COHERENT TRANSVERSE COASTING BEAM INSTABILITIES
IN THE ANTI ProTON ACCUMULATOR

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ABSTRACT

The existing theory of transverse instabilities of coasting beams due to wall impedance is applied to the Antiproton Accumulator in a rather detailed way. The vacuum chamber of the machine is carefully examined, taking into consideration its geometry and physical characteristics, together with, as far as possible, all the many items contained therein. The rectangular metal wall shape, which is the predominant type of vacuum chamber, is treated with particular emphasis. The dispersion relation coefficients and the transverse impedances at low frequencies are numerically evaluated by computer, obtaining the coherent tune shift and the instability growth-rate. Furthermore, the stability diagram is also calculated by computer for measured stack frequency distribution and compared with the threshold of stability given by the Schnell-Zotter criterion. The effect of the chromaticity on the stability limit is also analysed.

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The coherent transverse deformation of the beam has the form

\[ y = \tilde{y}_0 e^{j(\omega t - ks)} \]

where \( \tilde{y}_0 \) is the peak amplitude and \( s \) is the longitudinal coordinate along the particle orbit. The disturbance wavelength is \( \lambda = 2\pi/k \). The average collective motion of a coasting beam can be described as an oscillating closed sinewave with \( n \) wavelength per turn, which rotates around the machine. Therefore, at any time \( t \), there are \( n \) deformations around the ring, which means that the ring circumference must be a multiple of the wavelength, that is

\[ k = \frac{n}{\lambda} \]

where \( \lambda \) is the equivalent radius of the ring and \( n \) is an integer called the mode number. If all the particles have the same revolution frequency \( Q_0 \), then their velocity is \( v = Q_0 \lambda \), and in the frame of reference moving with the particles, the coherent displacement is

\[ \tilde{y} = \tilde{y}_0 e^{j(\omega - Q_0) t} \]

Assuming that the tune \( Q_0 \) is identical for all particles, the latter relationship is just the perturbed betatron motion

\[ y = y_0 e^{jQ_0 \Omega_0 t} \]

Combining the last two equations, one finds

\[ \omega = (n \pm Q_0) \Omega_0 \]

The phase velocity of the deformation wave is \( v = \omega / 2\pi \), or equivalently

\[ \beta = \left( 1 \pm \frac{Q_0}{n} \right) \beta_0 \]

which shows that this phase velocity will be close to the particle velocity for large mode numbers.
1. INTRODUCTION

This work applies the theory of transverse instabilities of coasting beams in the presence of a wall impedance, to the Antiproton Accumulator. We take the opportunity to summarise the standard theory for the benefit of the non-specialist reader. Anyone already familiar with the theory might care to skip to section 4.1.

Early theory of this effect formulated by Lasslet, Neil and Sessler \(^1\), was refined later by K. Hübner \(^3\), E. Jones \(^4\), A. Ruggiero \(^3\), V. Vaccaro \(^3\) and B. Zotter \(^2\). When a small coherent disturbance occurs locally in a charged particle beam, arising from statistical noise for instance, the beam undergoes oscillatory deformation around its equilibrium orbit. These oscillations induce an electromagnetic field in the walls of the vacuum chamber, which will react back coherently on the particles through the Lorentz forces. Because of the resistivity of the walls, these forces may have such a phase as to reinforce the oscillations and give rise to instability. This phenomenon is called transverse resistive wall instability.

The environment surrounding the circulating beam not only consists of a smooth stainless steel vacuum pipe but comprises also ceramic and ferrite structures built inside the vacuum chamber (i.e. stack tail kickers, injection and ejection kickers, precooling pick-ups and kickers), along with some other elements or irregularities such as clearing electrodes, bellows, pipe discontinuities, etc. These complex geometrical wall configurations with different material properties are responsible for generating non-uniform fields which contribute all together to the unstable coherent beam motion.

2. TRANSVERSE INSTABILITY AND IMPEDANCE FORMALISM

In this study, we only consider coherent dipole perturbation of the beam, that is transverse oscillations of the beam as a whole, where its cross-section remains constant. A simplified analysis of the transverse instability mechanism may be given as follows.

Let us assume that the coherent beam disturbance is at a single frequency and that its peak amplitude is small enough so that the relevant equations may be linearized. Thus, any arbitrary perturbation can be expressed as a superposition of the single frequency solution. Such a disturbance will oscillate with an angular frequency \(\omega\) and travels along the beam with a wave number \(k\).
The impedance thus defined is nothing else than the voltage given by integrating the Lorentz force per unit charge along the particle orbit, divided by the perturbed beam current displacement or dipole moment (times $\beta_0$). It has the dimensions of impedance divided by length ($\Omega/m$).

Following the original theoretical papers \(^5\), the impedance may be expressed in terms of the dispersion relation coefficients $U$ and $V$, namely

\[ z_L(s) = \frac{Z_0 e^\gamma Q_0}{2\pi R} \frac{1}{r_0 I} \left[ V(s) + j \left( U(s) + V(s) \right) \right] \quad [\Omega/m^2] \quad (10) \]

where $Z_0$ is the free space impedance, and $r_0$ the classical proton radius. The parameters $U$ and $V$ describe the interaction of the perturbed beam with its surroundings. They generally vary along the longitudinal direction, depending on the vacuum chamber geometry, the wall property and the beam characteristics. Using equations (7) and (10), and remembering that $r_0 = e^2/(4\pi\varepsilon_0 m_0 c^2)$ and $Z_0 = 1/\varepsilon_0 c$, the average force acting on a particle arising from the coherent disturbance $\bar{v}$ may be written as

\[ \langle F_L \rangle = -2j \frac{Z_0}{\varepsilon_0} m_0 \gamma Q_0 \bar{v} \left[ V + j(U + V) \right] \quad (11) \]

Let us rewrite the transverse linear density function in terms of the total number of circulating particles $N$ rather than in terms of the beam current $I$

\[ z_L = \frac{Z_0 \gamma Q_0}{N \frac{r_0}{\beta_0} c} \left[ V(s) + j(U(s) + V(s)) \right] \quad [\Omega/m^2] \quad (12) \]

Then, the transverse impedance turns into

\[ z_L = \frac{2\pi Z_0}{N \frac{r_0}{\beta_0} c} \left[ V + j(U + V) \right] \quad [\Omega] \quad (13) \]

with the average dispersion relation coefficients

\[ \bar{U} = \frac{1}{2\pi R} \int U(s) \, ds \quad \text{and} \quad \bar{V} = \frac{1}{2\pi R} \int V(s) \, ds \quad [s^{-1}] \quad (14) \]
These perturbation waves induce in the vacuum pipe image charges of opposite sign and image currents flowing in the opposite direction to the beam. It will be shown that, in presence of resistive walls, the electromagnetic field arising from the wave with frequency \( \omega = (n+Q_o) \omega_b \), moving faster than the beam, produces a transverse Lorentz force component which leads to damped oscillations. On the other hand, the wave with frequency \( \omega = (n-Q_o) \omega_b \), which moves slower than the beam, gives rise to a growth of the transverse oscillations.

