PHASE DISPLACEMENT
ACCELERATION

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Longitudinal Phase Space

- Particle motion described by:

\[ \phi = \frac{\omega_m h \eta}{B^4 E} \quad \eta = \frac{\delta T}{\delta p} \quad \dot{\epsilon} = \frac{\omega_m e V_{ee}}{2\pi} (\sin \phi - \sin \phi_s) \]

- For small amplitudes w.r.t. synchronous particle:

For \( \Omega^2 = \frac{eV_{ee} h \eta \cos \phi}{2\pi E B^3} \) positive - stable motion

\[ \delta \phi = a \delta E \quad \delta \epsilon = -b \delta \phi \]

\[ \Omega^2 = ab = \frac{eV_{ee} h \eta \cos \phi}{2\pi E B^3} \]

- For large amplitudes:

\[ \Delta \dot{\phi} = -\frac{\Omega^2}{\cos \phi_s} (\sin \phi - \sin \phi_s) = 0 \]

- See phase plane trajectories:

\( \Rightarrow \) Stationary bucket

\( \Rightarrow \) Moving bucket
Fig. 2.3 Stationary RF bucket above transition

Fig. 2.4 Accelerating RF bucket $\Gamma = 0.5$
Phase Displacement

- RF bucket moving through a coasting beam (stack) results in an overall displacement in the opposite direction:
  - Particles cannot enter bucket and move round to new energies.
    (Phase space density inside coasting beam cannot increase)
  - For bucket height << stack 'width' the net displacement is the bucket area.

- Illustrated in Figure and Simulation.

- Scattering effect - Dependent on $r$

- Motion complex, Solutions need computation.
  Usually analysed by computer simulations of particle motion.
Fig. 2.3. Stationary RF bucket above transition

\[ \Delta E = \frac{A \lambda}{2\pi} \]
Particle energy vs. Time

\[ \Gamma = 0.10 \]

Particle energy vs. Time

\[ \Gamma = 0.20 \text{ PhaseStep } = 0.31 \text{ } \text{MT2 } = 0.84 \]
Phase Displacement

\[ \gamma = 0.20 \]

Synchronous particle at centre of moving bucket.

Traversal of bucket through stack.
Factors Affecting Efficiency of Phase Displacement

- **Particle Scattering**
  
  For each sweep
  \[ \Delta E_{\text{sc}} = \Gamma \Delta \omega / 2\pi \quad \Delta E_{\text{d}} = \Gamma \]
  
  Result obtained from simulations, confirmed by experiment (ISR)

- **RF Noise**

  - Allows particles to enter bucket during sweep

  e.g. modulation near synchrotron frequency causes particles to spiral in and out of bucket

  - Results in:
    - Increased energy spread
    - Reduced energy displacement
    - Larger rms width increase per sweep

  - Cures:
    - Phaselock
    - Careful design of low level RF (reduce phase noise)
• Variation in Bucket Parameters

Assumption of constant bucket area during sweep not completely valid.

Change of real bucket area during the sweep will produce the same effects as RF noise.

Special case: CERN ISR, \( \gamma \gg \gamma_c \) and \( |\eta| << 1 \)

For the variation in area of a stationary bucket:

\[
\frac{\Delta A_n}{A_n} = \frac{\Delta E}{\sqrt{2}E}
\]

i.e. the area increases with decreasing bucket energy

For fixed rate frequency sweep the real value of \( \Gamma = \sin \phi \), changes:

\[
\frac{\Delta \Gamma}{\Gamma} = -\frac{\Delta E}{E}
\]

i.e. The real value of \( \Gamma \) increases, resulting in a decreasing bucket area.

The two effects are opposite and can be made to cancel out.

If \( \sigma(\Gamma) \) is the ratio of the area of a moving bucket to a stationary bucket, then for approximately constant bucket area through the sweep:

\[
\frac{\Delta A}{A_n} = \frac{\Delta A_n + \Delta \sigma(\Gamma)}{A_n} = 0
\]

This satisfied for \( \Gamma = 0.25 \)
Fig. 4.2 ISR stack density profiles before and after phase displacement acceleration (R933P, Ring 1, 1978)

Fig. 4.1 rms radial beam width $\sigma$ vs. momentum (ISR)

Ph. Disp Accn 1000p Gamma 0.1

Sweep num.
Conclusions

- Simple means of acceleration or displacement of coasting beam

- Some overall loss in phase space density inevitable (occurs mostly at start of process)

- Losses, blow up per sweep become stable after a number of sweeps

- Care needed in RF system design (Noise, program linearity..)

- Simulations play main role in analysis (Straightforward nowadays..)