AN APPROACH TO THE DESIGN OF SPACE-CHARGE LIMITED HIGH INTENSITY SYNCHROTRONS

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Summary and Introduction

A design procedure for storage rings has been reported previously. The approach proposed here concerns mainly synchrotrons which accelerate intense beams of protons or heavier ions; it could also be useful for the design of compressors or stretcher rings or similar facilities. The approach is based on experience gained in the involvement with half a dozen machines and in the design and development of the CPS Booster, which by now accelerates in one ring (vacuum chamber apertures 132 x 63 mm) B x 10^4 ppp with 50 MeV injection.

The approach is presented as a commented (looped) flow chart (Fig. 1). Key limitations are: space-charge induced tune shifts, longitudinal coasting beam instabilities, and beam-loss induced radioactivity. Other items dealt with include the determination of the mean machine radius, RF parameters, beam emittances and energy spread required for stability, vacuum pressure, transverse focusing, etc. "Factories", accelerating or compressing up to ~10^16 particle/sec, have been designed in the last fifteen years. Changes of emphasis at the design stage due to very high intensity are pointed out.

The information presented falls into three categories: (i) accelerator physics items of "permanent" nature, (ii) typical or limiting design values geared to machines with a mean radius of up to about 100 m and Bp up to 10 T, which could evolve further with technological progress, and (iii) suggestions based on personal experience. MSK units are used throughout unless stated otherwise. The information provided (including key references) together with formula collections and a pocket calculator, should allow one to arrive at a first basic parameter list not too much different from the final. The design proper would follow.

Mean Machine Radius \( \bar{R} = \frac{C}{2\pi} \)

If \( \bar{R} \) is not imposed, one starts from the bending radius:
\[
\rho = \frac{P(eV/c)}{Q_{cB}(\text{max})},
\]
where \( P \) is max. particle momentum, \( c \) = velocity of light, \( B \) = magnetic induction in the bending magnets (on orbit), and \( Q \) = number of electrical charges per particle. \( B < 1.3 \text{T} \) is usual for fast-cycling synchrotrons, 1.3 - 1.8 T for slowly-cycling gradient magnets, and up to 1.8 T or even 2 T for flat-field magnets. Conservative values are recommended to preserve field quality. Figure 2 gives \( R/p \) values for (a) light, and (b) conservative patching. \( n = 4Q \) is recommended for long life component rating, avoidance of significant overlapping of magnetic end fields (stopbands), and space for local shielding and remote handling. Costs, notably for the RF accelerating and damping systems, and the machine enclosure, increase from a) to b).

Tunels, Amplitude Functions \( B \), Periods \( N \). To start the iteration, approximate values of the betatron tunes \( Q \) can be taken from Fig. 2. Further, one assumes \( \beta = R/Q, \beta = 3\beta, N = 4Q \) and \( \beta = 1.2 \text{ m} \).

Beam Injection

In case the injector pulse length is \( T_{\text{inj}} > 3 \text{ to } 5 \times \text{rev} \) (= revolution period), almost lossless (a few percent) injection is only feasible by charge-exchange, where e.g. \( H \) ions are stripped to protons in a thin foil. For classical betatron stacking, the achievable injection efficiency can be estimated roughly (neglecting the width D \( \Delta p/p \) with D = dispersion):
\[
\eta_{\text{inj}} \simeq 0.9 \left[ 1 - \exp \left\{ -\left[ \frac{\pi^2}{2} \left( \frac{n_t}{\Delta n_q} \right) \right]^2 \frac{\eta_{\text{eff}}}{2} \right\} \right]
\]
where \( \eta_p \) = horizontal acceptance, \( n_t = T_{\text{inj}}/\text{rev,} \eta_{\text{eff}} \) = effective septum thickness, and the emittance \( \epsilon_{\text{qf}} \) contains 63% of the injector beam. For low injection currents and long pulses with small \( \Delta p/p \) stacking in longitudinal phase space is an alternative.

RF Parameters

RF Voltage

Acceleration. As in all synchrotrons with negligible synchrotron radiation, the energy gain per turn is given by:
\[
\frac{\nu}{\nu_{\text{RF}}} \cdot \sin \phi = C_B \beta
\]
where \( \nu_{\text{RF}} \) is the peak RF voltage per turn, \( \phi \) is counted from the zero crossing, and \( B = \text{dB/dt} \).