To describe the nature of the electromagnetic forces which act on the perturbed beam, it is convenient to use the transverse coupling impedance formalism. For non-smooth vacuum pipe with cross-section variation in the longitudinal coordinate, the Lorentz forces are varying along the particle trajectory. Let us define the transverse impedance linear density function by

\[
\mathcal{Z}_\perp(s) = \frac{j}{\varepsilon \beta_0 \bar{y}_0} \langle F_\perp \rangle \quad [\Omega m] \quad (7)
\]

which has the dimensions of impedance per unit area. \( J \) denotes the beam current, \( \bar{y}_0 \) the peak amplitude of the transverse beam oscillations and \( \langle F_\perp \rangle \) is the average of the transverse force component over the beam cross-section, of area \( A \), at location \( s \). This force is proportional to the exponential factor \( \exp \left[j(\omega t - ks)\right] \). Dropping out this factor, one gets the peak amplitude of the average force.

\[
\langle F_\perp \rangle = \frac{1}{A} \int_A \int (E^+ + v^+ x B^+) \perp \, dx \, dy \quad (8)
\]

where \( E^+ \) and \( B^+ \) are the transverse electromagnetic fields generated by the beam displacement \( \bar{y}_0 \) and \( v^+ \) is the particle velocity, \( v = \beta_0 c \). Thus, the transverse impedance seen by the particle beam over one machine revolution is the integral of the transverse impedance density function along a particle trajectory. In formula

\[
\mathcal{Z}_\perp = \int_0^{2\pi R} \mathcal{Z}_\perp(s) \, ds = \frac{j}{\varepsilon \beta_0 \bar{y}_0} \int_0^{2\pi R} \langle F_\perp \rangle \, ds \quad [\Omega m] \quad (9)
\]

The complex factor \( j \) has been chosen to ensure a purely reactive impedance for a lossless vacuum pipe. Note that early papers on transverse instabilities defined the impedance, using the imaginary unit \( i=-j \) in order to ensure a purely capacitive impedance for lossless structure.
If $|U|$ and $|V|$ are much less than $Q_o R_o$, we approximately find the angular frequency of the deformation wave

$$\omega = (n \pm Q_o) \frac{Q_o}{\alpha} \pm \left[ U + (1 - j)V \right]$$

(22)

Therefore, the motion equation of the coherent transverse beam displacement is

$$- \frac{1}{\tau} j \omega_o \left[ Q_o - (U+V)/\alpha \right] t = Y_t$$

$$y = y_0 e^{-\frac{1}{\tau} v_t}$$

(23)

Assuming that $V$ is of the same sign as $\omega$, with the $\omega$ given by eq.(5) as a first approximation, the top signs of eqs (22) and (23) represents a fast wave which is damped since $V$ is positive, whereas the bottom signs, for $n > Q_o$, represents a slow wave which grow exponentially, because $V$ is still positive, with an e-folding time $\tau$, and produces in addition a coherent tune shift $\Delta Q$

$$\frac{1}{\tau} = V$$

and

$$\Delta Q = \frac{U+V}{\alpha}$$

(24)

When $n < Q_o$, the bottom signs represents a wave with negative phase velocity which is damped since $V$ is negative. Thus, for $V > 0$, the slow waves are always unstable. Equivalently, the average growth rate and tune shift are, using (13) and (24).

$$\frac{1}{\tau} = \frac{N r_o Q_o}{2 \pi Z_o Q_o} \text{Re}(\mathcal{L})$$

$$\Delta Q = \frac{-N r_o}{2 \pi Z_o Q_o} \text{Im}(\mathcal{L})$$

(25)

For high intensity particle stack, care must be taken to avoid the tune to be shifted to a resonant value.

The foregoing instability analyses was made under the assumption that all particles in the beam have the same betatron frequency, and led to the conclusion that instabilities are always present for slow wave disturbances. In reality, there are momentum dispersion and oscillation amplitude spread among particles which may stabilize the beam.

Let us write eq.(22) for the fast wave determination in the form

$$1 = \frac{U + (1-j)V}{\omega - (n-Q_o)Q_o}$$
3. DISPERSION RELATIONS AND STABILITY DIAGRAM

The equation of motion of one particle with betatron frequency \( Q_0 R_0 \) oscillating inside a particle beam is

\[
\frac{d^2 y_i}{dt^2} + (Q_0 R_0)^2 y_i = \frac{\langle F_i \rangle}{m_0 y_i} \tag{15}
\]

where we have assumed that all the particles in the beam have the betatron tune \( Q_0 \) and \( \langle F_i \rangle \) is the force due to the beam induced fields. We suppose also \( U \) and \( V \) constant around the machine so that \( \dot{U} = U \) and \( \dot{V} = V \). Using (3) and (11), the latter equation takes the form

\[
\frac{d^2 y_i}{dt^2} + (Q_0 R_0)^2 y_i = 2 Q_0 R_0 \overline{y}_0 \left[ U + (1-j)V \right] e^{j(\omega-n R_0)t} \tag{16}
\]

Looking for a solution of the form

\[
y_i = y_{i0} e^{j(\omega-n R_0)t} \tag{17}
\]

we find

\[
[-(\omega-n \Omega_0)^2 + (Q_0 \Omega_0)^2] y_{i0} = 2 Q_0 \Omega_0 \overline{y}_0 \left[ U + (1-j)V \right] \tag{18}
\]

The mean displacement of these particles is nothing else than the coherent deformation of the beam, that is

\[
\frac{1}{N} \sum_{i=0}^{n} y_{i0} = \overline{y}_o \tag{19}
\]

