Pileup Subtraction for Jet Shapes

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Jets in high energy hadronic collisions often contain the fingerprints of the particles that produced them. Those fingerprints, and thus the nature of the particles that produced the jets, can be read off with the help of quantities known as jet shapes. Jet shapes are, however, severely affected by pileup, the accumulation in the detector of the residues of the many simultaneous collisions taking place in the Large Hadron Collider (LHC). We introduce a method to correct for pileup effects in jet shapes. Relative to earlier, limited approaches, the key advance resides in its full generality, achieved through a numerical determination, for each jet, of a given shape’s susceptibility to pileup. The method rescues the possibility of using jet shapes in the high pileup environment of current and future LHC running, as we show with examples of quark-gluon discrimination and top tagging.

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Our approach is rooted in the framework of the area-median method, which has been found to be beneficial in both ATLAS [24] and CMS [25] (see also Refs. [30–32]). It is intended to be valid for arbitrary jet algorithms and generic infrared and collinear safe jet shapes [33], without the need for dedicated analytic study of each individual shape variable. It also involves an extension of the original area-median prescription to account for hadron masses.

The first ingredient is a characterization of the average pileup density in a given event in terms of two variables, $\rho$ and $\rho_m$, such that the four-vector of the expected pileup deposition in a small region of size $\delta y \delta \phi$ can be written as

$$[\rho \cos \phi, \rho \sin \phi, (\rho + \rho_m) \sin \eta, (\rho + \rho_m) \cos \eta] \delta y \delta \phi,$$

(1)

where $\rho$ and $\rho_m$ have only weak dependence on $y$ (and $\phi$). One way of determining $\rho$ and $\rho_m$ is the area-median method [23]. The inclusion of the $\rho_m$ term is one novelty of this Letter: $\rho_m$ arises because pileup consists of low-$p_t$ hadrons, and their masses are not negligible relative to their $p_t$ (cf. also Refs. [36,37]). It is important mainly for observables sensitive to differences between energy and 3-momentum, e.g., jet masses, as we will see below.

The second and main new ingredient is a determination, for a specific jet, of the shape’s sensitivity to pileup. Let the shape be defined by some function $V(\{p_i\})$ of the momenta $p_i$ in the jet. Among these momenta, we include a set of “ghosts” [21], very low momentum particles that cover the $y$-$\phi$ plane at high density, each of them mimicking a pileup-like component in a region of area $A_g$. We then consider the derivatives of the jet shape with respect to the transverse-momentum scale $p_{t,g}$ of the ghosts and with respect to a component $m_{\delta g}$.

$$V_{\text{jet}}^{(m,n)} = A_g^{m+n} \frac{\partial^n}{\partial p_{t,g}^m} \frac{\partial m_{\delta g}}{d m_{\delta g}} V(\{p_i\}).$$

(2)

The derivatives are to be evaluated at $p_{t,g} = m_{\delta g} = 0$, and by scaling all ghost momenta simultaneously.

Given the level of pileup, $\rho$, $\rho_m$, and the information on the derivatives, one can then extrapolate the value of the jet’s shape to zero pileup,

$$V_{\text{jet,sub}} = V_{\text{jet}} - \rho V_{\text{jet}}^{(0,1)} - \rho_m V_{\text{jet}}^{(0,1)} + \frac{1}{2} \rho^2 V_{\text{jet}}^{(2,0)} + \frac{1}{2} \rho^2 m_{\delta g} V_{\text{jet}}^{(2,0)} + \frac{1}{2} \rho m_{\delta g} V_{\text{jet}}^{(1,1)} + \cdots,$$

(3)

where the formula takes into account the fact that the derivatives are evaluated for the jet including the pileup.

Handling derivatives with respect to both $p_{t,g}$ and $m_{\delta g}$ can be cumbersome in practice. An alternative is to introduce a new variable $r_{t,g}$ and set $p_{t,g} = r_{t,g}$ and $m_{\delta g} = \frac{\rho_m}{\rho} r_{t,g}$. We then take total derivatives with respect to $r_{t,g}$,

$$V_{\text{jet}}^{[n]} = A_g^n \frac{d^n}{d r_{t,g}^n} V(\{p_i\}).$$

(4)

so that the correction can be rewritten

$$V_{\text{jet,sub}} = V_{\text{jet}} - \rho V_{\text{jet}}^{[1]} + \frac{1}{2} \rho^2 V_{\text{jet}}^{[2]} + \cdots.$$  

(5)

The derivatives $V^{(m,n)}_{\text{jet}}$ or $V^{[n]}_{\text{jet}}$ can be determined numerically, for a specific jet, by rescaling the ghost momenta and reevaluating the jet shape for multiple rescaled values. Typically this is more stable with Eq. (4) and this is the approach we use below.

To investigate the performance of our correction procedure, we consider a number of jet shapes.

1. Angularities [12,38], adapted to hadron-collider jets as $\theta^{(\beta)} = \sum_j p_{t,j} \Delta R_{t,j}^\beta / \sum_i p_{t,i}$, for $\beta = 0.5$, 1, 2, 3; $\theta^{(1)}$, the “girth,” “width,” or “broadening” of the jet, has been found to be particularly useful for quark-gluon discrimination [17,39].

