ON THE MIKHILEV-SMIRNOV-WOLFENSTEIN (MSW) MECHANISM
OF AMPLIFICATION OF NEUTRINO OSCILLATIONS IN MATTER

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ABSTRACT

Mikhilev and Smirnov have recently proposed a novel and plausible solution of the solar neutrino problem, based on the resonant amplification of the neutrino oscillations in matter. We comment on several aspects of this mechanism. (a) For the values of neutrino masses and mixing angles predicted by the seesaw model of grand unified theories, the MSW effect may take place most naturally in the Sun leading to a considerable reduction of the flux of solar electron neutrinos, the dominant transition being $\nu_e \leftrightarrow \nu_x$ (rather than $\nu_e \leftrightarrow \nu_x$). (b) Oscillations between the ordinary neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) can affect primordial nucleosynthesis, but the effect is small (i.e., the abundance of $^4\text{He}$ is predicted to change by less than $1.3 \times 10^{-3}$). (c) A comparison of some of the general properties of neutrino oscillations in matter and in vacuum is given.

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INTRODUCTION

For some years the event rate in the $^{37}$Cl-$^{37}$Ar solar neutrino experiment\(^1\) (2.0±0.3 SNU) has been considerably below the prediction\(^2\) 5.8±2.2 SNU of the standard solar model. One explanation for the discrepancy is the existence of vacuum oscillations\(^3\) of the $\nu_e$ into other neutrinos. These could be important for neutrino mass-squared differences $\Delta m^2 \equiv m^2_{31} - m^2_{12}$ as small as $\Delta m^2 \sim (10^{-11}-10^{-10})$ eV\(^2\), but only if the mixing angles are large. Other canonical explanations involve non-standard solar models. The $^{37}$Cl detector is mainly sensitive to the relatively high energy (from 0.81 MeV up to 14 MeV) neutrinos from $^8$B decay. The flux of these $^8$B neutrinos depends very sensitively on the temperature of the solar core and could be changed significantly by modifications of the standard solar model.

A $^{71}$Ga-$^{71}$Ge experiment could distinguish between these two possibilities. Most of the expected $^{71}$Ga event rate is from the low energy pp neutrinos, the flux of which can be inferred from the over-all solar luminosity and is relatively insensitive to the temperature of the solar core. The predicted $^{71}$Ga event rate of =107 SNU can be reduced at most to around 78 SNU in most non-standard solar models\(^2,4\). The traditional view has been that a flux lower than this would imply large vacuum oscillations, which would reduce the $^{71}$Ga rate by a factor comparable to the $^{37}$Cl event rate reduction for most oscillation parameters (e.g., to around 40 SNU).

Recently, Mikheyev and Smirnov\(^5\) have proposed an elegant new solution to the solar neutrino problem\(^*\). The mechanism of reduction of the solar neutrino flux suggested by them is based on Wolfenstein's observation\(^7\) [see also Ref. 8] that neutrino oscillations are modified for neutrinos propagating through matter because the $\nu_e + e^- \rightarrow \nu_e + e^-$ forward scattering amplitude, which has both charged and neutral current contributions, is different from that for $\nu_\mu - e^-$ or $\nu_\tau - e^-$ scattering. If an appropriate relation between the vacuum oscillation length

\(^*\) Let us note that if the $\nu_e$ is a Dirac particle and has a magnetic (or electric dipole) moment much larger than predicted in the standard theory with right-handed (RH) neutrinos a part of the $\nu_e$ flux may be transformed on its way out of the Sun by the solar magnetic field into a flux of (sterile or active) RH neutrinos not detectable in the $^{37}$Cl experiment\(^6\). Neutrino magnetic (electric dipole) moments which may cause such a transition are possible in the gauge theories with RH currents\(^6\).
\[ \mathcal{E}_v = \frac{\sqrt{E_e}}{|\Delta m^2|} \]  

and the electron number density \( n_e \) is satisfied then a resonance occurs in which even a tiny vacuum mixing is amplified to a maximal (45°) mixing. If the electron density in the core of the Sun (where the \( v_e \) are produced) is higher than the resonance density the \( v_e \) must encounter a resonance region on their way out of the Sun, and, as Mikheyev and Smirnov showed, if the electron density decreases slowly enough there will be an essentially total conversion of the electron neutrinos into another flavour as they pass through the resonance layer.