Thus, averaging eq. (18), we get

\[
-(\omega-n \Omega_0)^2 + (Q_0 \Omega_0)^2 = 2 Q_0 \Omega_0 \left[ U + (1-j)V \right] \tag{20}
\]

Although the quantities \( U \) and \( V \) are not independent of \( \omega \) as we will see in the next section, we assume their values constant for the sake of simplicity. Then, the two roots of this quadratic equation in \( \omega \) are

\[
\omega = n \Omega_0 \pm Q_0 \Omega_0 \left( 1 - \frac{2}{Q_0 \Omega_0} \left[ U + (1-j)V \right] \right)^{1/2} \tag{21}
\]
The dispersion relation for slow waves takes the form

\[ -1 = (U' - jV') \int_{-\infty}^{\infty} \frac{F(x)}{x-x_1} \, dx \quad (31) \]

The threshold of instabilities is now given by the condition \( \text{Im}(x_1) = 0 \). Solving eq. (31) for any complex number \( x_1 \), constitutes a mapping of the complex \( x_1 \) plane onto the \((U'V')\) plane. The graph of this mapping is the stability diagram. For \( x_1 \) on or slightly above the real axis, we may consider the pole to be on the axis provided we make the contour of interation miss the pole by going around \( x_1 \) on a little semicircle above. Then, the integral (31) may be expressed as follows:

\[ \int_{-\infty}^{\infty} \frac{F(x)dx}{x-x_1} = P \int_{-\infty}^{\infty} \frac{F(x)dx}{x-x_1} + j \pi F(x_1) \quad (32) \]

where the letter \( P \) denotes the Cauchy principal value integral.

In a similar fashion, eq.(31) may be expressed in terms of the transverse impedance rather than in terms of the dispersion parameters. Using eq.(13), we get

\[ -j = Z'_{\perp} \int_{-\infty}^{\infty} \frac{F(x)dx}{x-x_1} \]

where \( Z'_{\perp} \) is the normalized transverse impedance

\[ Z'_{\perp} = \frac{N f_0 a_0}{2 \pi Z_0 a_0} \frac{Z_{\perp}}{\gamma} \quad \text{with} \quad Z_{\perp} = \text{Re}(Z_{\perp}) + j\text{Im}(Z_{\perp}) \quad (33) \]

and consequently

\[ U' = \text{Im}(Z'_{\perp}) \quad V' = \text{Re}(Z'_{\perp}) \]

Examination of stability diagrams for a wide range of momentum distributions led Schnell and Zotter 5) to formulate an approximate stability criterion.

\[ |U' - jV'| < \frac{Z}{\pi} F \quad (34) \]

where \( F \) is a form factor of the order of unity for most reasonable distributions (i.e. symmetric with a central plateau). Combining eqs (13), (29) to (31) and bearing in mind that \( |U + (1-j)V| = |V + j(U+V)| \), we get

\[ |Z'_{\perp}| < \frac{4 Z_0 a_0 \gamma Q_0}{N r_0} |(n-Q_0 r_0 - Q_0 \beta | \frac{\theta}{p} \frac{\hbar}{n} \quad (35) \]
Taking into account the normalized momentum distribution function \( f(p) \) of the particle beam, the previous equation leads to the dispersion relation for the slow wave, including momentum spread only

\[
1 = \left[U + (1-j)V\right] \int_{-\infty}^{\infty} \frac{f(p)}{\omega - (n-Q)\Omega} \, dp \quad \text{with} \quad \int f(p) \, dp = 1
\]

(26)

where \( \Omega \) and \( Q \) are the revolution frequency and the tune of the particle beam, which depend on momentum \( p \). Expanding \( \Omega \) and \( Q \) to the first order, we get

\[
\Omega = \Omega_0 + \frac{(p - p_0)}{p} \Omega_0 \eta \quad \text{and} \quad Q = Q_0 + \frac{(p - p_0)}{p} Q_0 \xi
\]

(27)

where \( \eta \) is the transition parameter and \( \xi \) the chromaticity. The index zero refers to a reference particle with momentum \( p_0 \). Hence the dispersion relation takes the form

\[
1 = \left[U + (1-j)V\right] \int_{-\infty}^{\infty} \frac{f(p)}{\omega - (n-Q_0)\Omega_0 - [(n-Q_0)\eta - Q_0 \xi] \Omega_0 (p - p_0)/p} \, dp
\]

(28)

Solving the above equation would yield the slow wave frequency function \( \omega \), generally complex valued, depending on the wave number \( n \). For slow wave one sees, eq. (3), that exponential growing waves occur if the imaginary part of \( \omega \) is negative. Thus, the threshold of instabilities is determined by solving (28) for \( \text{Im}(\omega) = 0 \). The integral can be analytically calculated only for the simpler distribution functions \( f(p) \). For more complicated functions, numerical integration is to be used. Following references \(^3,6\), it is convenient to introduce normalized quantities such that the integral depends only on the shape of the distribution function. Let us define first the half momentum spread \( \Delta p_{hh} \) at half height so that \( f(p_0 \pm \Delta p_{hh}) = 1/2 \, f(p_0) \). Then, let us introduce the frequency deviation and the betatron frequency spread

\[
\Delta \omega = \omega - (n-Q_0)\Omega_0 \quad \text{and} \quad \Delta \delta = \left[(n-Q_0)\eta - Q_0 \xi\right] \Omega_0 (\Delta p) / p_{hh}
\]

(29)

Furthermore, with the change of variables

\[
x = \frac{p - p_0}{\Delta p_{hh}} \quad x_1 = \frac{\Delta \omega}{\Delta \delta} \quad F(x) = f(p(x)) \Delta p_{hh}
\]

(30)

\[
U' - jV' = \frac{U + (1-j)V}{\Delta \delta}
\]
\[ U = \frac{N \rho_c}{Q \beta_o} \left( \frac{2}{Y^2 \Sigma g^2} \right) \]
\[ V = \frac{N \rho_c}{Q \beta_o} \tilde{R} \tilde{\text{Im}} \quad [\Omega^{-1}] \quad (36) \]

where \( \tilde{R} \) is the surface resistivity.

