LINEAR COLLIDERS: PROSPECTS 1985†

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Scaling laws for the design of electron linear colliders are obtained for both the familiar "classical" case, in which the particle energy lies well above the classical critical energy which characterizes the beamstrahlung, and the "quantum" case, in which the particle energy lies below it. The SLAC Linear Collider is considered as an example. Some of the problems which have had to be solved to build it are discussed in terms of colliders of the future.

1. INTRODUCTION‡

It was at the beginning of this decade that the linear collider emerged as the most promising machine with which to meet the challenge of colliding lepton beams in the TeV region of energies. Although the concept of the linear collider had been discussed in print earlier¹ and had been under study in Novosibirsk, the superiority of the linear collider over the storage ring at very high energies was first strongly emphasized in the report of the 1978 ICFA Workshop at Fermilab.² It was generally agreed that the linear collider would prove more economical to build than a storage ring of equivalent energy and at least as economical to operate for beam energies above several hundred GeV. This means, if it is true, that LEP will likely be the largest and the last electron-positron storage ring built for colliding beams. Opinion in the world has not changed much since that time, and much activity has developed in connection with linear colliders. A 50-GeV collider is being built at SLAC to prove the principle, to learn the fundamentals of the technology, and to study the physics of the region of 100 GeV in the center of mass. This machine is called the SLAC Linear Collider, or SLC for short. Also, since linear colliders are shorter and less expensive the higher the accelerating field, research on several new approaches to the production of very high fields has been spurred on.

In the following sections, we shall discuss the scaling laws of linear colliders and their consequences for accelerator design. We then shall report on the SLAC Linear Collider project and comment on experience gained on that project and its application to future colliders.

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2. THE SCALING LAWS OF LINEAR COLLIDERS

2.1. Round Beams

The scaling laws of linear colliders can be dealt with rather easily if some simplifying assumptions are accepted at the outset. We shall assume that the colliding beams have Gaussian distributions in both transverse degrees of freedom and the longitudinal degree of freedom at the interaction point. Moreover, we shall assume that the beams are round, by which is meant that their transverse distribution has cylindrical symmetry. This assumption is restrictive, because better performance might be attained with flat beams, as we shall discuss briefly later.\(^3\)

There are four scaling laws which relate experimental conditions for particle physics to accelerator parameters. Of these, three take a fixed form independent of the energy of operation of the collider:

\[ L = \frac{fN^2}{A}, \]

\[ P_b = fN\gamma m_e c^2, \]

\[ D = \frac{4\pi r_s \sigma_z N}{A \gamma}, \]

where \( L \) is the luminosity, \( P_b \) is the average power imparted to one of the two beams, \( D \) is the disruption parameter discussed below, \( f \) is the collider's repetition frequency, \( N \) is the population of a bunch (the number of particles in it), \( A \) is the effective interaction area, \( \sigma_z \) is the bunch length (the standard deviation of the Gaussian), and the other symbols have their customary meanings.

The disruption parameter can be understood by referring to Fig. 1, which shows a particle of one beam being deflected by the collective electromagnetic field of the counter-moving bunch of the opposing beam. If the incident particle is close to the axis, the fields of the opposing bunch are lenslike, with focal length \( F \), and the disruption parameter is just the ratio of the bunch length to the focal length, \( D = \sigma_z / F \). If this quantity is small, the beams do not alter each other's motion very much; if it is large, they do. Roughly, we may say that, compared to unity, small values of \( D \) are permissible, but large values are not, because they lead to unstable behavior of the bunches in collision.\(^4\) Intermediate values (1 < \( D < 10 \)) have advantages and disadvantages: They lead to the pinch effect described below, which can enhance the luminosity, but the same effect also results in a spraying-out of the beams after interaction to angles large compared to the incoming angles—a great increase in effective emittance. This in turn can cause difficult background problems in detectors.

The fourth scaling law gives the mean energy loss of particles in one bunch to bremsstrahlung in the electromagnetic field of the other, customarily called beamstrahlung (See Fig. 1). This energy loss is taken as a measure of the energy spread of the colliding particles, or more properly, a minimum value of it, since
the incoming beams from the accelerators will have some initial energy spread which will increase the net energy spread among the collisions. We call the fractional energy spread due to beamstrahlung $\delta_c$ or $\delta_q$ depending on whether the beamstrahlung's critical energy is below or above the beam energy. We view it as a performance-limiting parameter in the sense that, if it is too large, it deprives us of a knowledge of the collision energy. The classical equation is

$$\delta_c = 2.71 \frac{r_e^2 N^2 \gamma}{A \sigma_z}.$$  

The dividing line is determined by the critical energy of the beamstrahlung:

$$\epsilon_{\text{crit}} = \frac{3}{2} \frac{\hbar c \gamma^3}{\rho} = \frac{2}{4\sqrt{\pi}} \frac{\hbar c A^{1/2} D \gamma^3}{\sigma_z^2}.$$  