2. Energy-energy-correlation (EEC) moments, advocated for their resummation simplicity in Ref. [40], $E^{(\beta)} = \sum_i p_{t,i} \Delta R_{t,i}^\beta / (\sum_i p_{t,i})^2$, using the same set of $\beta$ values. EEC-related variables have been studied recently also in Ref. [41].

3. “Subjettiness” ratios, designed for characterizing multipronged jets [13–15]: one defines the subjettiness $\tau_N^{(\text{axes},\beta)} = \sum_i \Delta R_{t,i} \min(\Delta R_{t,i}, \ldots, \Delta R_{t,N}) / \sum_i p_{t,i}$, where $\Delta R_{t,i}$ is the distance between particle $i$ and axis $a$, where $a$ runs from 1 to $N$. One typically considers ratios such as $\tau_{21} = \tau_2 / \tau_1$ and $\tau_{32} = \tau_3 / \tau_2$ (the latter used, e.g., in a recent search for $R$-parity violating gluino decays [42]); we consider $\beta = 1$ and $\beta = 2$, as well as two choices for determining the axes: “$k_t$,” which exploits the $k_t$ algorithm [43,44] to decluster the jet to $N$ subjets and then uses their axes, and “$1k_t$,” which adjusts the $k_t$ axes so as to obtain a single-pass approximate minimisation of $\tau_N$ [15].

4. A longitudinally invariant version of the planar flow [11,12], involving a $2 \times 2$ matrix $M_{\alpha \beta} = \sum_i p_{t,i} (\alpha_i - \alpha_{\text{jet}})(\beta_i - \beta_{\text{jet}})$, where $\alpha$ and $\beta$ correspond to either the rapidity $y$ or the azimuth $\phi$; the planar flow is then given by $P_f = 4 \lambda_1 \lambda_2 / (\lambda_1 + \lambda_2)^2$, where $\lambda_{1,2}$ are the two eigenvalues of the matrix.

One should be aware that observables constructed from ratios of shapes, such as $\tau_{n,n-1}$ and planar flow, are not infrared and collinear safe for generic jets. In particular $P_f$ and $\tau_{21}$ are infrared and collinear safe only when applied to jets with a structure of at least two hard prongs, usually guaranteed by requiring the jets to have significant mass; $\tau_{32}$ requires a hard three-pronged structure [45], a condition not imposed in previous work, and that we will apply here through a cut on $\tau_{21}$.

For the angularities and EEC moments we have verified that the first two numerically obtained derivatives agree for their resummation simplicity in Ref. [40], $E^{(\beta)} = \sum_i p_{t,i} \Delta R_{t,i}^\beta / (\sum_i p_{t,i})^2$, using the same set of $\beta$ values. EEC-related variables have been studied recently also in Ref. [41].

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numerical derivative. The resulting discontinuities (or nonsmoothness) in the observable’s value would then result in nonsensical estimates of the derivatives. We find no such issue in our numerical method to evaluate the derivatives, but were it to arise, one could choose to force a fixed partitioning.

To test the method in simulated events with pileup, we use PYTHIA 8.165, tune 4C [46,47]. We consider three hard event samples: dijet, WW, and $t\bar{t}$ production, with hadronic $W$ decays, all with underlying event (UE) turned off (were it turned on, the subtraction procedure would remove it too). We use anti-$k_t$ jets [48] with $R = 0.7$, taking only those with $p_T > 500$ GeV (before addition of pileup). All jet finding is performed with FASTJET 3.0 [49]. The determination of $\rho$ and $\rho_m$ for each event follows the area-median approach [23]: the event is broken into patches and in each patch one evaluates $P_{\text{patch}} = \sum_{i \in \text{patch}} P_{i,i}$, as well as $m_{\delta,\text{patch}} = \sum_{i \in \text{patch}} (m_i^2 + p_{\text{jet},i}^2 - P_{i,i})$, where the sum runs over particles $i$ in the patch. Then $\rho$ and $\rho_m$ are given by

$$\rho = \text{median}_{\text{patches}} \left\{ P_{\text{patch}} / A_{\text{patch}} \right\}, \quad \rho_m = \text{median}_{\text{patches}} \left\{ m_{\delta,\text{patch}} / A_{\text{patch}} \right\},$$

(6)

where $A_{\text{patch}}$ is the area of each patch. To obtain the patches we cluster the event with the $k_t$ algorithm with $R = 0.4$. The median helps limit the results’ sensitivity to the presence of a handful of hard jets (cf. Refs. [50,51]).

For nonzero $\rho_m$ the formula for correcting a jet’s 4-momentum is

$$p_{\text{jet,sub}} = p_{\text{jet}} - \left[ (\rho A_{\text{jet}}^\mu) (\rho + \rho_m) A_{\text{jet}}^\nu - (\rho + \rho_m)^2 A_{\text{jet}}^\nu \right],$$

(7)

where the area four-vector $A^\mu$ is the four-vector sum of the momenta of the ghosts in the jet multiplied by $A^\mu / p_{\text{jet}}$ [21].