\[ \tilde{R} = \left( \frac{\mu \omega \varepsilon_o}{2\sigma} \right)^{1/2} \quad \text{[dimensionless]} \]

and

\[ \text{Re} = \Sigma m g^2 \left[ \frac{\mu^2}{4} + \frac{k^2 (\beta - \beta_o)^2}{\mu} \right] \text{cth} \left( \frac{\mu h}{2} \right) \]
\[ \text{Im} = \Sigma m g^2 \left[ \frac{\mu^2 \beta^2}{o} + k^2 (\beta^2 - \beta_o^2) + \frac{1}{\mu^2} \left( \frac{k \omega_c^2}{c} (\beta - \beta_o)^2 \right) \csc^2 \left( \frac{\mu h}{2} \right) \right] \quad (37) \]

\[ \mu^2 = \left( \frac{m \pi}{w} \right)^2 + k^2 - \left( \frac{\omega}{c} \right)^2 \]

The quantities \( k \) and \( \omega \) denote the wave number and slow-wave angular frequency given by eqs (1) and (5) (with the lower sign). Assuming a parabolic horizontal particle density, we find

\[ g_m = 24 \left( \frac{w}{m \pi \Delta} \right)^2 \sin \left( \frac{m \pi \alpha}{w} \right) \left[ \frac{w}{m \pi \Delta} \sin \left( \frac{m \pi \Delta}{2w} \right) - \frac{1}{2} \cos \left( \frac{m \pi \Delta}{2w} \right) \right] \quad (38) \]

For that distribution, we have \( \Sigma m g^2 = \frac{3w}{5 \Delta} \).

Note that another particle density (e.g. uniform) would not have a great effect on the numerical results.

Equivalently, using eq. (10), the transverse resistive wall impedance density function takes the form

\[ z = \frac{Z_o}{\beta_o^2} \left[ \tilde{R} \tilde{\text{Im}} + j \left( \text{Re} - \frac{6 w}{5 \beta_o^2 \Delta} \right) \right] \quad [\Omega m^2] \quad (39) \]
4. EXPRESSIONS FOR THE DISPERSION RELATION COEFFICIENTS AND IMPEDANCES

Analytical formulae are given below for the dispersion relation coefficients $U$, $V$ and the transverse impedances $Z_\perp$ for different wall structures and materials, in relation with the AA ring configuration. These expressions, taken from papers of L. Laslett et al. 1), E. Jones 4) and B. Zotter 2), apply when the disturbance wavelength is long compared with the transverse dimensions of the vacuum pipe. Furthermore, they are only valid for wall extending to infinity. Thus to take into account the numerous vacuum chamber irregularities around the machine, the additional impedances of cross-section variations should be evaluated. However, according to Zotter's paper 7), the contribution of these transition regions for the long wavelength limit may be neglected compared to the smooth wall effect. Therefore, the total impedance of the piecewise vacuum pipe will be approximated by summing up the product of the impedance linear density function, eq. (7) of each element with constant cross-section times the element length.

The rotating beam encounters mainly two geometries. The first is that in which the vacuum chamber pipe has a circular cross-section. To facilitate the analytical calculations, the beam is supposed to be cylindrical. Such wall configurations are located in quasi-zero dispersion regions and a uniform transverse beam density is assumed. The second geometry is that in which the vacuum chamber has a rectangular cross-section. The beam is assumed to be of the same shape. Such wall structures characterize non-zero dispersion regions and a uniform vertical beam density is assumed whereas the horizontal density is let to the user convenience. The electromagnetic field induced by the coherent oscillating beam, the transverse Lorenz force component, eq. (8) and then the parameters $U$, $V$ and the transverse impedance, eqs (10-11), are calculated by solving the Maxwell's equations with the boundary conditions. The results are just summarized below, for some simple configurations.

4.1 Rectangular metal wall (resistive)

The AA vacuum chamber mainly consists of rectangular stainless steel walls ($\varepsilon_r=1.004$; $\alpha=7.69 \times 10^5 \, [\Omega^{-1} \, m^{-1}]$).
To ensure continuity of the vacuum chamber shape, rectangular screen structures have been installed in almost all the circular vacuum tanks, in order to maintain a nearly constant impedance density function around the machine. For this reason, the AA ring contains only a small number of circular vacuum chamber sections.

4.3 Circular lossy ferrite wall

This wall configuration is a crude approximation for the geometrical structures encountered in the non-zero dispersion regions of injection and ejection kickers.

\[
U = \frac{N r_0 c}{2 m Q \beta_0} \left[ (1 + \beta^2) b^2 - 1 \right] \quad V = \frac{N r_0 c}{\pi a Q \beta_0} \left[ \frac{\tan \delta_c}{\epsilon_\tau} + \frac{\tan \delta_\mu}{\mu_\tau} \right] \quad [\text{s}^{-1}] \quad (45)
\]

where \(\tan \delta_c\) is the dielectric loss tangent \((\tan \delta_c / \epsilon_\tau = 2.1 \times 10^{-4})\) and \(\tan \delta_\mu\) the magnetic loss tangent \((\tan \delta_\mu / \mu_\tau = 2.5 \times 10^{-4})\). Since the beam is in general not concentric to the ferrite structure because of the dispersion, the "radius \(a\)" is taken as the minimum distance between the beam centre and the ferrite wall.

In terms of impedance, we get

\[
z = \frac{Z_0}{2 \pi} \left[ \frac{1}{a^2} \left( \frac{1}{\beta_0^2} + \frac{1}{\epsilon_\tau} + \frac{1}{\mu_\tau} \right) + j \left[ \frac{1}{a^2} \left( \frac{1}{\beta_0^2} - \frac{1}{\epsilon_\tau} - \frac{1}{\mu_\tau} \right) \right] \right] \quad [\Omega/m^2] \quad (46)
\]

4.4 Circular lossy ceramic wall

The ceramic vacuum chamber is a good representation of momentum stack tail kicker structures installed in zero dispersion region. Hence the beam centre is concentric to the pipe centre.

\[
U = \frac{N r_0 c}{2 m Q \beta_0} \left[ \frac{\epsilon - 1}{\epsilon + 1} \right] \quad V = \frac{N r_0 c}{\pi a^2 Q \beta_0} \left[ \frac{\epsilon}{\epsilon + 1} \right] \quad [\text{s}^{-1}] \quad (47)
\]

The impedance takes the form \((\epsilon_\tau = 9.5, \tan \delta_c = 10^{-3})\)

\[
z = \frac{Z_0}{2 \pi} \left[ \frac{1}{a^2} \left( \frac{\epsilon}{\beta_0^2 (1 + \epsilon_\tau)} \right) + j \left[ \frac{1}{a^2} \left( \frac{\epsilon}{\beta_0^2 (1 + \epsilon_\tau)^2} - \frac{1}{\beta_0^2} \left( \frac{1}{\epsilon_\tau - 1} \right)^2 \right) \right] \right] \quad [\Omega/m^2] \quad (48)
\]
Note that to derive the latter expressions, the top and bottom walls have been supposed resistive and the side walls perfectly conducting. The thickness of the walls was also assumed to be greater than the skin depth

\[ \delta = \left( \frac{2}{\mu_0 \mu_r \omega \sigma} \right)^{1/2} \text{ [m]} \] (40)

and the metal wall is considered as quasi non-magnetic \((\mu_r = 1)\).

