The Beam-Beam Effect
In the SPS

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1. Introduction

The SPS was operated for the first time as a proton-antiproton storage ring in the latter part of 1981. Since it is the first hadron storage ring to operate with bunched beams and head-on collisions there has been considerable speculation as to the importance of the beam-beam effect which influences so strongly the maximum luminosity achievable in electron-positron storage rings. Extrapolation of electron data to very long damping times predicted a very low upper limit of the maximum beam-beam tune shift achievable. On the other hand, simulation experiments performed in the ISR 1) and the SPS 2) as well as considerations based on classical nonlinear resonance theory 3) suggested that a linear beam-beam tune shift of $\delta = 3 \times 10^{-3}$ per intersection should be achievable as long as the working point could be carefully controlled to avoid resonances up to at least 10th order.

During 1981, the collider was operated with proton bunches with intensity and emittance very close to the design values but with anti-proton bunch intensity normally a factor of 20-50 below the nominal value. In this 'weak-strong' situation, the antiproton bunches experienced the nominal beam-beam tune shift whilst their effect on the proton bunches was negligible. Early experiments showed that the antiproton bunch lifetime was considerably lower than that of the protons. Subsequent measurements showed conclusively that this was due to the beam-beam effect. It was found that a reasonable antiproton lifetime could only be achieved when the tenth order sum resonances were avoided.

In electron machines, considerable progress has been made in understanding the beam-beam phenomenon by numerical simulation. However, in proton machines, this approach is much more limited due to the very long time scale of interest. Consequently, in this paper, we develop a simplified approach to the problem, which is based on the classical theory of one-dimensional nonlinear resonances 6). Using this theory a 'beam-beam' limit for stochastic behaviour in a weak-strong situation is derived. In the stochastic régime the phenomenon can be considered as a diffusive process with a diffusion coefficient which is strongly amplitude dependent.
In the first part of this paper the operation of the SPS as a hadron collider is very briefly sketched. In the second part some of the experimental results obtained during the commissioning period are described. A more detailed report covering this period is available 4). Finally the theory is developed and compared with the experimental results.

2. The SPS as a p-\(\bar{p}\) Collider

The SPS normally accelerates protons between 10 GeV/c and 400 GeV/c at harmonic number 4620, corresponding to an RF frequency of around 200 MHz. Operation in p-\(\bar{p}\) mode requires that two counter-rotating beams of protons and antiprotons, each consisting of only 6 bunches of 1011 particles are accelerated and stored at 270 GeV/c for times of the order of 24 hours. The detailed procedure for producing these beams is described elsewhere 5).

The two beams are injected on a magnetic flat bottom at 26 GeV/c in order to avoid instabilities encountered with such dense bunches during transition crossing 7) (\(\gamma_T = 23.2\)). If the low-beta insertions are in operation, they are detuned from their storage values of \(\beta^*_H = 1.5\text{m}\), \(\beta^*_V = 0.75\text{m}\), to \(\beta^*_H = 7\text{m}\), \(\beta^*_V = 3.5\text{m}\) in order to liberate injection aperture and to facilitate chromaticity correction at low energy. The two beams are captured by separate RF systems, each consisting of two 200 MHz travelling wave cavities. Once the injection is completed the magnetic field rises to the 270 GeV/c level in approximately 5 seconds. The \(\beta^*_i's\) are squeezed to their final values during the first second of the flat top and the closed orbit in the insertion regions is dynamically corrected using special pulsed dipoles. A detailed report on the design and operation of the low beta insertions and chromaticity correction is in preparation 8).