To see this in detail, consider the case of two neutrino weak eigenstates, e.g., the \( v_e \) and \( v_\mu \) for definiteness, which are mixtures

\[
\begin{align*}
|\psi_e \rangle &= |\psi \rangle \cos \theta_v + |\psi_2 \rangle \sin \theta_v \\
|\psi_\mu \rangle &= -|\psi \rangle \sin \theta_v + |\psi_2 \rangle \cos \theta_v
\end{align*}
\]

with mixing angle \( \theta_v \), of the (vacuum) mass eigenstates \( \psi_1 \) and \( \psi_2 \) with masses \( m_1 \) and \( m_2 \), respectively. It is easy to show that for \( E_v \gg m_1, m_2 \) the state

\[ |\psi(t) \rangle = |\psi_e(t) \rangle |\psi_e \rangle + |\psi_\mu(t) \rangle |\psi_\mu \rangle \]  

will propagate in vacuum according to the Schrödinger-like equation

\[ i \frac{d}{dt} \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \end{pmatrix} = M_0 \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \end{pmatrix} \]

where

\[
M_0 = \begin{pmatrix}
\frac{\Delta m^2}{2p} \cos 2\theta_v & -\frac{\Delta m^2}{4p} \sin 2\theta_v \\
-\frac{\Delta m^2}{4p} \sin 2\theta_v & 0
\end{pmatrix} = \frac{1}{2E_v} \begin{pmatrix}
\pm \frac{\cos 2\theta_v}{E_v} & \mp \frac{\sin 2\theta_v}{E_v} \\
\mp \frac{\sin 2\theta_v}{E_v} & \pm \frac{\cos 2\theta_v}{E_v}
\end{pmatrix}
\]

\( p \) is the neutrino momentum and the \( \pm \) is the sign of \( \Delta m^2 \). In (5) we have subtracted off a multiple \( [p + (m^2 \sin^2 \theta_v + m^2 \cos^2 \theta_v)/2p] \) of the identity, which only contributes an over-all phase and does not affect the \( v_e \to v_\mu \) transition amplitude. From (4) one can easily derive the usual oscillation formula
\[ P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta_v \sin^2 \left( \frac{4m_e^2}{4p} \right) \]  \hspace{1cm} (6)

where \( r \) is the distance travelled by the neutrinos.

In matter \( M_\nu \) is replaced by\(^7\)

\[ M' = \mathcal{I} \begin{pmatrix} \pm \frac{\cos 2\Theta_v}{2\nu} + \frac{1}{2\nu_0} & \mp \frac{\sin 2\Theta_v}{2\nu} \\ \mp \frac{\sin 2\Theta_v}{2\nu} & 0 \end{pmatrix} \]  \hspace{1cm} (7)

where\(^9\)

\[ \frac{\mathcal{I}}{\nu_0} = \sqrt{2} G_F n_e = \langle \nu_e^- | H_{\text{eff}} | \nu_e^- \rangle = - \frac{i \mathcal{I} F(0) n_e}{p} \]  \hspace{1cm} (8)

is the charged-current contribution to the \( \nu_e^- e^- \) forward scattering amplitude in a medium with electron density \( n_e \) (the neutral current contribution is the same for \( \nu_e \) and \( \nu_\mu \) and has again been subtracted off). In the language of optics, the weak amplitude contributes a relative contribution

\[ n - 1 = \frac{i \mathcal{I} F(0) n_e}{p} = - \frac{\sqrt{2} G_F n_e}{p} \]  \hspace{1cm} (9)

to the \( \nu_e \) index of refraction. In (8) and (9), \( F(0) \) is the forward scattering amplitude normalized such that the total cross-section is \( 4\pi ImF(0)/p \). Equation (7) implies matter eigenstates \( |\nu_1^m\rangle \) and \( |\nu_2^m\rangle \), which are related to \( |\nu_e^e\rangle \) and \( |\nu_\mu^e\rangle \) by an equation analogous to (2), only with \( \Theta_v \) replaced by the effective matter oscillation angle \( \Theta_m \) defined by

\[ \sin^2 \Theta_m = \frac{\sin^2 2\Theta_v}{1 + 2 \frac{m_e^2}{\nu_0} \cos 2\Theta_v + \frac{m_e^2}{\nu_0^2}} \]  \hspace{1cm} (10)