In the limit \(k \ll 1/\omega \) and \(1/h \) (long wavelength limit), and if the width of the vacuum chamber is much larger than its height, we get

\[ U = \frac{6 \ N \ r_o \ c}{5 \ \xi Q \ \beta_o \ \gamma^3 \ \Delta} \left( \frac{1}{h} - \frac{1}{\tau} \right) \quad \text{V} = \frac{12 \ N \ r_o \ c^2 \ \beta}{5 \ h^2 \ \Delta Q \ \gamma \ \omega} \hat{R} \text{ [s}^{-1}] \] (41)

The transverse impedance becomes

\[ z_{\perp} = \frac{6 \ Z_o}{5} \left[ \frac{\mu_r \delta}{h^2 \Delta} + j \left( \frac{\mu_r \delta}{h^2 \Delta} + \frac{1}{\beta_o^2 \gamma^2 \Delta \ h} \left( \frac{1}{h} - \frac{1}{\tau} \right) \right) \right] \text{ [V/m}^2] \] (42)

4.2 Circular metal wall (resistive)

This kind of vacuum chamber being located in quasi zero dispersion regions, the beam is concentric to the pipe.

\[ U = \frac{N \ r_o \ c}{2 \pi \ Q \ \beta_o \ \gamma^3} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \quad \text{V} = \frac{N \ r_o \ c^2 \ \beta_o}{\pi \ a^3 \ Q \ \gamma \ \omega} \hat{R} \text{ [s}^{-1}] \] (43)

Combining these two equations with eq. (10), (36) and (40), we find

\[ z_{\perp} = \frac{Z_o}{2\pi} \left[ \frac{\mu_r \delta}{a^3} + j \left( \frac{\mu_r \delta}{a^3} - \frac{1}{\beta_o^2 \gamma^2 \ a^3} \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \right) \right] \text{ [V/m}^2] \] (44)
\[ z_\perp = \frac{j z_0}{\varepsilon g a} \left( \frac{\lambda^2 - 1}{\lambda^2 + 1} - \frac{0.8526}{\beta_0^2 \varepsilon} \right) \quad [\Omega/\text{m}^2] \quad (52) \]

where \( \lambda = \frac{d}{a} \) and \( \varepsilon = \frac{m a}{q} \)

For the AA, the beam sees over one machine revolution, about 2 m of bellows with typically \( a \approx 160 \text{ mm}, \ d \approx 180 \text{ mm}, \ g \approx 10 \text{ mm} \). Thus, we find \( z_\perp \approx 232 \approx \Omega/\text{m}^2 \)
which is not important in comparison to the resistive smooth pipe effect (see the numerical results below).

**Remark**

The above formulae are roughly valid for a bunched beam on ejection orbit provided the peak particle current is considered rather than the average current \( I \). Assuming a Gaussian bunch, we have \(^9\)

\[ I_{\text{peak}} = 1.6 I \frac{L}{L_0} \quad \text{with} \quad I = \frac{Ne \beta_0 c}{2\pi R} \quad (53) \]

where \( L \) is the ejection orbit length and \( L_0 \) is the bunch length. For the AA, we approximately have \( I_{\text{peak}} \approx 10.7 I \) (for SPS fill).

5. **NUMERICAL RESULTS**

5.1 **Transverse impedance evaluation**

The mean relation dispersion parameters and the transverse impedance of the AA ring are calculated by averaging the local values at about 200 locations of the machine circumference. Integrals (9) and (14) are replaced by finite sums

\[ \bar{U} = \frac{\sum U L_i}{\sum L_i} \quad \bar{V} = \frac{\sum V L_i}{\sum L_i} \quad Z_\perp = \frac{\sum z_{\perp i} L_i}{\sum L_i} \quad (54) \]

where \( L_i \) is the section length along the particle orbit and \( U_i, V_i \) are computed using eqs (36) to (51), taking into account as far as possible the many structures encountered around the ring.
4.5 Rectangular ferrite and metal walls

This compound structure, proposed by E. Jones \(^{4}\), is a simplified model of a precooling pick-up or kicker (ferrite loops). The geometry is that of a square lossless ferrite configuration with perfectly conducting metal side walls.

\[
U = \frac{N r_o c \sigma_o \tau}{2ab \gamma} \epsilon \sum_{m=1}^{\infty} \frac{\sin^2(m \pi b/a)}{(m \pi/a)^2 - \nu^2} \quad V = 0 \quad [\text{a}^{-1}] \quad (49)
\]

where

\[
\nu^2 = \omega^2 \epsilon_r \frac{c^2}{c^2}
\]

The transverse impedance density function is

\[
z_\perp = j \frac{\omega Z_0}{2ab} \epsilon \sum_{m=1}^{\infty} \frac{\sin^2(m \pi b/a)}{(m \pi/a)^2 - \nu^2} \quad [\text{a}^{-1} \text{m}^2] \quad (50)
\]

When the disturbance frequency \(\omega\) is sufficiently low (long wavelength limit), we get

\[
z_\perp = j \frac{\omega Z_0}{2ab} \quad [\text{a}^{-1} \text{m}^2] \quad (51)
\]