Longitudinal Acceptance. Below transition energy the space-charge field reduces the zero-particle longitudinal acceptance (least for \( h = 1 \)). For fixed RF voltage required for providing longitudinal acceptance goes up. An acceptance margin of, say, 30% is recommended for low loss trapping.

Harmonic Number

Choice of the harmonic number (\( h = f_{\text{acc}}/f_{\text{rev}} \)) leads to (i) the lowest peak voltage, because the voltage required for providing longitudinal acceptance increases with \( h \); (ii) the absence of coupled bunch instabilities (still true for \( h = 2 \)); (iii) the longest rise time and hence to least stress (and cost) for any fast ejection kicker magnet; but also to (iv) only one bunch, not dividable "losslessly" among several users; (v) the smallest bunching factor \( \eta_p \) (\( \phi \)), because, for fixed \( \nu_{\text{RF}} \), the lowest \( \nu_{\text{RF}} \) leads to the highest \( \sin \phi \) and hence the shortest bunch (in RF radians); (vi) the need for a more complex phase control system if more than two cavities (assuming N even) are required.

To bring the peak line density further down towards the value of the mean density, a higher harmonic RF system can be added. Within certain limits this may also provide extra Landau damping.

Beam Loading Compensation

At the high intensities under discussion (RF Fourier component of beam current > RF cavity drive current) such a compensation is likely to be needed, particularly during adiabatic trapping and any other "gymnastics" with reduced RF. Servo-loop techniques and reduced RF. Servo-loop techniques are more cost effective than high power solutions.
Beam Emittances (Space-Charge Detuning)

During injection and initial acceleration beam emittances \( \varepsilon \) (in \( \text{m rad} \)) have to be controlled in such a way that the space-charge detuning \( \Delta Q_{\text{SC}} \) does not drive the beam into stopbands causing unacceptable beam loss. The largest \( \Delta Q_{\text{SC}} \) is experienced by particles with vanishing betatron amplitudes at the bunch centre. For the usual beams (width > height because of magnet field cost) the largest (negative) detuning is given by:

\[
\Delta Q_{\text{SC}} = \frac{r_p N_q \langle Z^2 \rangle \langle A \rangle}{\pi (\varepsilon_A)^2 \beta_y^2 B_f}
\]

where \( r_p \) = classical proton radius, \( N_q \) = number of ions circulating, \( A \) = ion mass in multiples of proton mass \( m_p \), \( \beta \) and \( \gamma \) = usual relativistic factors, and \( F_Y, G_Y, H_Y \) = form factors discussed below.

Bounding Factor. \( B_f \) (= average/peak line density) decreases with increasing stable angle \( \varepsilon_A \) (in rad) and is

\[
B_f = \frac{2}{\pi} \left[ 1 - \left( \varepsilon_A + 3.6 \frac{\beta}{\gamma} \right) \frac{1}{\pi} \right]
\]

for a full bucket (and 10% less in practice); \( n = \varepsilon_A \). The term \( \beta \gamma B_f \) in (4) can well reach its minimum some time after acceleration starts, particularly in fast-cycling synchrotrons.

\( \Delta Q_{\text{SC}} \) and Form Factors. The maximum admissible value for \( \Delta Q_{\text{SC}} \) depends on the stopband respectively beam loss situation, and the \( \Delta Q \) margin to be reserved notably for chromaticity and amplitude dependent Q-shifts. \( \Delta Q_{\text{SC}} \) = 0.25 is a starting value. \( F_Y \) contains the latest image coefficients \( \varepsilon_A \), for \( \gamma = 1 : F_Y = 1 \).

Table I. G for Various Distributions in Physical Space.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>( G_{Y,V} )</th>
<th>% beam in ( \varepsilon_{V,Y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1</td>
<td>100%</td>
</tr>
<tr>
<td>Parabolic density</td>
<td>2</td>
<td>100%</td>
</tr>
<tr>
<td>Gaussian</td>
<td>1.56</td>
<td>95%</td>
</tr>
<tr>
<td>PSB, best measured</td>
<td>1.37</td>
<td>95%</td>
</tr>
<tr>
<td>(Proton Injection)</td>
<td>1.23</td>
<td>95%</td>
</tr>
</tbody>
</table>

