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NON LINEAR WIGGLERS FOR LARGE e⁺e⁻ STORAGE RINGS

by

John M. Jowett

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John M. Jowett
CERN, Geneva, Switzerland

Summary

Combined function non-linear wiggles offer additional degrees of freedom in the control of bunch shape and energy distribution in large e⁺e⁻ storage rings. The relative merits of dipole-octupole and quadrupole-octupole wigglers are discussed. The former wiggles substantially modifies the zero-current beam parameters. On the other hand, because the energy lost by synchrotron radiation is smaller in the latter wiggler, it is much easier to reduce the energy spread at the price of a reduction in damping aperture.

A convenient method for calculating moments and other quantities from a "partition function" introduction. Asymptotic expansions of this function are particularly useful for calculating perturbations of the cases of completely linear or completely non-linear damping.

The energy range over which useful moulding of the bunch is achieved is limited by the maximum gradients and RF voltage. Calculations of lifetime and RF voltage have been incorporated in a computer program which supplies self-consistent sets of machine parameters. Radiation from the wigglers and consequent power requirements are included. Possible benefits of these wigglers are discussed.

Introduction

For small enough beam currents and particle amplitudes, the phase space distribution and beam parameters of an e⁺e⁻ storage ring are determined by linear radiation damping and quantum excitation as well as the applied RF voltage. The distribution is then Gaussian in each of the three degrees of freedom and is centred on an attracting fixed point (closed orbit for central momentum). If the linear damping is changed (usually by small variations of the RF frequency) the stable fixed point goes unstable at a critical electric field passes through zero.

It has been shown that combined function dipole-octupole wigglers may modify this by providing additional damping which increases quadratically with amplitude in the longitudinal phase plane. While \( \lambda \) > 0, the phase-plane trajectories are qualitatively unchanged but, as \( \lambda \) passes through zero, a stable limit cycle is born out of the fixed point as it goes unstable (Hopf bifurcation).

Dipole-Octupole Wiggles (DOW)

The radial variation of the magnetic field in a wiggler of length \( L_w \) is taken to be

\[
B_w (x) = \frac{e (B_w + \frac{1}{6} B_w^3 x^3)}{s_t (G_w + \frac{1}{6} K_w^3 x^3)} .
\]  

(1)

The non-linear damping effect is characterised by a parameter

\[
b = G_w K_w^2 / L_w - G_w / L_w + 6 G_w^2 / K_w^3 .
\]  

(2)

where \( G_w \) is the value of the dispersion at the DOW. We use the notation \( i = 1, 2, \ldots \) \( G_w \) for synchrotron integrals, with \( L_w \) referring to the contributions of the lattice when all wigglers are switched off. The quantity \( b \) (which coincides with the mean-square energy deviation \( \sigma_e^2 \)) only when \( \lambda = 2 \) and \( b = 0 \) is a measure of the spectral density of the fluctuating part of the synchrotron radiation power, i.e. the strength of quantum excitation:

\[
\sigma_e^2 = \frac{55 \pi \hbar}{64 \lambda^3 m_c} \left( \frac{e D_w}{mc^2} \right)^2 I_3 \lambda .
\]  

(3)

Since \( I_3 = I_3^0 + S |Z| L_w \), \( \sigma_e^2 \) will increase with \( B_w \) and \( b \) will decrease to a maximum value and then decrease. In addition, the dipole field of this wiggler causes a radiative energy loss \( U_w \), inflation of transverse beam dimensions and other phenomena well-known from simple dipole wigglers. These effects limit the maximum energy at which non-linear damping due to a DOW can be effective. They can be avoided to a large extent by using another multiple combination.