The oscillation length in matter is

\[ l_m = \frac{\nu}{(1 + 2 \frac{\nu^e}{\nu_0} \cos 2\Theta_v + \frac{\nu^e}{\nu_0^2})^{\frac{1}{2}}} \]  \hspace{1cm} (11)
Resonance occurs when the diagonal element in $M'$ vanishes, namely for

$$\frac{\ell_\nu}{\ell_o} = \cos 2\theta_\nu$$  \hspace{1cm} (12)

with $\Delta m^2 < 0$ (one can take $0 < \theta_\nu < 45^\circ$ without loss of generality). At resonance one has a maximal mixing $\theta_m = 45^\circ$, and a matter oscillation length $\lambda_m^{res} = \frac{\lambda_0}{\sin 2\theta_\nu}$. From (10) and (11) one observes that the resonance half-width in $n_e$ over which there is a significant enhancement is given by

$$\Delta \left( \frac{\ell_\nu}{\ell_o} \right) = \sin 4\theta_\nu.$$  \hspace{1cm} (13)

For small vacuum mixing angle $\theta_\nu$ the oscillation length is very large and the resonance is very narrow.

For neutrinos, resonance only occurs if $\Delta m^2 < 0$. Assuming a small vacuum mixing angle this implies that resonance occurs if the dominant component of the $v_e$ is lighter than the dominant $v_\mu$ component. This is expected intuitively and is given by most specific models, but it is by no means certain. The sign of the $\bar{\nu}_e^-e^-$ scattering amplitude is opposite to that of the $\nu_e^-e^-$ scattering amplitude [i.e., $\lambda_0$ is replaced by $(-\lambda_0)$ in (7), (10) and (11)]. Hence, resonance can occur either for neutrinos or for antineutrinos, depending on whether $\Delta m^2$ is negative or positive, but not for both.

Equation (8) is valid for oscillations between $v_e$ and another doublet neutrino such as $v_\mu$ or $v_\tau$ (first-class oscillations). For (second-class) oscillations between $v_e$ and a sterile neutrino [a hypothetical SU(2)$_L \times$U(1) singlet neutrino with no weak interactions except those generated by mixing with the $v_e$] one must include the neutral current contribution to the $v_e^-e^-$ scattering amplitude. The factor $\sqrt{2}G_F n_e$ in (8) is replaced by

$$\sqrt{\lambda} \frac{\sum a G^{\alpha}_\nu n_\alpha}{\lambda}$$  \hspace{1cm} (14)

where the $g^{\alpha}_\nu$ are the vector couplings for $v_e^-a$ elastic scattering. For a neutral target with density $n_e$ of electrons and protons and density $n_n$ of neutrons this implies

$$\frac{\Delta \ell}{\ell_o} = \sqrt{\lambda} G_F \left( n_e - \frac{n_n}{2} \right)$$  \hspace{1cm} (15)
Resonance still requires $\Delta m^2 < 0$ as long as $n_n < 2n_e$. Corrections to the index of refraction due to the neutrino mass (if it is not negligible compared to $p$) or for a polarized medium may be found in Ref. 10).

Mikheyev and Smirnov also considered the passage of a neutrino through a medium of varying density. If the density varies sufficiently slowly an adiabatic approximation applies. In particular, for $\Delta m^2 < 0$, $\nu$ coincides with $\nu^\mu_{\text{e}}$ for $n_\text{e}$ much greater than the resonance density $n_\text{Res}$ and with $\nu_\text{e}$ for $n_\text{e} \ll n_\text{Res}$. Hence, $\nu_\text{e}$'s produced in the core of the Sun will be almost totally converted to $\nu^\mu$'s for small $\theta_\nu$ provided that the density at the core exceeds $n_\text{Res}$ and the adiabatic condition

$$\frac{\Delta (\ell_{\nu})}{\ell (\ell_{\nu}/\ell_{0})} = \frac{\sin \theta_\nu}{n_\text{e}} \frac{\Delta n_\text{e}}{\Delta \ell} > \ell_{\text{res}}^n = \frac{\ell_{\nu}}{\sin \theta_\nu}$$

(16)

holds at the resonance*).

