LONGITUDINAL ASPECTS
OF SLOW EXTRACTION

Combined notes on:
Stochastic and other means of rf acceleration to 'feed' the resonance, 'Empty bucket' stabilisation of the spill and General longitudinal strategy

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Example: \[ \omega = 2\pi \cdot 500 \, \text{Hz}, \quad \epsilon = 10^{-4}, \quad \phi = 3 \cdot 10^{-3} \]

Then for \( v_0 = \omega r \): \( T_S = 100 \, \text{ms} \)

**Duty Factor** \( F \)

\[
F = \frac{\langle \phi \rangle}{\langle \phi^3 \rangle} = \frac{1}{1 + \frac{1}{2} \left( \frac{\omega r}{v_0} \right)} \quad 0 < F < 1
\]

If \( \omega r = v_0 \Rightarrow F = 2/3 \)

A mechanical analogy:

**1st Analogy: Molecular Diffusion**

Remarks:

1. one has a molecular diffusion when the spatial distribution of \( p \) is not uniform.
2. diffusion is always towards the low concentration

\[ n = \# \text{ of } p/\text{m}^3 = \text{concentration} \]

\[ j = \text{current density} \]

\[ j = \# \text{ of } p \text{ transferred in one second} \]

\[ \text{over area} \left[ \text{m}^2 \right] \]

**NB:** if \( n = \text{constant} \Rightarrow j = 0 \)

\[ j = -D \frac{dn}{dx} \quad \text{(1)} \]

where \( D = \text{diffusion coefficient} \)
\[ \frac{\partial p}{\partial t} = \nabla \cdot \mathbf{j} \]

\[ \nabla \cdot \mathbf{j} = \frac{j}{\partial x} \]

**Input Flow:** \( j_{\text{in}} \)

**Output Flow:** \( j_{\text{out}} \)

**Accumulation Rate:**

\[ j_{\text{in}} - j_{\text{out}} = \left( j \right)' \frac{dx}{dx} \]

\[ = \frac{2}{\partial x} \frac{dx}{dx} \]

**Differential Equation:**

\[ \frac{\partial n}{\partial t} = \frac{2}{\partial x} \frac{\partial^2 n}{\partial x^2} \]

**Example 1:**

\[ t = 0 \]

\[ n = n_0 \]

\[ t = \infty \]

\[ n = n_f \]

**steady state**
Example #2

\[ t = 0 \]

\[ \frac{\partial n}{\partial x} \]

\[ \frac{\partial^2 n}{\partial x^2} \]

\[ t \gg 0 \]

... Back to steady state ...

\[ n = n(x, \pi) \]

i.e. the concentration is stationary

\[ \frac{\partial n}{\partial t} = 0 \Rightarrow \frac{\partial}{\partial x} \frac{\partial^2 n}{\partial x^2} = 0 \]

i.e. \( D \frac{\partial n}{\partial x} = \text{constant} = j = \text{constant flux} = \text{no accumulation} \)

\[ j = -D \frac{\partial n}{\partial x} = \frac{D n_0}{L} \]

\[ \text{example:} \]

[Diagram of a ventilator system with a tank and flow control]
Remarks:
How the steady state is reached?

**Case #1: OPEN PIPE**

![Diagram of open pipe with temperature profiles.]

**Case #2: CLOSED PIPE**

![Diagram of closed pipe with temperature profiles.]

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2nd Analogy: THERMAL DIFFUSION

That is a transfer of energy (or heat).

**Remarks:**
1. One has thermal diffusion when there is a temperature difference.
2. The direction is from HOT to COLD $(T > T_{cs})$.

As in molecular diffusion exchanging $v \rightarrow T$ or per unit area, the energy density current $J_e = \frac{\partial T}{\partial x}$.

And the diffusion Eq.

$$\frac{\partial T}{\partial t} = \frac{D}{\partial x^2} \frac{\partial T}{\partial x}$$

The steady state, that is, $\frac{\partial T}{\partial t} = 0$, means $\frac{\partial T}{\partial x} = -J_e = \text{constant}$.