\( G_Y \) (Table I) describes the transverse density distribution. A well designed H- charge-exchange injection should yield \( G_{Y,V} = 1.2 \) to 1.3; classical multturn injection results in \( G_Y = 1.5 \) to 2. \( H_Y \) takes into account the beam aspect ratio width/height, and its variation with the amplitude function \( H_Y(v) \) is:

\[
H_Y = [1 + \sqrt{\varepsilon_A^2 + D \varepsilon_A / (\varepsilon_A^2 + 1)}]^{-1}.
\]

In smooth approximation, for \( \varepsilon_A / \varepsilon < 0 \), and averaging over a (super)period:\n
\[
H_Y = (1 + \sqrt{\varepsilon_A^2 / \varepsilon_A^2 + 1})^{-1}.
\]

Using \( H_Y = 1 + r_p \),

\[
\Delta Q_{\text{SC}} \Delta Q_{\text{SC}} = \frac{e q \varepsilon_A}{h \beta_y^2 B_f}.
\]

(7a)

Thus, with \( \Delta Q_{\text{SC}} ^v \) more critical (\( q < q_Y \)), raising \( q_Y / Q_h \) helps. For \( \Delta Q_{\text{SC}} ^v = 0.25 \) to 0.5, \( q_Y / Q_h = 1.5 \) to 2 is recommended for avoiding the stopband intersections on the diagonal \( \Omega = H_Y \) = integer. (7) thus yields \( q_Y / Q_h \) and with (4) \( q_Y \). In a machine with a sizeable relative beam width due to momentum spread, \( q_Y \) may be reduced correspondingly.

Longitudinal Stability\(^8,17,18\) (Momentum Spread)

Only the coasting beam is dealt with as long bunches can be stabilised by an electronic feedback system\(^9\). To estimate the growth during the critical phase (prior to capture) one computes the impedance (at \( \omega = \omega_{cav} \) due to the space-charge and various resonators:

\[
Z_{\Omega}(n) = -\frac{1}{2} Z_0 \frac{1}{2} \frac{1}{1 + j \omega / \omega_{cav}}.
\]

For a round beam pipe (space-charge + resistive wall) and resonators the transverse coupling impedance \( Z \) contains the Laslett coefficients \( \varepsilon_A \).

\[
Z(n) = 1 - j \frac{N_q \beta_y}{\beta_y} \frac{1}{1 + j \omega / \omega_{cav}}.
\]

For a well engineered machine, or possibly lower with extra effort\(^{20}\), known or likely parasitic resonators like a fast kicker magnet\(^{20}\) with \( Q = 1 \) to 2, \( R = \text{tens of} Q \) and \( \omega - \omega = 10 \text{ MHz} \). As the first term is dominant at lower energies, application of the Kell-Snell-Boussard criterion:

\[
\left| \frac{Z(n)}{n} \right| < \frac{1.56}{10^3} \text{FWHM}
\]

should be checked around \( n = h \) (effect of RF cavity) and \( n = z_{cav} / \sqrt{q} \) (microwave instability). If the beam costs for more than 3 to 4 e-folding times, the momentum spread will increase at least until the stability criterion (9) is fulfilled. Acceptance (physical and in terms of RF voltage) must be planned accordingly.

Transverse Stability\(^8,17,18\)

For a round beam pipe (space-charge + resistive wall) and resonators the transverse coupling impedance is:

\[
Z_L(\omega) = -j R_0 \beta_y^2 \frac{1}{1 + j \omega / \omega_{cav}}.
\]

With \( \omega = \omega_{cav} \) and \( I_0 \) = the (local) ion particle current may yield values too pessimistic by a factor up to three depending on the distribution. The growth rate (neglecting Landau damping):

\[
\frac{d \langle v \rangle}{dt} = \frac{d \langle v \rangle}{dt} + (6)
\]

should be checked around \( n = h \) (effect of RF cavity) and \( n = z_{cav} / \sqrt{q} \) (microwave instability). If the beam costs for more than 3 to 4 e-folding times, the momentum spread will increase at least until the stability criterion (9) is fulfilled. Acceptance (physical and in terms of RF voltage) must be planned accordingly.