A number of authors 5, 4, 11, 12 have performed detailed analyses of the implications of the MSW effect for the $^{37}$Cl experiment. The observed suppression can be accounted for by two bands of parameters: (a) for $-\Delta m^2 = 5 \times 10^{-5}$ eV$^2$, and any $\sin^2 2\theta_\nu > 4 \times 10^{-4}$, all neutrinos with energies $>5$ MeV encounter a resonance layer on their way out of the Sun. The adiabatic condition is satisfied, so the high energy neutrinos are converted, accounting for the $^{37}$Cl results. Lower energy neutrinos do not encounter a resonance layer and are not affected. Hence, the $^{71}$Ga experiment should see an event rate almost as large as if there were no oscillations. (b) A second set of solutions 12 satisfies $-\Delta m^2 \sin^2 2\theta_\nu = 10^{-7.5}$ eV$^2$. This set of solutions intersects with the first set of solutions for small $\sin^2 2\theta_\nu$. For relatively large $\sin^2 2\theta_\nu$, essentially all solar neutrinos encounter a resonance layer, but the adiabatic condition is only marginally satisfied, leading to a suppression of the $^{37}$Cl rate by a factor $=3$. The suppression for $^{71}$Ga depends on the value of $-\Delta m^2$. These two sets of solutions are similar in their implications for $^{71}$Ga to non-standard solar models and large vacuum oscillations, respectively.

*) Numerical studies indicate that substantial conversions still occur if (16) is relaxed somewhat. For example, $\Delta r = 0.1 m^2$ typically yields a 50% conversion probability.
THE SEESAW MODEL FOR THE NEUTRINO MASSES AND THE SOLAR NEUTRINO FLUX

It is interesting to compare the parameter ranges required to explain the solar neutrino problem with theoretical expectations. There are many classes of models of neutrino mass\(^{13}\), but the ones with the most predictive power are the seesaw models\(^{14}\) of Gell-Mann, Ramond and Slansky and of Yanagida. For one family the neutrino mass term is

\[
\mathcal{L}_M = - \frac{1}{2} \left( \overline{\nu}_L \begin{pmatrix} 0 & m_D \end{pmatrix} \nu_R \right) + \text{h.c.} \tag{17}
\]

In (17), \(\nu_L\) is the field of the ordinary [SU(2)\(_L\) doublet] left-handed (LH) neutrino and RH antineutrino, \(\nu_R^c = C \nu_L^T\) (C is the charge conjugation matrix) is a RH field\(^*\), and \(\nu_R\) and \(\nu_L^c = C \nu_R^T\) are SU(2)\(_L\)×U(1) singlet neutrino fields describing a RH neutrino \(\nu_R\) and LH antineutrino \(\overline{\nu}_L\). The mass \(m_D\) is an ordinary Dirac mass, generated by the vacuum expectation value of a Higgs doublet, while \(M\) is an SU(2)\(_L\)×U(1) singlet Majorana mass\(^**\) for the RH neutrino \(\nu_R\). \(M\) can be a bare mass or can be generated by a Higgs singlet. No tree level Majorana mass term for the \(\nu_L\) is allowed unless Higgs triplets are introduced into the theory.

Diagonalization of (17) yields two mass eigenstate Majorana neutrinos \(\nu_1\) and \(\nu_2\), with masses \(m_1\) and \(m_2\) (\(m_1 > 0\)) respectively, and with left-handed components given by

\[
\begin{align*}
\nu_L &= J_L \cos \theta + J_{2L} \sin \theta \\
\nu_L^c &= -J_L \sin \theta + J_{2L} \cos \theta
\end{align*}
\]

where \(\tan^2 \theta = m_1/m_2\). (A \(\gamma_5\) transformation has been performed to make \(m_1 > 0\).) The case \(M \gg m_D\) is particularly interesting. Then,

\[
\begin{align*}
m_1 &\approx \frac{m_D^2}{M} \ll m_D, \\
m_2 &\approx M \gg m_1.
\end{align*}
\]

\(^*\) The RH neutrino field \(\nu_R^c\) carries the quantum numbers of the field \(\nu_L\) (i.e., of the Dirac conjugate of \(\nu_L\)) and transforms as \(\nu_L\) under the proper Lorentz transformations.