Remark

In addition to the wall configurations reported above, the circulating beam interacts with many other equipments and obstacles, such that pick-up plates, clearing electrodes, RF cavity, bellows. The impedance of these objects is in general not tractable analytically. Hopefully, for some of them, such as the RF cavity \(^7\), the impedance can be ignored because a feedback system keeps it low. For some others, the impedance is completely negligible compared to the smooth wall contribution. For instance, the impedance density function of circular bellows extending to infinity, assuming a perfectly conducting metal, is at low frequency \(^8\).
For an antiproton stack during the accumulation process with the following characteristics:

\[
\begin{align*}
N &= 2.93 \times 10^{11} \bar{p} \\
Q_H &= 2.2708 \\
\xi_H &= 0.10 \\
\epsilon_H &= 7.9 \times \text{mm.mrad} \\
\delta p/p &= \pm 2.9 \times 10^{-3} \quad \text{(at 95\%)} \\
(\delta p/p)_{95\%} &= \pm 1.8 \times 10^{-3}
\end{align*}
\]

The transverse impedances, the instability growth times and the tune shift, considering the lowest mode number \(n = 3\) are:

\[
\begin{align*}
Z_L &= 3.6 \times 10^4 - j \times 6.2 \times 10^6 \ [\Omega/m] \\
\tau_V &= 0.11 \ [s] \\
U'_{V} &= -1.2 \\
V'_{V} &= 6.0 \times 10^{-3} \\
Q_V &= 1.4 \times 10^{-4} \\
Z_L &= 3.4 \times 10^3 + j \times 2.6 \times 10^7 \ [\Omega/m] \\
\tau_H &= 1.1 \ [s] \\
U'_{H} &= 5.0 \\
V'_{H} &= 6.6 \times 10^{-4} \\
Q_H &= -5.6 \times 10^{-4}
\end{align*}
\]

The absolute value of the impedance is almost independent of the mode number \(n\) at low frequency, since the real part of the impedance is much less than the imaginary part and the latter is independent of \(n\).

For another antiproton stack, left cooling down until stable, whose characteristics are:

\[
\begin{align*}
N &= 2.63 \times 10^{11} \bar{p} \\
Q_H &= 2.2646 \\
\xi_H &= 0.03 \\
\epsilon_H &= 2.6 \times \text{mm.mrad} \\
\delta p/p &= \pm 1.8 \times 10^{-3} \quad \text{(at 95\%)} \\
(\delta p/p)_{95\%} &= \pm 1.2 \times 10^{-3}
\end{align*}
\]

The transverse impedances, the instability growth times and the tune shift are for \(n=3\):

\[
\begin{align*}
Z_L &= 4.1 \times 10^4 - j \times 1.2 \times 10^7 \ [\Omega/m] \\
\tau_V &= 0.10 \ [s] \\
U'_{V} &= -3.1 \\
V'_{V} &= 1.1 \times 10^{-2} \\
Q_V &= 2.3 \times 10^{-4} \\
Z_L &= 3.4 \times 10^3 - j \times 4.5 \times 10^7 \ [\Omega/m] \\
\tau_H &= 1.3 \ [s] \\
U'_{H} &= 12.0 \\
V'_{H} &= 9.1 \times 10^{-4} \\
Q_H &= -8.9 \times 10^{-4}
\end{align*}
\]
For the region where the beam is assumed to be circular, we take

\[ b = \max \left( \sqrt{\frac{R_x}{x}}, \sqrt{\frac{R_y}{y}} \right), \]  

(55)

and for those where the beam is considered as rectangular, we define

\[ \Delta = 2 \left( a \frac{\Delta_p}{p} + \sqrt{\beta_x} \frac{\epsilon_x}{x} \right); \quad \tau = 2 \sqrt{\beta_y} \frac{\epsilon_y}{y}; \]  

(56)

The inner wall coordinates of the vacuum chamber measured from the central orbit are taken from BEPO plot produced by A. Poncet \(^{10}\). The lattice parameters and the beam characteristics are mostly taken from reference \(^{11}\). BEPO output not only gives the transverse vacuum chamber dimensions but include also the aperture limitation due to metal plates such that clearing electrodes and pick-ups.

For a rectangular vacuum chamber cross-section, the horizontal coherent beam oscillations have been studied by simply rotating the geometry by \(\pi/2\), which might perhaps be a too crude approximation.

The computations of the transverse impedances and all the related quantities have been performed with a FORTRAN program running on a CYBER 875 computer. According to the foregoing formulae, we note that the dispersion parameters \(U, V\), the growth rate \(\tau^{-1}\) and the coherent tune shift \(\Delta Q\) depend linearly on the total number of particles in the accumulator whilst the transverse impedance is independent of this number at low frequencies. Therefore, all the computations will be performed taking a nominal stack of \(N = 10^{12}\) antiprotons.

Figs 1 and 2 show the real and imaginary part of the transverse impedances \(Z_{\perp}\), the instability growth rate \(1/\tau\) and the tune shift \(\Delta Q\) versus the mode number \(n\) (for the long wavelength limit). Figs 3 and 4 display the transverse impedances, respectively versus the emittances \((\epsilon_x = \epsilon_y)\) and the relative momentum spread.
Retaining the results given by the stability diagram, all vertical modes above n=3 and horizontal modes above n=6 lie within the stable region provided the chromaticity is kept null or positive to avoid the cancellation of betatron frequency ΔS, as n is negative for the Antiproton Accumulator.

Remark

Remember that U' and V' are proportional to the number of particles so that it is easy to modify the final results for a higher stack, assuming that the shape of frequency distribution is unchanged. For instance, considering N = 10^{12} \rho, vertical modes would be unstable up to n=6 (f=6.9 MHz) and horizontal modes would be unstable up to n=19 (f=31.0 MHz).

6. DISCUSSION

The previous results concerning the stability limit of a dense antiproton stack are too optimistic because the small tune variation in the stack region was not taken into consideration as a first approximation. Unfortunately, a negative chromaticity may give rise to instabilities for different mode numbers, if the betatron frequency spread ΔS becomes small enough to push the values of U' and V' out of the stable region. This can occur for mode numbers n close to Q_0(1+ξ/n), since

\[ ΔS = \left\lfloor \frac{nQ}{Q_0} - Q_0 \right\rfloor ξ \frac{ΔS}{ΔS} \]

partially cancels and thus makes the beam unstable according to eqs. (30-31). ΔS is zero for a chromaticity limit ξ_0 and a chromaticity frequency Ω_0,

\[ ξ = \frac{n}{Q_0} - 1 \quad Ω = \frac{Q_0 ξ_0}{n} = - \left( \frac{dQ}{dΩ} \right)_0 η \]