For example:

- [Diagram of a heater with ice and temperature profile.]
- [Diagram of a thermometer with temperature reading.]
- [Diagram of a bed with thermal energy flux.]
PERTURBATION PROPAGATION

Given a perturbation like:

\[ n = n_0 \sin(wt) \]

we'll this perturbation propagates like a travelling wave like

\[ n(x,t) = n_0 + n_2 \sin(wt - kx) \]

\[ \frac{\partial n}{\partial t} = n_0 w \cos(wt - kx) \]

\[ \frac{\partial^2 n}{\partial x^2} = -k^2 n_0 \sin(wt - kx) \]

These do not satisfy:

\[ \frac{\partial n}{\partial t} = \frac{2}{D} \frac{\partial^2 n}{\partial x^2} \]

The answer is: \( n_0 \). BUT

\[ n = n_0 + n_2 \exp \left( -\frac{kx}{\sqrt{2D}} \right) \sin \left( wt - kx \right) \]

For

\[ k = \frac{2\pi}{\lambda} = \frac{\sqrt{w^2}}{2D} \]

\[ v = \Delta p/w = \sqrt{2D\omega} \]

\[ \Delta t \ll \Delta x \Rightarrow \text{he damping is small} \]

1) The group velocity: \( v_g = \frac{\partial w}{\partial k} = \sqrt{2D\omega} > v \)

BACK TO OUR ACCELERATOR

\[ n \left[ P/P_{\infty} \right] \rightarrow \psi \left[ P/P_{\infty} \right] = \text{proton density in space} \]

\[ i \left[ P/P_{\infty} \right] \rightarrow \phi \left[ P/P_{\infty} \exp (1/s) \right] \]

\[ \dot{E} = \frac{2}{D} \frac{\partial n}{\partial x} \]
Initial conditions

Remarks:
- For a p. in Brownian motion, the rms distance from the origin after a time t (where x0 = ar t0) is:
  \[ \langle x^2 \rangle = 2D t \]
- The rms energy gain given by a noise with bandwidth W (overlapping only one harmonic of the modulation frequency) and rms voltage Vm is
  \[ \frac{\Delta E}{\Delta E} = \frac{f_0}{W} \left( \frac{c V_m}{V_0} \right)^2 = \frac{1}{W} \left( \frac{c V_m}{V_0} \right)^2 \]
  from \[ D = \frac{1}{2} \int \frac{d (e P P)^2}{d \phi} \]
  and knowing that \[ \Delta P = \frac{1}{\beta \gamma} \Delta E \]
  we obtain
  \[ D = \frac{1}{2W} \left( \frac{V_m}{\text{Peak BW}} \right)^2 \]
  \[ F = \frac{1}{\frac{1}{2} \Delta P^2} = \frac{1}{\Delta P^2} \]
  \[ \Rightarrow D > \omega P \]
  \[ \text{or (Fig. 3)} \]
Fig. 2. Stochastic extraction in LEAR
Beam distribution during extraction

Sweep 10s
\[ P_A = 0.6 \, \text{W} \]

Sweep 100s
\[ P_A = 0.16 \, \text{W} \]

Fig. 3. First stochastic extraction in LEAR
Top trace: circulating beam current
Bottom: extracted flux
a) Phase jump debunching (16 GeV/c)

180° jump in unstable phase

leave the beam to

stretch

back on

stable phase

wait for

debunching
to take place

\[ T_d = \frac{2\pi}{k \cdot T_s} \]

1) The beam is 'pushed' to the resonance by decreasing the B field

\[ w_p \approx 2\pi s \cdot 10^{-4} \approx 0.03, \quad v_{w_p} = \frac{2\pi s}{2.6} \approx 0.01 \quad \Rightarrow F \approx 0.2 \]
Fig. 4: Empty bucket channeling

1) No RF
Servo loop current
Norm. = 100 mA/div.

2) RF on

3) RF on only during the 2nd half of extraction
(6th trace: losses in a n. 61)

NOISY BUCKETS

1) Moving, with a perturbation on the radial loop, the beam close to the extr. resonance
2) Shaking, with noise in the phase loop, the bunch to increase $\varepsilon_c$

Should provide a fair control of the spill

NB: - tails during Fc blow-up are welcome!
- the strong RF structure should not be a problem for medical applications
  - no dewaxing
  - no gap relays
  - feasible beam diagnostic (intensity, position,...)
SUMMARY

- **STOCHASTIC EXTRACTION**
  + good F for long spills (>> 1 sec.)
  + hardware simplicity
  + operational
    - moderate spill control
    - problems for short spill (< 1 sec.)

- **PHASE JUMP DEBUNCHING**
  + operationally simple & fast
  - distribution not very rectangular
  - hardware complicated

- **RESONANCE FEEDING WITH A SLOPE**
  + hardware simplicity (?)
    - poor F
    - poor spill control

- **EMPTY BUCKET CHANNELLING**
  + good for short spills (< 1 sec.)
    - hardware complicated
    - poor spill control

- **NOISY BUCKETS**
  + hardware simplicity
  + good spill control
  + no debunching
  + facile instrumentation
    - strong RF structure (may be acceptable ?)

- **UNSTARRING**
  + facile instrumentation
    - RF structure (may be acceptable ?)
    - hardware complicated
    - operationally
      - poor F

References
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