A method to evaluate RRR of superconducting cavities

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Abstract

The Residual Resistivity Ratio (RRR) is defined as $\rho(300 \text{ K})/\rho(10 \text{ K})$ where $\rho$ is the surface resistivity in the normal conducting state of the material. RRR is related to the impurity content in the material and then it represents a strong indication on how good the material is. In this paper we will develop an useful method to measure RRR for superconducting cavities during cryogenic tests.

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1 Introduction

The RRR is usually measured on material samples by applying a DC voltage. This measurement is particularly important for sputtered technology. The value of RRR on samples gives an indication on how good is the sputtering. Obviously even if the values of RRR for these samples are satisfying, this does not imply automatically that a more complex geometry as a cavity will show the same properties. The RRR can be extensively written as

\[
RRR = \frac{\rho_{ph}(T) + \rho_{imp}}{\rho_{imp}}
\]

where \(\rho_{ph}(T)\) is the resistivity due to the interaction between electrons and phonons which is dominant at high temperatures and \(\rho_{imp}\) is the resistivity due to the impurity in the material.

The purpose of this paper is to evaluate the RRR of a cavity during cryogenic tests. In fact using the equations developed in [1], it is possible to derive a relation between the frequency shift and the variation of the penetration depth, \(\lambda\), by which is possible to derive \(\rho_{imp}\).

2 Relation between frequency shift and variation of the penetration depth \(\lambda\)

Figure 1 shows a pictorial view of the two fluid model for superconductors.

![Lumped circuit model of the two fluid model for superconductors](image)

Figure 1: Lumped circuit model of the two fluid model for superconductors

In a superconducting material when the critical temperature \(T_c\) is reached, it is possible to imagine that the total current \(J\) flowing on the walls is composed by two parts: a “normal” current \(J_{NC}\) and a dominant “supercurrent” \(J_{SC}\) that becomes more and more important for \(T \to 0\). Following this model, we may write [2]
\[ \sigma = \sigma_N - j \sigma_S \]  \hspace{1cm} (2)

where \( \sigma_N \) and \( \sigma_S \) are the normal and superconducting conductivity and, in superconducting state, it will result \( \sigma_S \gg \sigma_N \).

According to the calculation developed in [1], \( \sigma_S \) is related to the penetration depth \( \lambda \) as follows

\[ \sigma_S = \frac{1}{\omega \mu_0 \lambda^2} \]  \hspace{1cm} (3)

It is worth noting that \( \lambda \) is also a function of the temperature. In next section we will delve deeper into the matter and we will give the full expression of \( \lambda \).

The surface impedance \( Z_s \) for a superconductor can be written in terms of \( \sigma \) as

\[ Z_s = R_s + jX_s \approx \sqrt{\frac{\omega \mu_0}{\sigma_S}} \frac{\sigma_N}{2 \sigma_S} + j \sqrt{\frac{\omega \mu_0}{\sigma_S}} \]  \hspace{1cm} (4)

or, equivalently, considering eq. 3, in terms of \( \lambda \) as

\[ Z_s = R_s + jX_s \approx \frac{\omega^2 \mu_0^2 \sigma_N \lambda^3}{2} + j \omega \mu_0 \lambda. \]  \hspace{1cm} (5)

The link between surface impedance and frequency shift is given in [1, 3] where we may derive the relation between the surface reactance and frequency shift\(^1\) as

\[ X_s = -2 \Gamma \frac{\Delta \omega}{\omega_0} \]  \hspace{1cm} (6)

where \( \Gamma \) is the geometrical factor.

Making the difference\(^2\) \( X_s(\omega_{T_n}) - X_s(\omega) \) with \( \omega_{T_n} \) the angular frequency at the initial temperature, \( T_n \), and using the same logic as in [1], we may get

\[ \frac{\Delta \omega}{\omega_{T_n}} = \frac{\mu_0 \omega_{T_n}}{2 \Gamma - \mu_0 \omega_{T_n} \Delta \lambda} \Delta \lambda \]  \hspace{1cm} (7)

which states the relation between a shift in frequency and a variation of the penetration depth. A similar formula can be found in [4].

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\(^1\)It is worth noting that eq. (6) is very general and it states that the presence of a surface reactance can be expressed in terms of a frequency shift induced by a finite conductivity. The angular frequency \( \omega_0 \) in the formula is the frequency in case of a perfect electric conductor (PEC) or, equivalently, for a zero penetration depth giving a vanishing surface impedance.