We have 17 observables and three event samples. Figure 1 gives a representative subset of the resulting 51 distributions, showing in each case the distribution (and average) for the shape without pileup (solid green line), the result with pileup (dashed line), and the impact of subtracting first and second derivatives (dotted and solid black lines, respectively). The plots for the distributions have been generated using a Poisson distribution of pileup events with an average of 30 events (comparable to typical 2012 LHC runs; our count includes diffractive and elastic events, and the analysis uses all particles from the event.

![Figure 1](color online) Impact of pileup and subtraction on various jet-shape distributions and their averages, in dijet, WW, and $t\bar{t}$ production processes. The distributions are shown for Poisson distributed pileup (with an average of 30 pileup events) and the averages are shown as a function of the number of pileup events $n_{PU}$. The shapes are calculated for jets with $p_T > 500$ GeV (the cut is applied before adding pileup, as are the cuts on the jet mass $m_j$ and subjettiness ratio $\tau_{21}$ where relevant).
FIG. 2 (color online). Left: Rate for tagging quark and gluon jets using a fixed cut on the jet width, shown as a function of the number of pileup vertices. Middle: Filtered jet-mass distribution for fat jets in $t\bar{t}$ events, showing the impact of the $\rho$ and $\rho_m$ components of the subtraction. Right: Tagging rate of an $N$-subjettiness top tagger for $t\bar{t}$ signal and dijet background as a function of the number of pileup vertices. All cuts are applied after addition (and possible subtraction) of pileup. Subtraction acts on $\tau_1$, $\tau_2$, and $\tau_3$ individually. See text for further details.

genera, leading to $\rho \approx 770$ MeV and $\rho_m \approx 125$ MeV per pileup event at central rapidities.

For nearly all the jet shapes, the pileup has a substantial impact, shifting the average values by up to 50%–100% (as compared to a 5%–10% effect on the jet $p_t$). The subtraction performs adequately: the averaged subtracted results for the shapes usually return very close to their original values, with the second derivative playing a small but sometimes relevant role. For the distributions, tails of the distributions are generally well recovered; however, intra-jet pileup fluctuations cause sharp peaks to be somewhat broadened. These cannot be corrected for without applying some form of noise reduction, which would, however, also tend to introduce a bias. Of the 51 combinations of observables and processes that we examined, most were of similar quality to those illustrated in Fig. 1, with the broadening of narrow peaks found to be more extreme for larger $\beta$ values. The one case where the subtraction procedure failed was the planar flow for (hadronic) $WW$ events: here the impact of pileup is dramatic, transforming a peak near the lower boundary of the shape’s range, $Pf = 0$, into a peak near its upper boundary, $Pf = 1$ (bottom-right plot of Fig. 1). This is an example where one cannot view the pileup as simply “perturbing” the jet shape, in part because of intrinsic large nonlinearities in the shape’s behavior; with our particular set of $p_t$ cuts and jet definition, the use of the small-$p_t$ expansion of Eq. (5) fails to adequately correct the planar flow for more than about 15 pileup events.

Next, we consider the use of the subtraction approach in the context of quark-gluon discrimination. In a study of a large number of shapes, Ref. [17] found the jet girth or broadening $\theta^{(1)}$ to be the most effective single infrared and collinear safe quark-gluon discriminator. Figure 2 (left) shows the fraction of quark and gluon-induced jets that pass a fixed cut on $\theta^{(1)} \leq 0.05$ as a function of the level of pileup—pileup radically changes the impact of the cut, while after subtraction the $q$-$g$ discrimination returns to its original behavior.

Our last test involves top tagging, which we illustrate on $R = 1$, anti-$k_t$ jets using cuts on the “filtered” jet mass and on the $\tau_{32}$ subjettiness ratio. The filtering selects the four hardest $R_{filt} = 0.25$, Cambridge-Aachen [52] subjets after pileup subtraction. The distribution of filtered jet mass is shown in Fig. 2 (middle), illustrating that the subtraction mostly recovers the original distribution and that $\rho_m$ is as important as $\rho$ (specific treatments of hadron masses, e.g., setting them to zero, may limit the impact of $\rho_m$ in an experimental context). The tagger itself consists of cuts on $\tau_{32} < 0.6$, $\tau_{21} \geq 0.15$ and a requirement that the filtered [6] jet mass be between 150 and 200 GeV. The rightmost plot of Fig. 2 shows the final tagging efficiencies for hadronic top quarks and for generic dijets as a function of the number of pileup events. Pileup has a huge impact on the tagging, but most of the original performance is restored after subtraction.

To conclude, this Letter has introduced a novel, fully general method that allows one to correct most jet shapes for the effects of pileup. The corrections allow shape-based jet substructure analyses to continue to perform well even in the presence of up to 60 pileup events, notably when combined with the corrections introduced here for hadron masses in pileup. This progress is likely to be key to the viability of shape-based substructure tools in LHC’s 2012 data set and in future running.

The software for the general shape subtraction approach presented here is available as part of the FASTJET Contrib project [53].

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