During the commissioning period of the collider covered in this report, initially only a single proton and antiproton bunch were stored. Later, when the two main experimental detectors became available, which are situated 1/6 of the machine circumference apart, two proton bunches were stored together with a single antiproton bunch. This is the minimum requirement to obtain simultaneously collisions in the two experimental intersections. The scenario described above remains a goal for the future.
3. **Experimental Results**

Earlier experiments with protons\(^2\) had revealed the importance of phase noise in the radio frequency system in limiting the lifetime of bunched beams. After a careful redesign of the low-level RF system, very long proton bunch lifetime was achieved (> 100 hours) and the lifetime was found to be almost independent of the machine tune as long as nonlinear resonances lower than 6th order are avoided. However, when a weak antiproton bunch was injected together with a proton bunch, the proton lifetime remained essentially unchanged whereas the antiproton lifetime was very low (~1 hour). Initially it was suspected that this bad lifetime was due to a high noise level in the \(\bar{p}\) RF system. To check this, an experiment was performed in which the proton beam was slowly debunched in order to progressively eliminate the resonance excitation whilst keeping the linear tune shift constant. The antiproton lifetime was immediately observed to rise to a very high value (> 20 hours), giving the first clear indication that the antiproton lifetime was beam-beam limited.

The next step was to measure the dependence of lifetime on tune without low-beta insertions in operation. Figure 1a) shows the tune diagram in the region for which the low-beta insertions have been optimized. All measurements up to now have been constrained to the small triangle bounded by the main diagonal and by the 3rd and 4th order resonances. Moving the working point parallel to the main diagonal gave a strong dependence of lifetime on tune shown in figure 1b). The strong effect of the 7th and 10th order resonance is somewhat surprising because, if the two beams collide exactly centred, the symmetry of the beam-beam force cannot drive the 7th order resonance. In addition, since these results were obtained for one proton against one antiproton bunch (with two intersections per revolution) and no low-beta insertions, the two-fold machine symmetry cannot drive the 10th order resonance \(10\nu = 267\). It was presumed that this resonance was driven by a slight asymmetry of the phase shift between intersections and if so, could only be weakly excited, but enough to have an appreciable effect on the lifetime.
Another somewhat surprising result was the catastrophic effect of moving the working point onto the main diagonal. The width of the linear coupling resonance, measured by kicking the beam in one plane and observing the coherent betatron oscillations in the other, was found to be $\Delta \nu = 0.013$. This was corrected using a set of zeroth harmonic skew quadrupoles but no improvement in the lifetime was observed. However, even with the linear coupling corrected, it is known that the nonlinear coupling resonance $2\nu_H - 2\nu_Y = 0$ is strongly excited in the SPS by a second order effect in the chromaticity correction sextupoles $^9$.

Another effect which was clearly observed was the fast diffusion of antiprotons at large amplitude. Figure 2 shows the result of an experiment where the Schottky electrodes (used for measuring the tune) were placed close to the beam defining an acceptance of $1\text{mm.mrad}$ long enough for an equilibrium lifetime to be reached. Then the plates were retracted to $2.5\text{mm.mrad}$ and initially the lifetime rose rapidly until the large emittance particles struck the new aperture limitation. This took of order of a few minutes with the beam on the $10^{th}$ order resonance which should only be very weakly excited anyway because of the machine symmetry.

The best lifetime obtained with the normal machine lattice was 10-20 hours with the working point VI of figure 1.

The next step to increased luminosity was to switch on the low-beta insertions. As this would destroy completely the machine symmetry, all azimuthal harmonics would be present and it was expected that the lifetime would be considerably worse. However, it was found that the main diagonal was much less destructive so the working point could be pushed further away from the $10^{th}$ order sum resonances (point VII) with the result that the lifetime improved to 20-40 hours. With the low-beta insertions the horizontal and vertical tunes are split by an integer ($Q_H = 27.675$, $Q_Y = 26.685$), and this should reduce the strength of the coupling resonances. There is no doubt that these difference resonances, which one would normally expect to be stable, do have an important influence on the beam lifetime.
Figure 3 summarizes the results of a long storage (20 hours) of two proton bunches against one antiproton bunch and the low-beta insertions at their injection value of $\beta_{H}^{*} = 7.5m$, $\beta_{V}^{*} = 3m$ at both experimental intersections. The two other intersections were at normal beta-values ($\beta_{H,V} \approx 50m$). The beam-beam tune shift experienced initially by the antiprotons was $2.5 \times 10^{-3}$ per intersection. Initially the antiprotons had an emittance about 50% larger than the protons and their lifetime was low. This is a very unfavourable situation since the large emittance particles see the strong beam-beam resonances induced by the other beam and are rapidly lost. Consequently, the antiproton emittance shrinks during the first few hours of storage and the beam lifetime improves. During this time the background in the experimental detectors is very high. At the same time the proton emittance blows up linearly at a rate $d\varepsilon_{L}/dt = 1.67 \times 10^{-3}$ mm.mrad.h$^{-1}$, which is compatible with a very reasonable value of the nitrogen equivalent gas pressure of $1.3 \times 10^{-9}$ torr. Consequently, the tune shift decreases almost linearly (the proton lifetime is very high so there is almost no loss). After about 7 hours when the beam-beam tune shift is reduced to $\sim 1.8 \times 10^{-3}$ and the proton and antiproton emittances are almost equal the antiproton beam also blows up at a rate compatible with gas scattering, and the lifetime is reasonably high ($\sim 40-50$h).