\(^2\)Notice that \( \omega_0 \) cancels in the difference.
3 Measurement of $\lambda_0$

The penetration depth $\lambda$ in a superconductor can be written according to Pippard corrections \cite{5} as

$$\lambda = \lambda_L \sqrt{1 + \frac{\xi_0}{\ell}} \frac{1}{\sqrt{1 - (T/T_c)^4}}$$

(8)

where $\lambda_L$ is the London penetration depth, $\xi_0$ is the coherence length, $\ell$ is the electron mean free path and $T_c$ is the critical temperature of the superconductor. In general $\lambda_L$ and $\xi_0$ are constants given by the theory depending only on the pure material. For example for Nb it is $\lambda_L = 360\,\text{Å}$ and $\xi_0 = 640\,\text{Å}$ \cite{6}. The critical temperature $T_c$ and especially $\ell$ are not constants and they do depend on the purity of the material.

The value of $\lambda_0$ can be easily derived by fitting the frequency shift as a function of the temperature change by making use of eq. 7. Here below is the function we use for the fit expressed directly in terms of frequency coming from a frequency counter ($f_{\text{meas}}$ is the measured frequency and $f_{T_{\text{in}}}$ is the frequency at the initial temperature $T_{\text{in}}$):

$$f_{\text{meas}}(T) = f_{T_{\text{in}}} + \Delta f = f_{T_{\text{in}}} + \frac{\pi \mu_0 f_{T_{\text{in}}}^2}{\Gamma - \pi \mu_0 f_{T_{\text{in}}} [\lambda(T_{\text{in}}) - \lambda(T)]]} [\lambda(T_{\text{in}}) - \lambda(T)].$$

(9)

From the value of $\lambda_0$ it is possible to evaluate $\ell$ from eq. 8 that we explicitly retype here below

$$\lambda_0 = \lambda_L \sqrt{1 + \frac{\xi_0}{\ell}}.$$

(10)

Figure 2 shows an example of the fit. A very simple routine using MATLAB \cite{7} nonlinear regression has been written. The input data are, obviously, measured frequencies and temperatures as well as the initial temperature $T_{\text{in}}$ and geometrical factor $\Gamma$. The output data are $\lambda_0$, which is what we finally aim to derive, as well as the critical temperature $T_c$ and initial frequency, $f_{T_{\text{in}}}$, at $T = T_{\text{in}}$. This last can be in principle considered a known parameter if the frequency measurements are very stable, in case of some jitter it is preferable to let it as an unknown parameter.

4 From $\lambda_0$ measurement to RRR

From the theory it is

$$\rho = \frac{m_e}{n_u e^2 \tau}$$

(11)

where $m_e$ is the mass of the electron, $e$ is its charge, $n_u$ is the number of unpaired electrons in the materials and $\tau$ is the average time between collisions. While the first three quantities are constant depending only on the atomic properties of the pure material,
Figure 2: Best fit of frequency as a function of the temperature according to eq. 9: blue circles are the original data, red line is the starting curve of the nonlinear fit, green line is the best fit; data come from measurements done at CERN of HIE-ISOLDE cavity sputtered using bias technique; output fit data are in the yellow box.

it is $\tau = \ell / v_f$ where $v_f$ is Fermi’s velocity and is still a constant but $\ell$ depends on the impurities as mentioned in the previous section. It comes out that at low temperatures

$$\rho_{imp} \cdot \ell = constant$$

(12)

this constant is equal to $0.37 \times 10^{-11} \Omega cm^2$ for Nb [8]. This is the value of $\rho_{imp}$ that has to be inserted in eq. 1. Finally it is also possible to evaluate $R_{BCS}$ from eq. 5, that, in terms of $\rho_{imp}$ and $\lambda(T)$, becomes

$$R_{BCS} = \frac{\omega^2 \mu_0^2 \lambda^3(T)}{2 \rho_{imp}}.$$  

(13)

According to the quantities in Fig. 2, for example, we may get $\ell = 14.54 \text{nm}$, $\rho_{imp} = 2.55 \mu \Omega cm$, $\text{RRR}=6.7$ and $R_{BCS} = 8.29 n\Omega$.

5 Conclusions

A practical method to measure the penetration depth $\lambda_0$ has been discussed. From $\lambda_0$ it is possible to measure the electron mean free path $\ell$ which is proportional to $\rho_{imp}$ of the
material. Then it is possible to calculate the RRR for the whole cavity together with its $R_{BCS}$.

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References