(57)

where the signs of ξ and n are identical. Figs 9 and 10 plot the coefficients U'V' and V'V' versus the chromaticity for different modes n, in the case of the particular cooled stack studied in section 5.2. The measured vertical chromaticity around the stack core region being ξ = -0.03, computations indicate that the modes n=3 (already discussed) and n=4 are expected to be unstable, since the upper stability limit is roughly |U'| = 2 (for V' < 0.01). However, feedback stabilization by means of a transverse damping system with a bandwidth of about 40 MHz (n=24) is a cure for instabilities up to a stack intensity of at least 10^{12} \rho, provided the chromaticity is kept close to zero, more precisely ξ > -0.8 for n=24, according to eq.(57).
For a cooled bunched beam of about $2.4 \times 10^{10}$ circulating on the ejection orbit just before a SPS transfer, the transverse impedances are, in the approximation of formula (53):

$$Z_{LV} = 4.1 \times 10^4 - j \ 3.0 \times 10^6 \ [\Omega/m] \quad U'_V = -1.0 \quad V'_V = 1.3 \times 10^{-2}$$
$$Z_{LH} = 3.5 \times 10^3 - j \ 4.1 \times 10^7 \ [\Omega/m] \quad U'_H = -14.4 \quad V'_H = 1.1 \times 10^{-3}$$

5.2 Stability diagram computation

A NODAL program, for use on a front-end computer NORD 10, has been written to evaluate and plot the stability threshold given any frequency (or momentum) distribution of the particle beam. Fig. 5 shows the diagram of stability for an antiproton stack during the stacking process. The corresponding frequency distribution of the particle is shown in Fig. 6. The solution of the dispersion relation (31) for real frequencies gives a curve in the $(U',V')$ plane, which encloses the stable region (continuous curve). By comparison, the approximate Schnell-Zotter criterion (34) for a form factor equal to unity, gives a semicircle in the $(U',V')$ plane (dotted curve), which appears to be more restrictive than the effective stability limit. Fig. 7 shows the stability diagram of a well cooled stack, whose frequency distribution is shown in Fig. 8. Note in passing that both stability diagrams look like that of a triangular frequency distribution.

To avoid a too erratic behaviour of the stability diagrams, the acquired particle distribution functions have been smoothed by means of spline approximation techniques as a preliminary.

The transverse impedances corresponding to the stack parameters described above, once transformed into $U'$, $V'$ and $Z'_{\perp}$ normalized components by eqs. (12)-(30)-(33), are reported in Figs 5 and 7 for some mode numbers, assuming a chromaticity equal to zero. Thus, for the diagram of Fig. 7, only the mode $n=3$ ($f=1.4$ MHz) is unstable for the vertical plane, whilst modes up to $n=6$ ($f=6.9$ MHz) are unstable for the horizontal plane. On the other hand, the Schnell-Zotter criterion (35) gives

$$|Z_{LV,H}| < 3.2 \times 10^6 (n-Q_{V,H}) \quad \text{for} \quad N = 2.63 \times 10^{11} \ p$$

Consequently, with $|Z_{LV}| = 1.2 \times 10^7 \ [\Omega/m]$ and $|Z_{LH}| = 4.5 \times 10^7 \ [\Omega/m]$ for any mode numbers, all modes up to $n=5$ ($f=5.1$ MHz) and up to $n=16$ ($f=25.5$ MHz) are respectively unstable for the vertical and horizontal planes.
GLOSSARY

a  half aperture of circular vacuum chamber  [m]
A  beam cross-section area  [m^2]
b  beam radius (for circular beam cross-section)  [m]
B  magnetic field  [tesla]
c= 2.9979.10^8 light velocity  [m/s]
d  half aperture of enlarged pipe (bellows)  [m]
e= 1.6022.10^{-19} particle charge  [Coulomb]
E  electric field  [Volt/m]
fr  particle revolution frequency  [Hz]
F  form factor of particle distribution
FL electromagnetic force  [Newton]
g  length of the pipe enlargement (bellows)  [m]
gm  coefficient of the particle distribution Fourier
h  vertical aperture of rectangular vacuum chamber  [m]
I  circulating beam current  [A]
j  \sqrt{-1} imaginary unit
k  wave number  [m^{-1}]
A_b  bunch length  [m]
L,Li  length along beam trajectory  [m]
m_0  1.6725.10^{-27} proton rest mass  [kg]
n  mode number
N  particle number
P,P_0  particle momentum  [GeV/c]
Q,Q_0  betatron number
r_0  1.5347.10^{-18} classical proton radius  [m]
R  equivalent radius of the particle trajectory over one revolution
\hat{R}  surface resistivity
s  longitudinal coordinate  [m]
t  time  [s]
U,V  dispersion relation coefficients  [s^{-1}]
U',V' normalized dispersion relation coefficients
v  longitudinal particle velocity  [m/s]
w  horizontal aperture of rectangular vacuum chamber  [m]
x  horizontal transverse coordinate  [m]
y  vertical transverse coordinate  [m]
Concluding remarks

In 1982, experiments were performed on the AA with a cooled proton stack of characteristics 15):

\[ N = 7 \times 10^{11} \, \beta \]
\[ Q_H = 2.2650 \]
\[ \varphi_H = 6 \, \pi \, \text{mm.mrad} \]
\[ \Delta p/p = 2.1 \times 10^{-3} \, \text{(at 95%)} \]

\[ f_0 = 1055.2 \, \text{[kHz]} \]
\[ Q_V = 2.2559 \]
\[ \varphi_V = 4 \, \pi \, \text{mm.mrad} \, \text{(at 95%)} \]

Vertical instabilities were observed between 15 and 20 MHz, with a peak around 18 MHz (n=12), giving rise to vertical emittance blow-up. Measured growth times were in the range 2 to 10 [s], which were much larger than the theoretical prediction 4). One attempt of an explanation was that probably only a small function of the stack oscillates, because scraping 1% of the beam eliminated the unstable mode. Redoing the calculations for the above stack characteristics, yields for the mode n=12.

\[ z_{LV} = 1.1 \times 10^4 - j \times 5.2 \times 10^6 \, \text{[V/m]} \]
\[ \tau_V = 0.15 \, \text{[s]} \]
\[ \Delta \varphi_V = 2.7 \times 10^{-4} \]

which would seem to indicate that further experiments would be needed in order to check more carefully the reliability of the results predicted by the theory.