Stabilisation. (i) integer \( Q < Q \) half-integer is recommended as otherwise the resistive wall impedance may become uncomfortably high for the smallest frequency \( \omega = \omega_{cav} (n > Q) \); (ii) conventional Landau damping by octupoles enlarges stopbands and should be avoided; (iii) the Kell-Snell-Boussard criterion (7) is an electronic damper\(^9\), generating a transverse impedance\(^{20}\) offsetting the driving one \( \text{Re}(Z_{\text{sc}}) \) to a machine \( \text{Re}(Z_{\text{sc}}) \) beam to a machine \( \omega = 4/(2n = 50 \text{ MHz}(coasting beam)) \).
A simple approach to estimate the average properties of the radioactive isotopes produced in high energy particle interactions can be used to predict the induced radioactivity with sufficient accuracy, provided many different isotopes are found and the distribution of their half-lives is not too different from a natural one. Using these approximations, it is estimated that a thick target (beam dump) of iron or copper irradiated with a beam of $\Delta N$ protons/s of an energy greater than about 200 MeV (isotope production cross-section constant) will produce at 40 cm distance a dose-rate $D$:

$$D_{(\text{rem/h})} = 2.3 \times 10^{-12} \Delta N \log(T + 1/t) \text{ (12)}$$

with $T$ the irradiation time, and $t$ the cool down time. Thus, for $T = 30d$, $t = 1d$ and $\Delta N_1 = 3 \times 10^{11}$ p/s, $D = 1$ rem/h. For heavier ions, the constant in (12) has to be scaled by the ratio of the total inelastic cross-sections (for the target material). In practical situations the beam may first hit an accelerator component which does not represent a thick target. The dose rate calculated will then be smeared out over a certain distance rather than be concentrated. Dose-rates from lighter materials such as graphite or limestone, are up to two orders of magnitude lower. However, (12) will be scaled by the ratio of the total inelastic cross-sections (for the target material).

The damage to accelerator components resulting from irradiation depends on the type of material, the type of radiation, temperature, humidity, etc. Again, for a very rough first check, the yearly dose to a "thick target" can be estimated from:

$$D_{(\text{rad/y})} = 3 \times 10^{-10} \Delta N_2$$,

where $\Delta N_2$ is number of irradiating protons per year.

Control of Beam Loss and Its Effects

Main points are (i) studying and limiting beam loss to far below what would be acceptable by the beam users (beam injection, RF capture, betatron resonances, instabilities, injection, etc.); (ii) concentrating beam loss in beam lines, at injection, when dumping internally, at injection, etc. in tailormade beam dumps (made from low-radioactivity material like graphite or shielded e.g. by limestone; (iii) avoiding potential interventions in radioactive areas by housing whenever possible equipment outside the machine enclosure and increasing the lifetime of components housed inside (conservative ratings); use of high-grade radiation resistant materials, remedial actions against specific beam loss effects like local overheating, increase of the ozone content of the air with the consequent corrosion effects, degradation of demineralised cooling water with the resulting blockage of thin cooling passages, etc.; (iv) reduction of time for actual interventions through the use of rapidly exchangeable modular plug-in type components; (v) provision of remote handling of potentially all, but at least of critical components such as septum magnets (and their enclosure), beam dumps, etc. The radiation shielding is based on the well established techniques developed around accelerators and storage rings. At the interties under consideration air conditioning and water cooling have to be closed-circuit designs (for keeping the outside radioactivity at legally acceptable levels). Most of these questions have recently been studied for a $3 \times 10^{14}$ pps 1100 MeV proton linac$^3$. Betatron Resonances

A particle experiences unlimited betatron amplitude growth if its tunes $Q_h, Q_v$ satisfy

$$| m Q_h + n Q_v - p | < \pi e / 2, \ m, n, p \ \text{integers} \text{ (14)}$$

where $m + n$ is the order of the resonance and $\pi e / (m+n)$ the total stopband width in the $Q_h - Q_v$ diagram. Stopband considerations determine (i) $\Delta Q_{\text{SC}}$, (ii) the requirements on magnet field quality, and (iii) the need for harmonic correcting multipoles. Before going into a full analysis, a rough consideration of the one-dimensional resonances serves to assess the magnitude of the problem on hand.

Stopband Width and Amplitude Growth. Limiting oneself to the vertical plane ($m = 0$), the $p$th Fourier component of a magnet imperfection $B(n-l)B_x / \alpha(n-l)$ excites a stopband of width:

$$\Delta \alpha = \frac{3}{8} \frac{\pi}{\beta_2} \frac{1 \sqrt{E}}{\beta_2} \left( \frac{2 \pi}{\beta_2} \right) \cos \theta \left[ 2 \pi \frac{1}{\beta_2} \sqrt{E} \right]$$

$$n = 3 \ (\text{skew})$$

where $\beta_2$ is the amplitude function at the driving elements. Of particular concern here is the $e$-dependence of $\Delta \alpha$ for non-linear resonances.