Figures 4 and 5 show the transverse beam profiles of the three bunches in the horizontal and vertical planes measured with the fast wire scanner$^{11}$ which was used to accumulate the data in the previous curves. At the beginning of storage (figure 4) the antiproton emittance is larger than the proton emittance. All three bunches are 'round' ($\varepsilon_{H} = \varepsilon_{V}$) probably due to crossing the main diagonal during acceleration. Figure 6 shows the profiles after 20 hours of storage. The proton bunches remain identical and quite Gaussian in shape (the boxes are always normalised to $\pm 40$). However, the antiproton bunch develops a fairly pronounced discontinuity in its distribution at an amplitude corresponding to $1.6\sigma$ of the proton distribution, indicating a dilution of the large amplitude phase space.
4. Theory

It is generally agreed that the beam-beam effect is intrinsically a statistical phenomenon. It has been shown that an approach based on considering the beam-beam interaction as a series of random kicks\textsuperscript{12}) gives results in qualitative agreement with experiments in electron machines, although the underlying physical mechanism was not discussed. Another closely related theory\textsuperscript{13}) assumes that the random diffusion could be due to external noise.

It is shown below that the classical theory of repeated resonance crossing\textsuperscript{6}) due to coherent tune modulation contains all the ingredients necessary to produce stochastic behaviour for a large enough beam beam tune shift. A simple physical picture of the mechanism for achieving stochastic behaviour is discussed which is clearly related to the resonance overlap criterion\textsuperscript{14}).

It is well known that in a static situation, particle orbits are always stable in the presence of the beam-beam force due to the relative strengths of resonance excitation and nonlinear detuning. This has been verified by computer simulation \textsuperscript{13}). In order to explain beam blowup, time variation of some parameter is generally assumed. One source of modulation which is invariably present for bunched beams is energy modulation, which gives rise to tune modulation due to non-zero chromaticity, variation of arrival time or non-zero dispersion at the interaction point.

In the static case, beam-beam resonances give rise to complicated phase-space trajectories. An \( n \)th order resonance gives rise to a ring of stable islands at an amplitude at which the particles tune is equal to the resonance tune. In the presence of coherent tune modulation the original resonance splits into sidebands at \( \nu = p/n \pm l\nu_s/n \) where \( p \) and \( l \) are integers. Each of these resonances gives rise to a ring of islands at an amplitude corresponding to the sideband tune, but with a resonance width which is weaker by the factor \( J_l^\nu_s^\nu \) where \( \nu \) is the tune modulation amplitude. Now as long as these islands are well separated the phase space trajectories are still closed so the resonance does not cause beam blowup \textsuperscript{14}).
The sideband picture is valid if the phase space is observed on a unit time scale of the synchrotron period. On a much finer time scale the stable island structure of the static picture oscillates in amplitude, and it has been pointed out \(^{6,15}\) that if the modulation frequency is low enough, particles can remain adiabatically trapped in the islands and more or less rapidly transported to large amplitude where they can be lost to the vacuum chamber wall or can leave the island because the adiabaticity condition is not satisfied. This phenomenon has been examined in detail for beam-beam resonances \(^{15}\) and is generally agreed to give a reasonably good description in the régime of very slow tune change.