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REFERENCES


GLOSSARY (suite)

$Z'_{\perp}$ normalized transverse impedance  
$Z_{\perp}$ transverse impedance linear density function  \[ \text{[A/m²]} \]  
$Z_{\perp}$ transverse impedance  \[ \text{[A/m]} \]  
$Z_0 = (\mu_0 / \varepsilon_0)^{1/2} = 376.73$ free space impedance  \[ \text{(Ω)} \]  
$\alpha_p$ momentum compaction function  \[ \text{[m]} \]  
$\beta_0$ particle velocity (over c)  \[ \text{[m]} \]  
$\beta$ wave velocity (over c)  \[ \text{[m]} \]  
$\beta_x, \beta_y$ beam envelope function  \[ \text{[m]} \]  
$\gamma$ total particle energy (over rest particle energy)  \[ \text{[Hz]} \]  
$\gamma_t$ transition energy  \[ \text{[Hz]} \]  
$\delta$ skin depth  \[ \text{[m]} \]  
$\delta p / p$ relative orbit momentum from the central orbit  \[ \text{[m]} \]  
$\Delta p / p$ relative momentum spread at 95% of particles  \[ \text{[m]} \]  
$(\Delta p / p)_{hh}$ half relative momentum spread at half height  \[ \text{[m]} \]  
$\Delta Q$ coherent tune shift  \[ \text{[Hz]} \]  
$\Delta \omega$ frequency deviation  \[ \text{[Hz]} \]  
$\Delta S$ betatron frequency spread  \[ \text{[Hz]} \]  
$\Delta$ beam width (for rectangular beam cross-section)  \[ \text{[m]} \]  
$\epsilon_x, \epsilon_y$ transverse beam emittance (also written $\epsilon_x', \epsilon_y'$)  \[ \text{[mrad]} \]  
$\epsilon_0 = 10^{7/4} \pi^2$ free space permittivity  \[ \text{[Farad/m]} \]  
$\epsilon_r$ relative permittivity  \[ \text{[Farad/m]} \]  
$tg \delta_e$ electric loss tangent  \[ \text{[Farad/m]} \]  
$\eta = (df/dp) / (dp/p) = 1/\gamma^2 - 1/\chi^2$ transition parameter  \[ \text{[Farad/m]} \]  
$\lambda$ wavelength  \[ \text{[m]} \]  
$\mu_0 = 4\pi \times 10^{-7}$ free space permeability  \[ \text{[Henry/m]} \]  
$\mu_r$ relative permeability  \[ \text{[Henry/m]} \]  
$tg \delta_\mu$ magnetic loss tangent  \[ \text{[Henry/m]} \]  
$\xi = (dQ/Q) / (dp/p)$ chromaticity  \[ \text{[Hz]} \]  
$\sigma$ vacuum chamber wall conductivity  \[ \text{[Ω⁻¹m⁻¹]} \]  
$\tau$ beam height (for rectangular beam cross-section)  \[ \text{[Hz]} \]  
(also e-folding time)  \[ \text{[s]} \]  
$\omega$ wave angular frequency  \[ \text{[Hz]} \]  
$\Omega_r, \Omega_d$ particle angular revolution frequency  \[ \text{[Hz]} \]  
$\Omega_\xi$ chromaticity angular frequency  \[ \text{[Hz]} \]
FIG. 1

TRANSVERSE IMPEDANCES (Ω/m)

N = 10^{12} \ p

ε_h = 2.0 × 10^{-3} mm.mrad

ε_v = 1.5 × 10^{-3} mm.mrad

Δp/p = 1.5 × 10^{-3}

Re(Z_{LV})

Re(Z_{LN})

Im(Z_{LV}) = -1.4 × 10^{-7} (Ω/m)

Im(Z_{LN}) = 5.2 × 10^{-7} (Ω/m)

MODE NUMBER n

0.2

0.3

0.4

0.5

FIG. 2

GROWTH RATES AND TUNE SHIFTS

N = 10^{12} \ p

ε_h = 2.0 × 10^{-3} mm.mrad

ε_v = 1.5 × 10^{-3} mm.mrad

Δp/p = 1.5 × 10^{-3}

τ_Y' (s^{-1})

τ_N' (s^{-1})

ΔΩ_Y = 1.0 × 10^{-3}

ΔΩ_N = -3.9 × 10^{-3}

MODE NUMBER n
**FIG. 5**

AA STACK STABILITY DIAGRAM

(Stack during the accumulation)

\[ V' = \text{Re}(Z_{1}) \]

\[ U' = \text{Im}(Z_{1}) \]

Normalized transverse impedance \( Z'_{st} = V' + jU' \)

- \( \times \): Vertical \( Z'_{stv} \)
- \( \circ \): Horizontal \( Z'_{sth} \)

**FIG. 6**

CONSOLE - GRAPHIC SYSTEM HARD-COPY

Spectrum acquired at 1995-09-08-10:40:25

- RES BW 3 kHz
- VDG 30 kHz
- SWP 6.88 sec
- ATTEN 10 dB
- REF -95.7 dBm
- 5 dB
- CENTER 72.288 kHz
- SPAN 105 kHz

\( N = 2.93 \times 10^{11} \) p

\( \delta H = 7.9 \text{ mm, mrad} \)

\( \delta V = 3.5 \text{ mm, mrad} \)

\( \Delta p/p = 2.9 \times 10^{-3} \)

Core peak at: 1854.08 kHz
**FIG. 7**

**AA STACK STABILITY DIAGRAM**

(COLED STACK)

\[ V' = Re(Z'_1) \]

\[ U' = Im(Z'_1) \]

Normalized transverse impedance \( Z'_1 = V' + j U' \)

* x: VERTICAL \( Z'_{1V} \)
* o: HORIZONTAL \( Z'_{1H} \)

**FIG. 8**

**CONSOLE - GRAPHIC SYSTEM HARD-COPY**

SPECTRUM ACQUIRED AT 1995-10-10-09:48:29

- RCS ON 3 kHz
- VDM 30 kHz
- SUP 5.05 sec
- ATTEN: 10 dB
- REF: -95.7 dBm
- CENTER: 72.2 kHz
- SPAN: 195 kHz
- N = 2.63 \times 10^{11}
- c_{hi} = 2.6 \text{ mm.mrad}
- c_{iy} = 1.8 \text{ mm.mrad}
- kp/p = 1.8 \times 10^{-3}

Core peak at: 1855.09 kHz