Growth Rate for Coasting Beam. A very rough estimate of the initial loss rate, without considering amplitude dependent space-charge detuning, can be made assuming that (i) the aperture is completely filled, (ii) at the beam edge the number of particles $N$ contained up to amplitude $r$ is proportional to $r$, and (iii) the fraction of the particles locked on resonance is $(\Delta \alpha / N) / \Delta Q_{\text{SC}}$.

$$\frac{1}{N} \frac{dN}{dt} = \frac{\pi}{1 \ rev} \frac{\Delta \alpha}{N} \frac{\Delta \alpha}{N}$$

Growth Rate for Bunched Beam. The particle is repetitively swept over $m N_2 - p = 0$, twice during one synchrotron oscillation period. With the same assumption as in the coasting beam case, and supposing that all particles are swept over the stopband considered (but do not lock on), one can estimate the approximate beam loss during time $\Delta t$:

$$\frac{1}{N_1} \frac{dN_1}{dt} = \frac{\Delta \alpha}{N_1} \sqrt{\frac{\Delta \alpha}{2 R \Delta Q_{\text{SC}} \Delta t}}$$

Data. In Table 2 computed loss rates at "50 MeV for the PS Booster (4 rings, 16 magnet periods, imperfections in bending magnets $B(0/8) < 2 \times 10^{-6}$)" are shown, using $\Delta \alpha$ values computed from magnet measurements or measured in beam experiments$^2$. Assessment. (i) Assume a maximum field error $\Delta B / B$ at the edge of the chamber aperture (between $2 \times 10^{-6}$) and compute the relevant multipole coefficients; (ii) derive rms values of the width of (non-systematic) stopbands (produced by $k$ types of elements each having $N_k$ elements of length $l_k$) from (15) where the integral is replaced by the sum:

$$\left\{ \frac{1}{k} \left( \frac{N_k}{2} \right) \left( \frac{Z_k / l_k}{Z_k / l_k} \right) \right\}^{2}/2$$

$$\text{ (15a)}$$
Table 2 - Stopband Widths and Loss Rates in the PS Booster

<table>
<thead>
<tr>
<th>n</th>
<th>Stopband (syst.)</th>
<th>Loss rate 1/μ</th>
<th>β or if aperture filled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2ν = 11</td>
<td>0.0004</td>
<td>0.007</td>
</tr>
<tr>
<td>2</td>
<td>2ν = 12</td>
<td>0.0008</td>
<td>0.0005</td>
</tr>
<tr>
<td>3</td>
<td>3ν = 21</td>
<td>0.0001</td>
<td>0.011</td>
</tr>
</tbody>
</table>

(iii) compute loss rates for coasting and bunched beams (16, 17); (iv) if losses are too fast, consider improved magnet quality or plan for stopband compensation by appropriate multipoles; (v) from the start, PSB stopbands were narrowed by a factor 10 to 30 (n = 2, 3).

Vacuum Pressure

Avoiding significant emittance blow-up due to collision with the residual gas is usually no problem with modern all-metal ang ceramic vacuum systems achieving with some care 10⁻⁸ mbar or better. Depending on energy, charge state, ions may well require a lower pressure to avoid charge exchange.

Stability Against Electron-Proton (Ion) Oscillations. A low vacuum pressure would be required to ensure that the relative neutralisation stays below a threshold value ηthr. Otherwise electrons created by ionisation of the residual gas and trapped in the potential well of a coasting ion beam (and sometimes in the well of long bunches) give rise to rapidly growing transverse oscillations. For large Δμsc one has with (6):

\[ η_{thr} = (28.8 \beta^2)^{-1} \left( V_{max}^2 - V_{min}^2 \right) / V \]  

Measurements 32 suggest a neutralisation rate:

\[ h = 6 \times 10^{-7} \, \text{q}^2 \, \text{P} \, \text{(Torr)} \left( 10.1 + \ln q \right)^2 - 2 - \beta \right) / \beta \cdot 19 \]  

In general, highest ηthr values occur at injection. For a linear current build-up one has η = 0.5 \, T1νn. The pressure (19) must be such that η < ηthr (18).

Machine Lattice

Contrary to the other items, lattice selection and design cannot be solved approximately by a few (rough) calculations. Even a short lens approach would exceed the scope of this paper. However, one has here at least a check list of the items to be considered (except correction elements) and a reminder of certain consequences of parameter variations. This should help to fix the initial configuration for the iterative computer-aided optimisation 32.