4.1 Emittance growth due to fast random crossings

The other régime, which we wish to develop here is the limit of fast random crossings \(^{6}\). First we outline the theory and afterwards we will discuss in more detail the criterion for 'randomness' and the range of validity of the theory.

We consider only a weak-strong case produced by the interaction of a round Gaussian beam of \(N\) particles per unit length with a test particle. The angular deflection of the particle on traversing the strong beam is

\[
\Delta x' = \frac{N e^{(1 + \beta^2)} L}{2\pi r} \frac{\varepsilon_0 B_0}{\beta c \rho} \left( 1 - e^{-r^2/2\sigma^2} \right)
\]

We assume that the interaction is localised about a small distance \(L\). The linear tune shift experienced by the test particle is then

\[
\xi = \frac{\beta^*}{8\pi} \frac{L}{B_0^2} \frac{N e}{\pi \varepsilon_0 \beta c \sigma^2}
\]
The resonant invariant for the particle motion in a linear machine near an \( n^{th} \) order resonance with the nonlinear perturbation 1) is 15)

\[
c = (\nu - p/n) \alpha + M \xi U(\alpha) + \overline{M} \xi V_n(\alpha) \cos n\psi
\]  (3)

where \( M \) is the number of interactions per revolution and \( \overline{M} \) is the coefficient of the \( p^{th} \) azimuthal component of the beam-beam force. \( \alpha \) is the normalised 'emittance' of a particle \( \alpha = r^2/2\sigma^2 \). The functions \( U \) and \( V_n \) are given below.

The amplitude and phase equations are then

\[
\frac{d\alpha}{d\psi} = -\frac{3c}{3\psi} = n \overline{M} \xi V_n(\alpha) \sin n\psi
\]  (4a)

\[
\frac{d\psi}{d\theta} = \frac{3c}{3\alpha} = (\nu - p/n) + M \xi U'(\alpha) + \overline{M} \xi V_n'(\alpha) \cos n\psi
\]  (4b)

The function \( \xi dU(\alpha)/d\alpha \) is just the nonlinear detuning function. \( V_n'(\alpha) \) is identified as the equivalent of the 'resonance width'. The functions \( U' \) and \( V_n' \) 16) corresponding to the beam-beam kick 1) are

\[
U'(\alpha) = \frac{2}{\alpha} \left[ 1 - e^{-\alpha/2} I_{0}(\alpha/2) \right]
\]  (5a)

\[
V_n'(\alpha) = (-1)^n \frac{n}{2} + 1 \cdot \frac{4}{\alpha} e^{-\alpha/2} I_{n/2}(\alpha/2)
\]  (5b)

Equivalent series forms of these functions have been derived by
decomposing the beam-beam force in a multipole expansion and isolating all slowly varying terms in the phase equation 16), giving

\[
U'(\alpha) = \sum_{k=1}^{\infty} \frac{(-1)^k (2k)! \alpha^{k-1}}{2^{2k-1} (k!)^3} \tag{6a}
\]

\[
V'_n (\alpha) = \sum_{k=\frac{n}{2}}^{\infty} \frac{(-1)^k (2k)! \alpha^{k-1}}{2^{2k-2} k! (k + \frac{n}{2})! (k - \frac{n}{2})!} \tag{6b}
\]

These forms of the function \(U'\) and \(V'\) are convenient because their derivatives and integrals are trivial. We also need the function \(d^2 U(\alpha)/d\alpha^2 = U''(\alpha)\). The functions \(U'(\alpha), U''(\alpha), V(\alpha)\), are shown in graphical form in figures 6 - 8.