If the critical energy does not lie well below the beam energy, a quantum mechanical treatment of the radiation process must be used. For the case in which the beam energy lies below the critical energy, a rather good approximation to the beamstrahlung energy spread can be expressed in a simple formula which, however, is quite different in form from Eq. (4):  

$$\delta_q = 1.63 \left( \frac{\alpha^4 r_e N^2 \sigma_z}{A \gamma} \right)^{1/3},$$  

where $\alpha$ is the fine structure constant. The reader should be warned that the formulas for $\delta_c$ and for $\delta_q$ were derived from somewhat different models of the physical bunch and with different intended accuracies; however, their dependencies on the variables of the problem are reliable.
Comparing Eq. (4) with Eq. (6), we find that the combination $N^2/A$ appears in both. It is equal to $L/f$. For the same values of that quantity and of the bunch length, the quantum formula is far less punitive than the classical formula as the energy is increased. The suppression of the classical radiation by quantum mechanics is beneficial to us; the energy spread at high energies is less than it would otherwise be.

To design a linear collider for high-energy physics, the set of parameters we need to know is the set $(N, A, f, \sigma_z)$, in addition to $\gamma$. Thus, for accelerator design we need to solve Eqs. (1), (2), and (3) and either Eq. (4) or Eq. (6) in terms of the desired values of the set $(\gamma, L, \delta_c$ or $\delta_q)$ and feasible values of $D$ and $P_b$. To do so in either case is simple, but to know which case applies—high energy/quantum or low energy/classical—one must know the result. We must calculate one case first and then compare the critical energy with the beam energy to find our if we have calculated the right case. If not, we must calculate the other case. The two solutions are as follows:

**Classical ($\epsilon_{\text{crit}} \ll E$)**

$$N = \frac{1}{2.71(4\pi)} \left( \frac{P_b^2 D \delta_c}{r_b^2 E^2 L^2} \right)$$  \hspace{1cm} (7)

$$A = \frac{1}{2.71(4\pi)} \left( \frac{P_b^3 D \delta_c}{r_b^2 E^3 L^3} \right)$$  \hspace{1cm} (8)

$$f = 2.71(4\pi) \left( \frac{r_b^2 E L^2}{P_b D \delta_c} \right)$$  \hspace{1cm} (9)

$$\sigma_z = \frac{1}{4\pi} \left( \frac{P_b D}{r_e m_e c^2 L} \right)$$  \hspace{1cm} (10)

**Quantum ($\epsilon_{\text{crit}} > E$)**

$$N = \frac{4\pi}{(1.63)^3 \alpha^4} \left( \frac{\delta_q^3}{D} \right)$$  \hspace{1cm} (11)

$$A = \frac{4\pi}{(1.63)^3 \alpha^4} \left( \frac{P_b \delta_q^3}{EDL} \right)$$  \hspace{1cm} (12)

$$f = \frac{(1.63)^3 \alpha^4}{4\pi} \left( \frac{P_b D}{E \delta_q^3} \right)$$  \hspace{1cm} (13)

$$\sigma_z = \frac{1}{4\pi} \left( \frac{P_b D}{r_e m_e c^2 L} \right)$$  \hspace{1cm} (10)

Here, $E = \gamma m_e c^2$. Neither set of formulas is reliable when the critical energy is near the beam energy, because neither Eq. (4) nor Eq. (6) is valid there.

To aid us in understanding what these equations mean, we shall consider a specific example of a linear collider which would be an interesting machine if we could build it. We choose a beam energy of 5 TeV, at which the effective energy for the production of high-mass states is greater than that at a 20-TeV hadron collider like the SSC. We choose a luminosity of $10^{34}$/cm$^2$/s to produce useful
TABLE I
Values for Collider Parameters for the Conditions Defined in the Text

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual value</th>
<th>If $\hbar = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$2.8 \times 10^8$</td>
<td>$2.2 \times 10^3$</td>
</tr>
<tr>
<td>$A$ (cm$^2$)</td>
<td>$3.5 \times 10^{-14}$</td>
<td>$2.7 \times 10^{-19}$</td>
</tr>
<tr>
<td>$f$ (Hz)</td>
<td>4,500</td>
<td>$5.7 \times 10^8$</td>
</tr>
<tr>
<td>$\sigma_z$ (cm)</td>
<td>$3.4 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

event rates for a more-than-exploratory experimental program, and we choose a beamstrahlung energy spread of 0.3, which results in a spread of collision energies of about 10% when appropriately averaged. We choose a small value of $D$, 0.1, because of its influence on the interaction area and the frequency, and we choose $P_b = 1$ megawatt, a conceivable amount of power to dissipate and to pay for. The design values which follow from these choices are given in Table I. The center column gives the actual values. The rightmost column gives values which would result if $\hbar = 0$; they are totally fictitious, of course, but they serve to emphasize how profoundly the quantum nature of the radiation affects the behavior of linear colliders. Although the “actual values” are not within the scope of our present technology, they are closer to it by far than those in the “$\hbar = 0$” column.