Superperiods or Not?

Providing a total of two lss for injection, ejection and possibly RF accelerating cavities, i.e. S = 2, would probably lead to the shortest circumference, but also to systematic (structural) stopbands. The decision about superperiods should thus be based on a Qs - νγ diagram showing all such stopbands p + k3 up to order 4 or even 5, and a loss assessment. For a number of periods N larger than, say, 12, S = N/2 could possibly be an acceptable compromise (the larger S, the fewer such stopbands) which would notable still allow equidistant distribution of the RF cavities. A lattice without superperiods may, however, be preferred for simplicity and possibly minimum magnet energy stored, and for easier B tracking and smaller likelihood of standing wave modes (leading to unequal magnet excitation) in the case of fast-cycling machines.

Minimum Length of Long Straight Section.

With a fixed machine radius and in the absence of superperiods one upper limit for N is reached when the lss becomes too short for housing the most critical piece of equipment.

Beam Ejection. If D0 is the beam displacement required to eject the beam from a straight section of length Lss by means of a septum magnet with an induction Bμ occupying the upstream fraction f Lss one has (for zero septum entrance angle) (Fig. 3):

\[ L_{ss} = \sqrt{\frac{D_0}{B_\mu (B_0 s)}} / (f - f_s/2) \]  

D0/Bμ = 0.05 m/T applies to ejection through a D gradient magnet or past a "septum" bending magnet; 0.2 to 0.3 m/T are normal. Larger Lss values may be required to eject (large emittance) beams with kicker and septum magnets on either side of a D lens (no aperture increase elsewhere.) f < 0.7 is recommended.

RF Accelerating Cavity. Modern tunable ferrite-loaded cavities are often designed as double-gap units about 2.5 m long (delivering approx. 20 kV in the MHz range, or 50 kV in the tens of MHz range) and f(ν)/ν(ν) < 2. Below about 2 GeV kinetic proton energy the RF cavity determines the straight section length, and beam ejection above (assuming that injection takes less space).

Number of Long Straight Sections and Periods N.

Space requirements for injection (1), acceleration (say, 1 to 16), ejection (1 to 3), beam observation (1) and spares (1 to 2) give about 6 to 22 lss (for one ejected beam). N = 6 would lead to excessive apertures for the large emittances and sizeable momentum spreads considered, and probably to an inconvenient γthr. Increasing N (and Q) for fixed machine radius and vacuum chamber dimensions increases, (i) the vertical and even more the horizontal acceptances, (ii) the range of betatron tuning, but also, (iii) the number and strength of quadrupoles (in case of a separate function magnet), (iv) the number of correction elements and beam position monitors, (v) the space taken up by the coil ends and the power dissipated in them, and (vi) the number of "unoccupied" lss. It decreases, (i) the straight section length available for injection and ejection, (ii) the length available for quadrupoles and correction elements). The fraction of the circumference taken up by the bending magnet yokes, RF cavities, pump manifolds and general beam diagnostics is left unchanged.

Thus, the choice of N could be guided by a cost optimisation: purchasing and operating cost for a lesser number of wider aperture elements and their power supplies vs a higher number of smaller aperture elements.

More on Q Values

Phase Advance of Betatron Oscillations/period 17.

In case of a PODO lattice, μ = 75° leads to a [broad] minimum of the maximum amplitude function (= 1.7 Βμsc (17)). For fast beam ejection the deflection needed by the kicker magnet is:

\[ θ_k = \sqrt{\frac{W_b [s(β_\text{septum}) + t_s + Δ]}{\text{Kicker} \cdot β_\text{septum} \cdot \sin β_\text{septum}}} \]  

where Wb = beam width at the septum magnet, ts = septum thickness, and Δ = safety margin. Apart from a large kicker, μ = (2k + 1) 90° between kicker and septum magnets is hence most favourable.
Table 3 - Some Types of Lattice Periods