\(^**\) Let us note that a neutrino mass Lagrangian containing both Dirac and Majorana terms was considered first by Bilenky and Pontecorvo\(^{15}\).
One therefore has a natural explanation for very small neutrino masses. In particular, in typical grand unified theories \cite[e.g., the simplest SO(10) model] \cite{13} one has $M = M_{\text{GUT}} = 10^{14}$ GeV and $m_D(M_{\text{GUT}}) = m_u(M_{\text{GUT}})$. Renormalizing down to low momenta this implies \cite{16,17} $m_D = m_u/k = 5$ MeV/K, where $k = 4.7$, yielding $m_1 = 10^{-11}$ eV. [Left-right symmetric SU(2)$_L \times$SU(2)$_R \times$U(1) models \cite{18}, on the other hand, typically have $m_D = m_e$ and $M = 1$ TeV, yielding $m_1 = 0.25$ eV. Therefore for the range of neutrino masses they predict, the resonant amplification of the neutrino transitions cannot take place in the Sun and we shall not consider them further.]

The seesaw model can easily be generalized to $n$ families. Equation (17) is replaced by

$$
\mathcal{L}_M = -\frac{1}{2} \left( \bar{\nu}_L \mathcal{L} \nu \mathcal{L} \right) \left( \begin{array}{c} \nu_L \\ M_D T \end{array} \right) \left( \begin{array}{c} \nu_L \\ M_R \end{array} \right) + \mathcal{L}_c.
$$

(19)

where $\nu_L$ and $N_L$ are now $n$ component vectors of doublet and singlet neutrinos, respectively, $M_D$ is an $n \times n$ Dirac matrix, and $M_R$ is a symmetric $n \times n$ Majorana mass matrix. Equation (18) implies the existence of $n$ light and $n$ heavy Majorana mass eigenstates. The light eigenstates are mainly mixtures of the $\nu_L$ [except for small $O(M_D/M_R)$ admixtures of the $N_L$] and have the mass matrix

$$
M_J = M_D M_R^{-1} M_D^T
$$

(20)

while the heavy neutrinos consist mainly of the $N_L$ and have a mass matrix $M_R$.

Let us now consider simple models such as the minimal version of SO(10) in which $M_D$, $M_e$, $M_u$, and $M_d$, which are the $n \times n$ Dirac mass matrices for neutrinos, electrons, $u$ quarks, and $d$ quarks, respectively, are symmetric and are generated by SO(10) 10's while $M_R$ is generated by the SU(3)$_C \times$SU(2)$_L \times$U(1) singlet component of a 126. Models of this type have well-known difficulties \cite{13} in their predictions for $m_d/m_s$ and for $m_s$, and of course for proton decay, but for the rough guidance we are seeking they should suffice. We will make the further simplifying assumptions that $M_R$ is a multiple $M$ of the identity and neglect CP violating phases in $M_D$. Because of the ad hoc nature of these assumptions the results must be regarded as illustrations of typical possibilities and not as concrete predictions.
In such models, $M_D(M_{\text{GUT}}) = M_u(M_{\text{GUT}})$ and $M_e(M_{\text{GUT}}) = M_d(M_{\text{GUT}})$. Hence the neutrino and electron mass matrices at low momenta are

$$M_\nu = M_u M_R^{-1} M_u^T / K^2, \quad M_e = M_d / K$$

(21)

where $K = 4.7$ if $M_u$ and $M_d$ are evaluated at 1 GeV$^*$. For $M_R = M_l$ and $M_u$ real this implies that $M_\nu$ has the eigenvalues

$$m_i = \frac{m_{u,i}^2}{MK^2}$$

(22a)

where $m_{u,i}$ are the eigenvalues of $M_u$. For three families with$^{17}$ $m_u(1 \text{ GeV}) = 5 \text{ MeV}$ and $m_c(1 \text{ GeV}) = 1.35 \text{ GeV}$, and arbitrarily taking $m_e(1 \text{ GeV}) = 50 \text{ GeV}$, (22) implies

$$m_1 \approx 10^{-14} \text{ eV},$$

$$m_2 \approx 8 \times 10^{-17} \text{ eV},$$

$$m_3 \approx 10^{-3} \text{ eV}.$$  

(22b)

Of course, for small mixing angles the $\nu_1$, $\nu_2$ and $\nu_3$ correspond approximately to the $\nu_e$, $\nu_\mu$, and $\nu_\tau$, respectively.