Following Schoch 6), we compute the change in emittance of a particle crossing the resonance by integrating equation 4a).

\[
\int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{V'_{n}(\alpha)} = \frac{\theta_2}{\theta_1} \int_{\alpha_1}^{\alpha_2} \sin \psi \, d\theta \tag{7}
\]

The procedure for performing the integral on the right hand side is well known 6). It gives

\[
\int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{V'_{n}(\alpha)} = n \bar{M} \xi \sqrt{\frac{2\pi}{n \text{d} \psi/\text{d}\theta}} = 2 \pi \bar{M} \xi \sqrt{\frac{n}{\Delta \nu_{\text{rev}}}} \tag{8}
\]
where \( \alpha_1 \) is the initial normalized emittance, \( \alpha_2 \) is the emittance after crossing the resonance and \( \Delta \nu_{\text{rev}} \) is the tune change per revolution. We assume that we are dealing with high-order resonances so that the emittance change per crossing is small. The change in radius \( r = \xi^2 \) in the phase plane due to a single crossing is then

\[
\Delta r = \pi M \xi V_n(\alpha) \sqrt{\frac{\varepsilon_0 n}{2 \alpha \Delta \nu_{\text{rev}}}} \tag{9}
\]

where \( \varepsilon_0 \) is the strong beam emittance. We now consider repeated crossings as a random walk process (the validity of this assumption will be discussed later), so that the change in a particle's amplitude after \( N \) crossings is given by

\[
\overline{\Delta r^2} = \frac{N}{2} \Delta r^2 \tag{10}
\]

We assume a tune modulation of the form

\[
\nu = \nu_0 + \hat{\nu} \cos \nu_S \theta
\]

where \( \nu_S \) is the synchrotron frequency and \( \hat{\nu} \) the maximum modulation amplitude. Then the mean value of the tune change per revolution is \( \Delta \nu_{\text{rev}} = 4 \hat{\nu} \nu_S \). Putting \( N = 2 \nu_S f_r t \), where \( f_r \) is the revolution frequency we get

\[
\overline{\Delta r^2} = \left[ \frac{\pi^2 M^2 \xi^2 V_n^2(\alpha) \varepsilon_0 n f_r}{8 \alpha} \right] t \tag{11}
\]
which is similar to the classical equation for Brownian motion, where the mean square amplitude varies linearly with time

$$\Delta r^2 = 4Dt$$

$D$ is a diffusion coefficient, which in our case is a very strong function of amplitude

$$D(\alpha) = \left[ \frac{\pi^2 M^2 \xi^2 V^2(\alpha) \varepsilon_{o n f r}}{32 \hat{\nu} \alpha} \right]$$ (12)

We see that $D$ is proportional to $\xi^2$ and is energy independent, as is required for the fit to electron data\textsuperscript{12}). It is also independent of both the synchrotron frequency and the nonlinear determining $U'(\alpha)$ although we will see below that both these parameters play a vital rôle in governing the stochastic nature of the process.

We would like to compare this diffusion rate with that induced by the other major mechanism for diffusive beam growth - Coulomb scattering due to the residual gas. The growth rate of the mean square amplitude is 17)

$$\frac{d\Delta r^2}{dt} = \frac{0.32}{\beta^3 \gamma^2} P$$ (13)

where $P$ is the nitrogen pressure in torr. The diffusion coefficient is then

$$D = \frac{0.08 P \beta}{\beta^3 \gamma^2}$$ (14)
where \( \beta = R/v \). The growth rate of the beam emittance, defined as the phase space area inside \( \pm 2\sigma \) of the projected distribution is approximately

\[
\frac{1}{\pi} \frac{dc}{dt} = 8D
\]  

(15)

From the experimental results shown in figure 4, we obtain a measured growth rate \( dc/dt = 1.67 \times 10^{-3} \) mm.mrad.h\(^{-1} \) from which we obtain a diffusion coefficient \( D = 5.8 \times 10^{-14} \) m.s\(^{-1} \).

In figure 9 the dependence of the beam-beam diffusion coefficient (eq. 12) is plotted as a function of amplitude together with the amplitude independent coefficient (eq. 15) for gas scattering. A glance at the figure explains at least qualitatively the experimental results shown in figure 3. The blowup of the central core of the antiproton beam is governed by gas scattering. The blowup due to the beam-beam effect is very much lower, which is intuitively obvious because these particles only see the linear part of the beam-beam force. However, large emittance particles are subjected to a diffusion which is very much faster than gas. For example, on the 10\(^{th}\) order resonance, beam-beam diffusion is dominant above about 1.7\(\sigma\) of the proton distribution, and results in a dilution of the large amplitude phase space for the antiprotons.