Quadrupole-Sextupole Wiggles (QSW)

The QSW has a magnetic field

\[
B(x) = \frac{e (B_w + \frac{1}{2} B_w^2 x^2)}{s_t (K_w + \frac{1}{2} K_w x^2)} .
\]  

(4)

and total length \( L_w \). The resultant damping is given by writing down the energy loss by a particle of instantaneous energy \( E_0 (1 + \epsilon) \)

\[
U(x) = \frac{1}{3} e \frac{B_0}{(mc^2)^2} (1 + \epsilon) \left( I_2 + \frac{2 S |Z| L_w}{E_0} B_0 (D_w)^2 L_w \right) .
\]  

(5)

and forming the \( \epsilon \)-dependent damping partition number

\[
J_3 (\epsilon) = \left[ U'(\epsilon) + U'(-\epsilon) \right] / 2 U(0) .
\]  

(6)

The coefficients of \( \epsilon^2 \) is the value of \( b \) for this wiggler, the damping has exactly the same form as for the DOW and the same formulae apply with the appropriate value of \( b \).

Now, however, \( b \) is simply proportional to the product \( (3b^2)^\frac{1}{3} \) (the second term in brackets in Eq. (5) usually dominates) at a given energy and is not diminished by extra energy loss. Furthermore \( \sigma_e^2 \) is not affected. The only adverse effect is a reduction in "damping aperture" through an increased sensitivity of \( J_3 \) to changes, \( E_0 \), in the central energy of the beam

\[
J_3 = \frac{d J_3}{d \epsilon} \frac{d^2}{d \epsilon^2} \log U(\epsilon) \bigg|_{\epsilon=0} = J_3^0 \frac{1}{12} + 2 Z K_w^2 / L_w .
\]  

(7)

Equilibrium Distribution

The fluctuations due to the quantised nature of synchrotron radiation result in an equilibrium distribution in synchrotron phase space given by

\[
f(\epsilon) = Z \left[ J_3 (\epsilon) \right]^{-1} \exp \left[ -b \frac{\epsilon}{3b^2} \right] .
\]  

(8)

where \( Z \) is chosen to normalise

\[
\int \epsilon f(\epsilon) d\epsilon = 1 .
\]  

(9)
and the amplitude $A$ is such that the smooth approximation to the fractional energy deviation, $\epsilon$, has a variation of the form

$$
\epsilon(t) = A \cos \left( \frac{2\pi Q \epsilon t}{T_0} + \phi \right) + O(Q \epsilon T_0/\epsilon) \tag{10}
$$

between photon emissions; $Q$, $T_0$, and $\epsilon$ being the synchrotron tune, revolution period and energy damping time respectively. It has been shown that adequate stability of the transverse distributions is assured if

$$
\frac{b \epsilon_x \beta_w}{\delta_w} \lesssim 4(3 - J_e)^2 - 2\Delta^4 b \tag{11}
$$

where $\epsilon_x$ is the horizontal emittance and $\beta_w$ the horizontal $\beta$-function at the wiggler.

The Partition Function $Z$

This is easily calculated from Eqs. (8) and (9)

$$
Z(J_e, b, \epsilon_x) = 4\pi \sigma_e \left( \frac{\epsilon_x}{2b} \right)^{1/2} \exp \left( \frac{1}{12} \frac{J_e}{\epsilon_x} \right) \tag{12}
$$

where $\omega$ is the usual error function for complex arguments. The distribution is stable when $b = 0$ and $J_e > 0$ or when $b > Q$; $Z$ has a finite value in both cases. Two asymptotic expansions of $Z^{-1}$ are very useful

$$
Z^{-1} \sim \frac{J_e}{4 \pi \sigma_e} \left( \frac{b}{4\epsilon_x} \right)^{1/2} \exp \left( -\frac{1}{8} \frac{J_e}{\epsilon_x} \right) \quad (b = 0, J_e > 0) \tag{13}
$$

$$
Z^{-1} \sim \frac{1}{\pi} \left( \frac{3 - 2b/\epsilon_x}{2b^3} \right)^{1/2} \left( \frac{J_e}{\pi} \right)^{3/2} \exp \left( \frac{1}{8} \frac{J_e}{\epsilon_x} \right) \quad (J_e > 0, b > 0) \tag{14}
$$

The expansion (13) applies when the non-linear damping can be regarded as a small perturbation of linear damping and the expansion (14) when the damping is predominantly non-linear. In the latter case, $J_e$ may be positive or negative.