For three or more neutrino flavours and small vacuum mixing angles, only two flavours will be important for a given density and neutrino energy (unless there are degeneracies). To see this, let us write the analogue of (7) for $n$ neutrinos:

$$M' = U^\dagger \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & 2p \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n^2 \end{pmatrix} U^\dagger + \frac{i\Gamma}{\lambda_0} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

(23)

where $U'$ is the unitary matrix which relates the LH components of the weak and (vacuum) mass eigenstate neutrino fields $\nu'_{\ell}$ and $\nu_{\ell}$:

$$\nu_{\ell L} = \sum_{i=1}^n U'_{\ell i} \nu_{i L}$$

(24)

For small mixing angles (i.e., $U'$ is close to the identity) resonance occurs when $\Delta m^2_{ij} / 2p = (m_j^2 - m_i^2) / 2p$ approximately equals $2\pi / \lambda_0 = \sqrt{2} G_F m_e$. Because of the degeneracy between $M'_{i1}$ and $M'_{j1}$, the eigenstates in matter will be approximately

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*) In the calculation of the running quark mass matrices we have made the simplifying assumption that the heavy quark threshold effects are negligible.
mixtures of $\nu_i$ and $\nu_j$ with maximal mixing angle ($\sim 45^\circ$) even for small off-diagonal elements in $M'$. Unless there are degeneracies in the $m_i^2$ the other neutrinos will play little role at the resonance (except for small effects involving the vacuum mixing angles), and one has an effective two-state problem. For the case $m_j^2 >> m_i^2$, which occurs in the seesaw model, the effective mixing angle is $|\theta| = |U_{ej}^V|$, as can easily be verified by comparing (5) and (23).

First, consider $\nu_e - \nu_\tau$ oscillations, which correspond to the mass difference $\Delta m_{21}^2 = m_2^2 - m_1^2$. The present model predicts $\Delta m_{21}^2 = 6 \times 10^{-13}$ eV$^2$, so that the adiabatic condition (16) is badly violated and essentially no $\nu_e - \nu_\mu$ conversions can occur$^{5),4),11,12)}$. In contrast, $\nu_e - \nu_\tau$ conversions are associated with $\Delta m_{31}^2 = m_3^2 - m_1^2 = 10^{-6}$ eV$^2$. This value is well within the range needed to account for the result of the $^{37}$Cl experiment (especially given that reasonable variations on the assumptions could easily change $\Delta m_{31}^2$ by a factor of 10$^2$ in either direction).

The relevant mixing angles can also be estimated in this simplified seesaw model. From (21) it is apparent that $M_\nu$ and $M_u$ are both diagonalized by the same unitary matrix $U_u^*$, while $M_e$ and $M_d$ are both diagonalized by $U_d$. Hence, the unitary matrix $U^V$ which relates the weak and mass eigenstate neutrinos in (24) is just $V^*$, where $V = U_u^*U_d^*$ is the Kobayashi-Maskawa mixing matrix. In particular, the effective mixing angle for $\nu_e - \nu_\tau$ oscillations is $|\theta| = |U_{e3}^V| = |V_{\tau d}|$. The phenomenological estimates$^{19)} |V_{\tau d}| = 0.005-0.02$ are well within the range needed to explain the result of the $^{37}$Cl experiment. It should be noted that while the numerical values of the neutrino masses (22a) depend sensitively on the simplifying assumption $M_{\nu} = M$, the order of magnitude of the predicted value of $\theta$ holds for a wide class of matrices $M_{\nu}$.

There are a number of uncertainties and ad hoc assumptions in these estimates. However, they suggest that the range of parameters needed to account for the solar neutrino results is perfectly consistent with typical predictions of the seesaw model. In particular, the seesaw model favours the small $\sin^2 2\theta_\nu$ region in which the two solutions described in the Introduction coincide and for which there should be little suppression of the flux of $\nu_e$ detectable in the $^{71}$Ga + $^{71}$Ge experiments. Moreover the relevant process may well be $\nu_e + \nu_\mu$ rather than $\nu_e + \nu_\tau$. 