As the beam-beam tune shift reduces due to the natural blowup of the proton beam these curves move to the right until finally all particles are limited by gas rather than by the beam-beam effect.

The curves also illustrate the importance of keeping the weak beam
emittance as small as possible in order to minimise the beam-beam diffusion. Up to now the antiproton emittance in the SPS has always been found to be larger than the proton emittance and a blowup of as much as factor 10 has been measured between the antiproton accumulator and the SPS stored. The reason is not yet completely clear but will be investigated in future experiments.

Equation 8) gives time for a particle's emittance to grow from $\alpha_1$ to $\alpha_2$:

$$ t = \frac{\hat{v}}{n \xi} \left[ \frac{1}{\frac{1}{\pi} \frac{\hat{N}}{z} \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{V_n(\alpha)} } \right]^2 $$

(16)

When this equation is used to compute the blowup from $1\pi$ to $2.5\pi$ mm.rad, as in the case of the experiment with the Schottky plates, we find times of the order of seconds and not minutes as measured. However, it has already been stated that the experiment was performed with a symmetric machine where the azimuthal harmonic driving the 10th order resonance could only be excited by machine imperfections, so should be weak.

4.2 The stochastic limit

There remains the question of the range of the assumption that the successive traversals of the resonance are made with random phase. The following simple argument will be used to derive a criterion for randomness. It will be shown that this is identical, apart from a numerical factor, to the criterion of overlap of synchrotron sidebands$^{14}$).

A single crossing produces a change of emittance which is statistically distributed with a maximum value

$$ \Delta \alpha = \pi \hat{N} \xi \sqrt{\frac{n}{\hat{v}} V^2} V_n(\alpha) $$

(17)
This, in turn, produces a tune shift

\[ \Delta \nu = M \xi \frac{d^2 \nu}{d \alpha^2} \Delta \alpha = \pi M \bar{M} \xi^2 \sqrt{\frac{n}{\nu_s}} V(n) U''(n) (18) \]

This tune shift produces a betatron phase shift \( \Delta \phi \) which, after half a synchrotron period amounts to

\[ \Delta \phi = \frac{\pi \Delta \nu}{\nu_s} (19) \]

and if this phase shift is larger than \( \pi/n \), a particle which was initially at a stable fixed point on resonance suddenly finds itself on the unstable fixed point on the next crossing, so clearly cannot move on a closed trajectory. An exactly identical condition to the above is to demand that the tune shift (eq. 18) is greater than the spacing \( \nu_s/n \) between synchrotron sidebands. Our condition for randomness or 'stochasticity' is then

\[ \pi M \bar{M} \xi^2 \sqrt{\frac{1}{\nu}} V(n) U''(n) = \left( \frac{\nu_s}{n} \right)^{3/2} (20a) \]

or

\[ \xi_{\text{max}} < \frac{\nu^{1/4}}{(\pi M \bar{M} V(n) U''(n))^{1/2}} \left( \frac{\nu_s}{n} \right)^{3/4} (20b) \]

This criterion can be used to define an amplitude-dependent 'beam-beam limit \( \xi_{\text{max}} \). Alternatively it can be used to define, for a fixed
beam-beam tune shift, the minimum value of the tune modulation frequency for which the motion is stable.

A stochasticity limit can be derived in a more rigorous way by applying the Courant-Chirikov \cite{14} criterion for the overlap of synchrotron sidebands.