Turning our attention to the actual values, we can see how the demands of frontier physics, taken together with these basic scaling laws for colliders, point out the directions in which our current technology must be developed to make linear colliders feasible in the TeV range of energies.

The spot size $A$ requires beam dimensions at the interaction region of a few angstroms—ten thousand times smaller than those presently being designed for in the SLC. The increased energy of the Table I collider, compared with the SLC, will automatically supply a tenfold smaller beam radius than in the SLC, but much more improvement is needed. Without going into detail, we can say that we shall require much smaller emittances and perhaps more sophisticated optical systems. These subjects are under study.

The repetition frequency $f$ is an order of magnitude beyond current linac technology, but there does not appear to be any fundamental physical reason that this parameter cannot be pushed to the required values. Several new accelerating technologies now under vigorous investigation are quite appealing for use in linear colliders, because they offer promise of much higher accelerating gradients than microwave linac technology does. Most of them involve high-power lasers, however, and high-power laser technology currently offers only repetition rates many orders of magnitude too low for collider use.

The bunch length $\sigma_z$ needs to be in the range of the wavelength of violet light. A rule of thumb tells us that if the bunches are to be accelerated by traveling sinusoidal waves the wavelength must be at least an order of magnitude longer than a bunch; such wavelengths lie in the infrared. The rule of thumb, however, is just that, and there is nothing to prevent much longer waves from accelerating
such short bunches, if the short bunches can be formed at all with such populations. This subject needs study.

Whether the required bunch population $N$ creates difficulties depends on the other bunch properties as well. We may remark that the current density implied by the values in Table I is of the order of $10^{18}$ amp/cm$^2$, which is about $10^8$ times higher than that expected in the SLC. So far, however, elementary space-charge force estimates have not revealed any threat to the feasibility of such current densities at the energy of 5 TeV.

2.2. Flat Beams

We must remember that all of the scaling laws and the results we have discussed up to this point have been for round beams. It has long been appreciated that flat beams—beams having their width much greater than their height, for example—offer significant advantages over round beams, at least in the classical regime. For the same cross-sectional area, a flat beam has lower peak electric and magnetic fields; in the classical regime, less energy is emitted as beamstrahlung. Thus, higher luminosities can be reached at a given $\delta_c$. In the quantum regime, it appears that the advantage of flat beams should diminish or disappear.

2.3. The Pinch

A phenomenon which is important in some collider designs is the pinch effect—a name we have adopted from plasma physics. When the opposing bunches of oppositely charged particles pass through one another, each bunch focuses the other, tending to make it smaller in transverse dimensions. The result is to raise the luminosity above the value it would have if the bunches retained their incoming lateral dimensions throughout the interaction.\textsuperscript{9} In very general terms, the pinch is beneficial when a collider has little beamstrahlung energy spread or, in other words, is starved for transverse particle density; it is less advantageous otherwise. The SLC expects to gain a substantial factor in luminosity from the pinch effect. It should be noted that the interaction area $A$ is defined in this paper as the area after the pinch has done its work—the actual effective interaction area. As a result, our definition of $D$[Eq. (3)] differs from the usage of some other authors. The reader is warned.

3. THE SLAC LINEAR COLLIDER

Shortly after the 1978 ICFA workshop mentioned in the introduction of this paper, SLAC began the study of a collider system which, although not strictly linear, could test many of the ideas embodied in linear colliders.\textsuperscript{10} The SLC takes advantage of the existing two-mile linear accelerator, using it as both of the linacs which are needed for a purely linear collider.
The two colliding bunches, one of positrons and the other of electrons, are accelerated in the linac, the electrons following the positrons by about twenty meters. They are separated to the left and right at the end of the linac (see Fig. 2). After separation the bunches are conducted around beam transport paths—the collider arcs—which aim them at one another, and then through final focus systems which powerfully demagnify the beams to tiny dimensions at the interaction point.

In fact, three bunches of particles are accelerated by the linac during each pulse of the SLC: The two bunches already mentioned—those intended to collide—are followed at another twenty-meter distance by another electron bunch whose destination is a heavy-metal target where the positrons are created to sustain the repetitive cycle of the collider. That target, the $e^+$ target, is located two-thirds of the way down the linac. The third bunch is plucked out of the linac by a fast-pulsed magnet which does not disturb the first two bunches. The positrons
created in the target are collected laterally by a high-field pulsed solenoidal focusing system and accelerated by a high-gradient accelerator to an energy of about 200 MeV. Then they are transported to the beginning of the linac.