<table>
<thead>
<tr>
<th>Period</th>
<th>Advantages</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>FODO</td>
<td>- Lowest gradients in lenses*</td>
<td>- Strong variation of amplitude functions in lss (inefficient use of aperture of elements in lss, unless divided into sections with different apertures).</td>
</tr>
<tr>
<td></td>
<td>- Two lenses per cell (only).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Chromaticity and stopband corrections with sextupoles well decoupled in H and V planes.</td>
<td></td>
</tr>
<tr>
<td>OFODO</td>
<td>- Longer lss than in FODO case</td>
<td>- Gradients higher than for FODO.</td>
</tr>
<tr>
<td></td>
<td>- $\alpha_H, \alpha_V$ variations over same distance in lss less marked than for FODO.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Two lenses* per cell</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Chromaticity and stopband corrections fairly decoupled.</td>
<td></td>
</tr>
<tr>
<td>Triplet (OFDO or OFDDO)</td>
<td>- Both $\alpha_H$ and $\alpha_V$ small in lss (smallest aperture for magnets, RF cavity, etc.)</td>
<td>- One extra lens* compared to FODO.</td>
</tr>
<tr>
<td></td>
<td>- If $\alpha_H/\alpha_V = \beta_H/\beta_V$ in central lens, efficient use of single lens aperture.</td>
<td>- Chromaticity and stopband corrections not very effective (coupled).</td>
</tr>
<tr>
<td>FDDO resp. FODFDO or FDDDO</td>
<td>- $\beta_H/\beta_V$ change least in lss (efficient use of single aperture).</td>
<td>- Two extra lenses* compared to FODO.</td>
</tr>
<tr>
<td></td>
<td>- Choice of appropriate $\alpha_H, \alpha_V$ values for injection and ejection.</td>
<td>- Higher gradients than in FODO.</td>
</tr>
<tr>
<td></td>
<td>- Chromaticity and stopband corrections well decoupled.</td>
<td></td>
</tr>
</tbody>
</table>

*Gradient magnets in case of combined function magnets, in which case "extra lenses" will not increase the total magnet yoke length.

Third order resonance excitation by sextupoles (of the same family) used for chromaticity correction is avoided if they are arranged in pairs with an odd number of half betatron wavelengths between them. Apart from $N =$ even, this favours $\mu =$ 90° or 60°.

Q-Splitting, $Q_H - Q_V = 1$ (or even 2) will raise the space-charge limit (7), avoid the stronger zeroeth harmonic non-linear coupling resonances, and is recommended. The price to be paid may be larger $\beta $ values at least in one plane, and a greater difficulty in finding a region free of systematic stopbands. In case high $\gamma $ is an overriding constraint, $Q_H > Q_V$.

Combined or Separate Function Magnet?

Combined function magnets are a cost-effective way to achieve high $N$ focusing structures even using quadruplets, and may well be the choice for a fast-cycling synchrotron where losses due to stopbands are less critical. A separate function magnet would probably be the choice for a slow-cycling or dc machine since (i) an experimentally optimized (dynamic) $Q_H - Q_V$ working area could be implemented more easily, and (ii) stopbands are likely to be narrower. Also, it can be "recycled" more easily. A hybrid solution (gradient magnets plus lenses) may combine the best features of the two.

Type of Lattice Period

For a first orientation, the information given in Table 3 may be helpful. A (separate function) OFDDO lattice could be a good compromise between the minimum FODO lattice and a more refined one. An OFDDO lattice with an auxiliary short dc lens has also been proposed to take advantage of the increased machine acceptance at low energies 11.

Transition Energy

While up to $2 \times 10^{13}$ protons per pulse are routinely accelerated through transition in the CERN PS due to the $\gamma $-jump, "lossless" crossing may become even more difficult at substantially higher intensities. Beam ejection becomes tricky or impractical around transition, thus there is an incentive to shift $\gamma $ to well above top energy. While the type of lattice chosen has some influence (say, ±15%), basically $\gamma $ or $Q_H$; it can thus be raised by raising $N$ and hence $Q$. If that is not enough, more sophisticated methods are available based either on extra lenses 18, 35, 36 or on creating an azimuthal pattern of bending magnet positions or lens strengths 34.

Acknowledgements

We thank our colleagues for a number of discussions which have been useful for this paper, and Mrs. A. Molat-Berbiers for typing it.

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Fig. 1 - Proposed Approach. () refers to eqn. in text.

Fig. 2 - $\hat{R}$ vs $\hat{R}/p$ (*) and $\hat{R}$ vs $Q$ ($\Delta$), for all AG synchrotrons in operation. (ES = electron synchrotron).

Fig. 3 - Magnetic rigidity $B_p$ (respectively kinetic proton energy) vs length of ejection straight section $L_{ss}$ for $f = 0.7$. 

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