Just as in elementary statistical mechanics, many important quantities are obtained from $Z$ by differentiation. For example, the even moments of the amplitude are

$$
<\Delta^2> = 2<\Delta^2> = -4\epsilon_x^2 \frac{1}{J_e} \log Z = -\frac{4\epsilon_x^2}{bZ} \frac{J_e}{b} \tag{15}
$$

$$
<\Delta^4> = -\frac{4\epsilon_x^2}{J_e} \left( 1 - \frac{2b/\epsilon_x}{2b^3} \right) \quad (b = 0^+, J_e > 0) \tag{16}
$$

$$
<\Delta^{2n}> = \left( -4\epsilon_x^2 \right)^n \frac{1}{b^{n/2}} \log Z \tag{17}
$$

The cumulants of $<\Delta^2>$

$$
\epsilon_n = \left( -4\epsilon_x^2 \right)^n \frac{1}{b^{n/2}} \log Z \tag{18}
$$

may conveniently be used to calculate cumulants of the energy deviation, e.g.

$$
<\Delta^2> = 3<\epsilon_x^2> = \frac{3}{8} \left( x_2 - x_2 \right) \tag{19}
$$

$$
-\frac{b^2 \sigma_e^2}{\delta_e^2} \left( b + b^* J_e > 0 \right) \tag{20}
$$

$$
6(1 - \frac{3}{b\sigma_e^2}) \frac{\sigma_e^2}{\sigma_e} + 12(3 - \frac{b}{b\sigma_e^2}) \frac{\sigma_e^2}{\sigma_e} \tag{21}
$$

$$
\pi \left( 2\sigma_e^2 \right)^{1/2} \left( J_e > 0, b > 0 \right) \tag{22}
$$

which characterizes the deviation from a Gaussian distribution in energy. All these formulae take on familiar forms when $b = 0$.

Quantum Lifetime

With an RF system working on the $n$th harmonic of the revolution frequency, with peak voltage $V$, we can follow the notations of Ref. 1 and define

$$
\tau = \frac{\varrho}{2n\sigma_e^2} F(E/\varrho U) \tag{23}
$$

which is half the squared bucket half-height in units of $\sigma_e$. Then the longitudinal quantum lifetime with non-linear wiggler's may be written

$$
\tau \equiv \frac{\epsilon_e \sigma_e J_e}{\sigma_e^2 (\sigma_e^2 b + 2J_e)} \exp \left( \frac{\epsilon_e}{\sigma_e^2} \frac{b}{b + \sigma_e^2} \right) \tag{24}
$$

and reduces to Eq. (5.137) of Ref. 1 when $b = 0$ and $J_e = 2$.

A computer program to calculate beam and RF parameters, RF voltages and wiggler excitations has been developed. It includes combinations of DOVs, QSVs and ordinary dipole wiggler, and will attempt to satisfy a variety of constraints, e.g. of fixed emittance, energy spread, synchrotron tune, bunch shape or lifetime, as the energy is ramped up, within the capabilities of the specified hardware. It also produces a variety of graphic output. Some examples are given in Figs. 1-3.

For Fig. 1, the DOVs were excited with constant fields

$$
B = 1 T, \quad B = 4000 Tm^{-3}
$$

and $J_e$ was chosen to make

$$
R = -J_e (2b\sigma_e^2)^{-1} = 0.5409
$$

which makes the longitudinal current density as flat as possible. This produces a very large energy spread and the RF system can no longer contain the bunch above about 30 GeV. The situation could be improved by reducing the dipole field and increasing $J_e$. However one can conclude that DOVs are little use above 30 GeV in LEP because of the large amount of power radiated in the dipole field.

On the other hand, Fig. 2 shows the effect of keeping

$$
B = 10 Tm^{-1}, \quad B = 400 Tm^{-1}
$$


Fig. 1 QSWS excited with constant fields and bunch shape parameter $R$.

**Conclusions**

Quadrupole-sextupole wigglers permit a degree of bunch-shape moulding up to quite high energies. In this respect they should be preferred over dipole-octupole wigglers which necessitate large amounts of RF power.

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**References**