NUCLEOSYNTHESIS AND NEUTRINO OSCILLATIONS IN MATTER

Primordial nucleosynthesis\textsuperscript{20} provides a probe of cosmological neutrino oscillations and conversions. Indeed, Kristev, Mikheyev and Smirnov\textsuperscript{21} have noted that since either $\nu_e$ or $\bar{\nu}_e$ transitions but not both may be affected by resonant amplifications in matter, oscillations may create a $\nu_e - \bar{\nu}_e$ asymmetry prior to nucleosynthesis. If such an asymmetry can be created while the reactions

$$\nu_e n \leftrightarrow e^- p, \quad \bar{\nu}_e p \leftrightarrow e^+ n$$

are still occurring, the neutron-to-proton ratio and, therefore, the predicted primordial abundance of $^4$He and the limits to the number of neutrino flavours\textsuperscript{22} can be affected\textsuperscript{*}.

Kristev, Mikheyev and Smirnov\textsuperscript{21} considered the conversions of $\nu_e$ into sterile [SU(2)$_L$×U(1) singlet] neutrinos (ν) and concluded the effects could be significant for $\sin^2 2\theta_\nu > 0.05$ and $(-\Delta m^2) = (10^{-6}-10^{-9})$ eV$^2$. Although oscillations of ordinary neutrinos into sterile neutrinos are a logical possibility, they may take place only under rather specific circumstances. In particular, the necessary masses and mixings could only occur if there are both Dirac mass terms (connecting the $\nu_e$ neutrino and the sterile neutrino) and Majorana masses (for $\nu_e$, the $\nu$, or both) in the range $(10^{-6}-10^{-4})$ eV$^2$. Such small Dirac and Majorana masses are not achieved simultaneously in any of the simpler models without fine tunings. Furthermore, if the small masses are generated by new Higgs fields with very small vacuum expectation values (vevs), then most likely there would be a phase transition in the early Universe, above which the vevs would be zero. Hence, for a vev $\ll 1$ MeV, the relevant temperature-dependent mass would actually be zero\textsuperscript{24} at a time relevant for nucleosynthesis and no oscillations could take place. Finally, we will argue

\textsuperscript{*} The possibility that the oscillations of $\nu_e$ and $\bar{\nu}_e$ into sterile neutrinos may generate an asymmetry between the densities of $\nu_e$ and $\bar{\nu}_e$ prior to nucleosynthesis and therefore may affect the predictions for the $^4$He abundance and, consequently, the limits on the number of neutrino flavours, was discussed without taking matter effects into account in Ref. 23).

\textsuperscript{**} The indicated range corresponds, of course, to the case of non-degenerate neutrino mass spectrum.
argue below that such oscillations would have to be into a different type of neutrino (if it is sterile) than those which are produced by the solar neutrino oscillations.

We have therefore considered instead the effects of ordinary conversions of the $\nu_e$ into the $\nu_\mu$ or $\nu_\tau$. It will turn out that resonance can be achieved prior to nucleosynthesis for $(-\Delta m^2)$ of order $10^{-7}$ eV$^2$, which is consistent with the expectations of the seesaw model. Furthermore, in the seesaw model, all mass terms are associated with Higgs fields with vevs very much greater than 1 MeV, so the masses and mixing angles can be regarded as constants during the relevant period. However, it will be argued that it is difficult to satisfy the adiabatic condition with mixing angles as small as those typically predicted by the seesaw model.

For simplicity, we will consider conversions involving two flavours only, and will usually assume that the second flavour is the same as the one that is relevant to the solar neutrino problem. We will assume that this is the $\nu_\tau$ for definiteness, but similar considerations would apply to the $\nu_\mu$. At resonance $\nu_e$'s are converted into $\nu_\tau$'s and vice versa, so a net change can only occur if there is a difference between the number (or momentum distributions) of $\nu_e$ and $\nu_\tau$ to begin with.* A small initial $\nu_e-\nu_\tau$ asymmetry is in fact expected25) because the $\nu_\tau$ (and $\nu_\mu$) decouple somewhat earlier than the $\nu_e$ (at $T^{*}_\tau = 5$ MeV as compared with $T^{*}_e = 3$ MeV). $e^+e^-$ annihilations between $T^{*}_\tau$ and $T^{*}_e$ therefore increase the $\nu_e$ temperature and density slightly compared to that of $\nu_\tau$. If a $\nu_e-\nu_\tau$ resonance conversion occurs the net effect will be to decrease the number of $\nu_e$ compared with $\overline{\nu}_e$ (which are not affected). The resulting $\overline{\nu}_e-\nu_e$ asymmetry will therefore increase the abundance of neutrons relative to protons, leading to an increased $^4$He abundance.