When the positrons arrive there, the linac's electron gun fires twice to add two electron bunches and produce a procession of bunches just like the one described before—a positron bunch followed by an electron bunch followed by another electron bunch. These bunches are accelerated to about 1 GeV by the linac and then removed from it to a pair of 1-GeV storage rings, one for positrons and one for electrons.

These storage rings are designed to produce the fastest possible radiation damping and a very small equilibrium emittance. Such rings, called damping rings, are essential for producing the tiny beam spots required at the interaction region. The bunches must be cooled to an extraordinarily high density of particle points in phase space. Although the design of storage rings optimized for this function is a new art, the requirements are closely related to those of high-brightness synchrotron radiation sources which have been under study for some years, and the density needed for the SLC has not pushed us up against fundamental physics limits. Nevertheless, the SLC damping rings are the first of the breed, and they have not proven easy to build or operate. Damping rings will be required for future, higher-energy linear colliders, and the densities in phase space needed for those machines will be formidable in comparison to those of the SLC. The problems and limits in reaching such densities are under study in several centers.

The performance goals of the SLC were suited to the existing linac and to the immediate particle-physics prospects. It appears quite feasible to raise the operating energy of the linac to 50 GeV—just above the energy needed to produce the $Z^0$—and to operate at a luminosity which will yield thousands of $Z^0$s per day. The ultimate performance goals are as follows:

\[
E = 50 \text{ GeV} \\
L = 6 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1} \\
P_b = 0.072 \text{ MW} \\
D = 2.5 \\
\delta_c = 0.002
\]

These figures give, from the scaling laws,

\[
N = 5 \times 10^{10} \\
A = 10^{-7} \text{ cm}^2 \\
f = 180 \text{ Hz} \\
\sigma_z = 0.1 \text{ cm}
\]

In fact, the value of $\delta_c$ given above was a result, not a predetermined requirement, in the design of the SLC. Since the linac existed before the collider design, its parameters $f$ and $\sigma_z$ were given a priori.
Many difficult problems have had to be solved in designing and building the SLC. Some of them are peculiar to the SLAC adaptation of the linear collider idea. For example, the optical systems embodied in the arcs are highly sophisticated and precise. We shall not dwell here on such problems, as they may have little application in future colliders. Rather, we shall discuss some of the problems which will certainly have to be dealt with in those machines. The damping ring has already been discussed as an example of such a problem.

A family of fundamental problems arises from the nature of the linear accelerator as an environment for a compact, low-emittance bunch. By its nature, it is a hostile environment. The corrugated internal structure of the accelerating pipe, which is tailored to support high longitudinal gradients in waves which propagate at the speed of light, is also a fertile bed for the excitation of waves which deflect the particles from their intended paths. These waves are excited by the bunch itself whenever it strays from the center line of the pipe, and they combine to form the “wake field” of the bunch. If we think of the bunch as having a head and a tail, the wake field of the head deflects the tail, and the whole bunch becomes larger in effective cross section, with the result that the interaction area increases and the luminosity falls.

To counteract this disruptive effect, we strive to keep the head (and consequently the tail) very near the axis of the pipe, and in order to do that, we have had to straighten the pipe, to install precisely located beam-position monitors at frequent intervals, and to install precisely aligned quadrupole focusing magnets, also at frequent intervals, to increase the transverse focusing, especially at low energy.

Wake-field effects will surely pose major threats to the performance of higher-energy colliders, regardless of the accelerating technologies used in them. Any medium which supports the desired longitudinal accelerating waves is likely to support transversely deflecting waves too, and their effects will have to be curtailed even more stringently than in the SLC, because smaller spots will be required.

Noise of all kinds tends to vitiate the performance of a linear collider. Electrical noise is especially troublesome, including that arising from discharges and arcs in the high-power rf equipment, from irreproducibility in thyratron firings, and from timing jitter. To compensate for noise at frequencies which are low compared to the repetition frequency of the collider, many feedback loops are needed. For the SLC, several dozen loops have already been planned and the list continues to grow. Noise at frequencies comparable to or above $f$ must be kept to tolerable levels, because feedback cannot be used to control its deleterious effects.

Chromatic aberrations in the final focus system will clearly create a lower limit on the spot size if no other effect does. Even with the micrometer-size beams of the SLC, the optical design is very complex and the tolerances on optical elements exacting. Colliders of the future will, as we have seen, demand beam spots ten thousand times smaller.

In summary, the problems we have encountered in building the SLC have pointed out some of the important directions which research must take if the next
generation of colliders is to be realized. There surely will be others, particularly if the acceleration technology chosen is not that of the microwave linac.

On the other hand, the problems have not proved insuperable for the SLC: As of early 1986, the project is on schedule. SLAC plans to put the first experiment (Mark II) on the beam line at the beginning of 1987.

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REFERENCES

7. See, for example, Ref. 3.