GEOMETRICAL SCALING AT THE $p\bar{p}$ COLLIDER

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ABSTRACT

Using the idea of geometrical scaling, predictions are made for $p\bar{p}$ scattering at $\sqrt{s} = 540$ GeV and compared to the first data from the collider.

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As there is some, although admittedly weak, evidence that the total cross-sections for $p\bar{p}$ scattering rises as $\ln^2 s$ asymptotically one may ask oneself if not for non-forward directions the asymptotic regime is already reached at ISR and collider energies. If so, the theorem of Auberson, Kinoshita and Martin \textsuperscript{1}) tells us that if $\sigma_{\text{tot}} \sim \ln^2 s$ then the function

$$\phi(\tau) = \lim_{s \to \infty} \frac{F(s,\tau)}{F(s,0)}$$

($\tau = -2\sigma_{\text{tot}}$; $F$ elastic scattering amplitude) is entire of the order $\frac{1}{\tau}$, i.e., analytic in $\tau$ and bounded by $\exp(ov|\tau|)$. Thus, a scaling behaviour - usually called geometrical scaling (GS) - of the elastic amplitude emerges as a consequence of first principles.

Of course, the first principles do not provide us with the scaling function $\phi$. It has to be calculated - in principle - from a dynamical theory (QCD ?) or as the situation is nowadays it has to be taken from experiment. Also the question how strong are scaling violations at a given finite energy has to be answered by theory or experiment. It may well turn out that GS is a general property of diffraction scattering and not only the asymptotic behaviour of it. Indeed, there are indications \textsuperscript{2)-4}) that the inelastic overlap functions of NN, nN and KN scattering scale even at a few GeV.

How can we test GS?

Neglecting crossing odd contributions, the diffraction amplitude may be written as

$$\text{Im} \ F(s,t) = \text{Im} \ F(s,0) \ \phi(\tau) \ ; \ \text{Re} \ F(s,t) = \text{Re} \ F(s,0) \ \frac{d(\tau\phi)}{d\tau}$$

(2)

where use has been made of asymptotic phase relations. Note that GS allows us to turn the derivative with respect to $\ln s$ as appearing in the phase relation, into a derivative with respect to $\tau$, i.e., at fixed energy into a derivative with respect to $t$ which can easily be calculated from data.

Combining the expressions for the real and the imaginary parts and neglecting spin effects one obtains

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{16\pi} \sigma_{\text{tot}}^2(s) \ (\phi^2(\tau) + \rho^2(s) (\frac{d}{d\tau}(\tau\phi))^2)$$

(3)
Where \( \rho(s) = \text{Re} \, F(s,0)/\text{Im} \, F(s,0) \) which may be interpreted as a differential equation for \( \phi \). Solving that equation numerically \(^5\) for pp scattering at \( \sqrt{s} = 52.8 \, \text{GeV} \) where the most accurate data are available, one obtains the results shown in Fig. 1. Using this solution and \( \sigma_{\text{tot}} \) as well as \( \rho \) data, the differential cross-section at other energies can be predicted and compared to data. Over the ISR energy region this works very well.

Before turning to collider energies I want to mention some particular features of GS. Firstly, to the extent that the real part is negligible, Eqs. (2) and (3) imply that \( \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \text{const.} \) and \( B/\sigma_{\text{tot}} = \text{const.} \) where \( B \) is the slope of \( d\sigma/dt \) at \( t = 0 \). Secondly, there is a correlation between \( \rho \) and the differential cross-section at the position of the dip where \( \phi \) is zero (compare Fig. 1). Thus, at \( \tau_d \)

\[
\frac{d\sigma}{dt}(s,t_d) = \frac{\frac{d\sigma}{dt}(t=0) - \rho^2(s)}{1+\rho^2(s)} K^2
\]

where \( K = \tau \frac{d\phi}{dt} \) (at \( \tau = \tau_d \)) is a known number obtained from the 52.8 GeV ISR data. That this relation is in agreement with both the ISR data \( d\sigma/dt \) and \( \rho \) from Coulomb interference measurements can be seen from Fig. 2. The dip is deepest where \( \rho = 0 \) and at high energies with a rising \( \rho \) the dip is filled in gradually. Below ISR energies this relation does not work quantitatively, indicating a breakdown of GS for the elastic amplitude (it is the approximation for the real part which goes wrong \(^3\)).

This success of GS at ISR energies gives us confidence in predictions for the collider. Of course, we assume pp = p\( \bar{p} \) at such energies. Using the value of \( \sigma_{\text{tot}} \) (66±7 mb) given in the first publication of the UA4 group \(^6\) and estimating from asymptotic phase relation \( \rho \) to be 0.15 we can predict \( d\sigma/dt \) at \( \sqrt{s} = 540 \, \text{GeV} \). For small \(|t|\) good agreement is obtained between prediction and the UA4 data (compare Fig. 3). The predicted average slope in the interval \( 0 \leq |t| \leq 0.2 \) is 17.5 \( \text{GeV}^{-2} \) whereas the UA4 group quoted 17.2±1.5 \( \text{GeV}^{-2} \). There is also agreement with the UA1 data \(^7\) after normalizing them so as to obtain agreement between the two sets of data.

Both the groups UA1 and UA4 presented new preliminary data at this meeting \(^8\),\(^9\) which differ slightly from the previous ones. For the small \(|t|\) region it seems to me that the new data are in agreement with GS.
The GS predictions for large $|t|$ are shown in Fig. 4. Note the dip seen in the pp data at ISR energies has turned into a shoulder. The new data presented by UA4 show a shoulder at the right place. However, the cross-section is larger than the GS prediction. Part of that discrepancy may have its origin in the fact that the value of $\rho$ we used is too small. Using, for instance, instead of 0.15 a value of 0.25, an admittedly somewhat large value, the GS prediction in the shoulder region is raised by about a factor of 3. From the experimental side, one should keep in mind that the normalization of the large $|t|$ data is done by linking the data in the three different $t$ regions together by means of exponential fits. Moreover, the data in the first $t$ interval have been normalized with $\sigma_{tot} = 71$ mb, which for experimental reasons is the value of $(1+\rho^2)\sigma_{tot}$. Using the reasonable value of 66 mb instead the differential cross-section goes down by almost 20%.

In summary, if $\sigma_{tot}$ rises as $\ln^2$ GS should hold asymptotically as a consequence of first principles. The proton-proton data at ISR energies give strong evidence for GS. The pp collider data at small $|t|$ are compatible with GS. The preliminary large $|t|$ data show the expected shoulder but the cross-section seems to be too large which needs further investigation both experimentally as well as theoretically.

REFERENCES

**Fig. 1** The scaling function $\phi$ and its derivative. The dashed lines represent error corridors.

**Fig. 2** Test of the energy dependence of $\frac{d\sigma}{dt} (pp)$ at the position of the dip. Figure taken from Ref. 5.

**Fig. 3** The $p\bar{p}$ differential cross-section at $\sqrt{s} = 540$ GeV. Data taken from UA1\(^7\) (o) and UA4\(^8\) (●). The solid line is GS prediction, using $\sigma_{\text{tot}} = 66$ mb.

**Fig. 4** The GS predictions for $d\sigma/dt$ at $\sqrt{s} = 540$ GeV. Solid line $\sigma_{\text{tot}} = 66$ mb, dashed line 73 mb, dashed-dotted line 59 mb, $\rho = 0.15$. 