The spacing between synchrotron-island emittance is

\[
\Delta \alpha = \frac{\nu_s}{M \xi n U''(\alpha)}
\]  

(21)

The island width for the \(l\)th sideband is

\[
\Delta \alpha = 4 \sqrt{\frac{M}{\bar{M}}} \frac{V_n(\alpha)}{U''(\alpha)} J_l \left( \frac{\nu}{\nu_s} \right)
\]  

(22)

Expanding the Bessel function for large argument

\[
J_l \left( \frac{\nu}{\nu_s} \right) \approx \sqrt{\frac{\nu_s}{\pi n \nu}}
\]

and equating (21) and (22) we get

\[
16 \ M \bar{M} \xi^2 \ V_n(\alpha)U''(\alpha) \ \sqrt{\frac{1}{\pi \nu}} = \left( \frac{\nu_s}{n \nu} \right)^{3/2}
\]  

(23)

Using (23) as a criterion for the maximum linear beam-beam tune shift gives
\[ \xi_{\text{max}} = \frac{1}{4} \frac{(\pi \hat{v})^{1/4}}{\sqrt{M \tilde{M} V_n(\alpha)U''(\alpha)}} \left( \frac{\nu_s}{n} \right)^{3/4} \]  

(24)

which differs only by a factor \((\pi^{3/4}/4)\) from the previous expression (20b).

Assuming \(\hat{v} = 0.0025, \nu_s = 0.0045, M = 6, \tilde{M} = \sqrt{\alpha}\) table 1 shows a comparison between 8th and 10th order resonances. The parameter \(x/\sigma\) is used instead of the emittance \(\alpha\), where \(\alpha = x^2/2\sigma^2\).

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<th>(U''(x/\sigma))</th>
<th>(V_8(x/\sigma))</th>
<th>(\xi_{\text{max}})</th>
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</table>

**Table 1**

We see that this theory predicts stochastic behaviour of large amplitude particles at values of the linear tune shift not far from that experimentally achieved. It is fully appreciated that the approximations involved considerably over-simplify the real world. In particular, an extension to two dimensions is necessary to try to explain the influence of the coupling resonances.

One important feature of the stochasticity criterion is the dependence on the tune modulation frequency. In the SPS, as well as the synchrotron
frequency at around 200 Hz, current ripple of the quadrupole power supplies in the range 1 - 2 Hz has been observed and this should indeed play an important part in the randomisation process, since it is well known that the effective resonance width is increased by magnetic field ripple. Experiments are in progress using a nonlinear lens with protons instead of rarely available antiprotons in order to investigate in more details the importance of these parameters.

5. Conclusions

The SPS has operated in a weak-strong regime where the linear beam-beam tune shift of the weak antiproton beam has been close to the design value of $3 \times 10^{-3}$ per intersection, and with up to 4 intersections. Strong beam-beam effects have been observed for the large emittance particles in the antiproton beam. Reasonable lifetime is only achieved when the working point is clear of $10^{th}$ order sum resonances.

A simple theory has been proposed which is based on multiple random crossings of isolated one-dimensional nonlinear resonances. The condition for randomisation has been given in terms of a beam-beam limit which depends on particle emittance. The fast diffusion of large emittance particles can be understood in this way. For the improvement of luminosity lifetime an obvious priority is to minimise the emittance blowup between the antiproton accumulator and the SPS.

* * *
Acknowledgements

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References

4. L. Evans, J. Gareyte, W. Scandale, L. Vos, CERN SPS/82-9 (DI-MST).
Fig. 1. a) Tune diagram with working points (hatched: antiproton tune spread) sum resonances are drawn up to order 13.

b) Lifetime versus tune (no low betas)
Fig. 3 Evolution of beam parameters during storage

- $\xi \times 10^3 \quad \pi \text{mm mrad}$
- $L_\bar{p}$: $\bar{p}$ lifetime
- $E_\bar{p}$: emittances
- $E_p$:
- $\xi$: beam-beam tune shift

The graph shows the evolution of beam parameters over time, with axes labeled as follows:

- Y-axis: $\xi \times 10^3 \quad \pi \text{mm mrad}$
- X-axis: Time into store (hours)
- $L_\bar{p}$ (dashed line)
- $E_\bar{p}$ (line)
- $E_p$ (line)
- $\xi$ (line)
Fig. 4 Horizontal (upper) and vertical (lower) profiles of two proton bunches and one antiproton bunch at the beginning of storage.
Fig. 5  Beam profiles of proton and antiproton bunches after 20 hours of storage.
Fig. 7
$D(x/\sigma)$

Fig. 9

Diffusion coefficient vs amplitude for beam-beam and Coulomb scattering