) with the shorting endplate at $z = 0$
b) with the shorting endplate at $z = -L$
Fig. 7: Standing wave longitudinal $E_x$-field 1 cm off axis in an 8-cell resonator slotted iris, with $\alpha = 65^\circ$, rotated by 90$^\circ$ from cell to cell.

a) $E_{11}$-like mode, higher passband, probes and needle as in Fig. 6a).

b) $E_{11}$-like mode, higher passband, probes and needle as in Fig. 6b).

c) $H_{11}$-like mode, lower passband, probes and needle as in Fig. 6a).

d) $H_{11}$-like mode, lower passband, probes and needle as in Fig. 6b).
Fig. 8: Dispersion diagram of the first 5 accelerating and 3 deflecting waves for the disk-loaded structure without slots (model 1:2)
Dimensions (see Fig. 1):

a = 22.5 mm, b = 70 mm, t = 5 mm, s = 2 mm, L = 46.75 mm

The labels in the first two passbands give the ratio R/Q of the corresponding space harmonics, in Ω/m.
Fig. 9: Dispersion diagram of the first four accelerating and three deflecting waves for the disk-loaded structure with slots in line ($\alpha = 65^\circ$), dimensions as in Fig. 8, $r_1 = 50$ mm, $r_2 = 60$ mm, $p = 5$ mm. The first polarization corresponds to excitation as in Fig. 4b; the second as in Fig. 4a.
Fig. 10: Dispersion diagram for the disk-loaded structure with slots $(\alpha = 65^\circ)$ rotated from cell to cell by $90^\circ$. Dimensions as for Fig. 9.
Fig. 11: Enlarged plot for frequencies between 2 and 3 GHz for the structure of Fig. 10.
Fig. 12: Dispersion diagram of the first accelerating passband of a disk-loaded structure with slots inline for different slot length. Dimensions as for Fig. 9.
Fig. 13: Dispersion diagram of the first accelerating passband of a disk-loaded structure with slots rotated by 90° from cell to cell. Dimensions as for Fig. 9.
Fig. 14: The R/Ω values for the fullscale model of the first accelerating passband in the structure with slots in-line or rotated as in Figs. 12 and 13, for different α over the whole passband.
Fig. 15: Dispersion diagrams of the slot passbands for the "inline" and "rotated" case (as in Figs. 12 and 13) for different $\alpha$.

Dimensions as for Fig. 9.
Fig. 16: Comparison between the resonant frequencies of different slot modes in the "rotated" case, and of a resonance on a line with slot length \( l_{\text{eff}} = \alpha \cdot (r_2 - r_1)/2 \).

Dimensions as for Fig. 9.

The points are given for \( \alpha = 30^\circ, 45^\circ, 60^\circ, 65^\circ \); they are connected by straight lines.
Fig. 17: Dispersion diagram of the first deflecting passband in the iris-loaded structure with slots in-line or rotated (as in Figs. 9 and 10), for different $\alpha$. For "inline" only the first polarization (Fig. 4b) is plotted.

Dimensions as for Fig. 9.
Fig. 18: The \( R/Q \) values for the first deflecting passband, both polarizations, for the slots in-line (as in Fig. 9); the 1st polarization corresponds to excitation as in Fig. 4b, the 2nd as in Fig. 4a.

Data for the full-scale structure.
Fig. 19: The R/O₄ values for modes near synchronism of the first deflecting passband as a function of α, for the slots rotated by 90° from cell to cell. Data for the full-scale structure.
Fig. 20: The R/Q values for modes near synchronism of the accelerating slot wave and the $H_{11}$-like deflecting wave (lower passband) as a function of $a$, for the slots rotated by 90° from cell to cell.

Data for the full-scale structure.
Fig. 21: The quality factor Q of the $6\pi/8$ mode of the $E_{11}$-like passband, for the slots inline and rotated. For the polarizations, see Fig. 18.

Dimensions as for Fig. 9 (half-scale, brass model).