*) In the case of sterile neutrinos considered in Refs. 23) and 21), one expects a large initial excess of $\nu_e$ over the sterile $\nu$ since the latter, which are not produced by ordinary weak processes, would have decoupled early in the evolution of the Universe.
If the assumption that the conversion is into the same flavour in the early Universe as in the Sun is relaxed, then there is another possibility). Namely, the dominant component of the $\nu_e$ could be the intermediate mass state in a three-neutrino hierarchy: i.e., $m_1 < m_2 < m_3$, where the $\nu_1$, $\nu_2$, and $\nu_3$ are the dominant components of the $\nu_a$, $\nu_e$, and $\nu_b$, respectively. In that case, the $\nu_e - \nu_b$ conversion could occur in the Sun and the $\bar{\nu}_e - \bar{\nu}_a$ conversion in the early Universe. Then the neutron and the $^4$He fractions would decrease. The decrease of the $^4$He abundance could be compensated by additional neutrinos or other new light particles. We will not consider this possibility further.

When, during the early evolution of the Universe, the bulk of the $e^\pm$ pairs annihilates, the neutrinos are to a good approximation decoupled. The photons, therefore, are heated with respect to the neutrinos; after $e^\pm$ annihilation is complete, $T_\gamma / T_\nu = (11/4)^{1/3}$. However, the weak rates, Eq. (25), which determine the n/p ratio and, therefore, influence the $^4$He abundance, are extremely sensitive to the neutrino temperature. It is, thus, valuable to consider the effects of the deviations from the approximation of perfect decoupling25). Because electron neutrinos $\nu_e$ have charged current as well as neutral current weak interactions, they remain coupled longer than do the $\nu_\mu$ or $\nu_\tau$. Dicus et al.25) find for the fractional change in the neutrino temperatures (with respect to the standard-decoupled-result), $\delta = \Delta T / T$,

$$
\delta(T) = 5.9 \times 10^{-3} \left( \frac{a T^3}{3} \right)^{2/3} \exp \left( \frac{a T^3}{3} \right) \int_0^\infty \frac{e^{-x}}{x^{2/3}} dx
$$

(26)

where for $\nu_e a = 0.8$ and, for $\nu_\mu$ and $\nu_\tau a = 0.2$. For $T = T_{n/p} = 0.7$ MeV, $\delta_e = 4 \times 10^{-3}$ whereas $\delta_\mu = \delta_\tau = 2 \times 10^{-3}$. For $T$ in excess of $T_{n/p}$, the $\delta$'s decrease and the difference between $\delta_e$ and $\delta_\mu$, $\delta_\tau$ becomes smaller. Since $\nu_e - \nu_\tau$ conversions rely on the $\delta_e$, $\delta_\tau$ difference, the cosmological effect of such conversions becomes negligible for $T > 2$ MeV; we, therefore, limit our considerations to $T \lesssim 2$ MeV.

Neglecting conversions, but accounting for the slight temperature differences between $\nu_e$ and $\nu_\mu$, $\nu_\tau$, the predicted $^4$He mass fraction changes by25)

*) It is necessary to relax this assumption in the case of conversions into sterile neutrinos [Ref. 21]) because of the much larger initial excess of electron neutrinos over the sterile neutrinos.
\[ \Delta Y \approx -0.10 \delta_e + 0.04 \left( \frac{\delta_{\mu} + \delta_{\tau}}{2} \right) \]  

(27)

If, now, MSW oscillations succeed in exchanging \( v_e \) with \( v_\tau \) this change in \( Y \) will be modified to

\[ \Delta Y^{(e)} \approx -0.06 \left( \frac{\delta_e + \delta_\tau}{2} \right) + 0.04 \delta_\mu. \]  

(28)

since the \( v_e \) and \( v_\tau \) are swapped but the antineutrinos are untouched.

There is, in the case of \( v_e - v_\tau \) conversions an additional effect. After MSW oscillations, the "new" \( v_e \) are less abundant than the \( \bar{v}_e \); there is a \( v_e - \bar{v}_e \) asymmetry which affects the rates (25):

\[ \frac{n_{\bar{v}_e} - n_{v_e}}{n_{\bar{v}_e} + n_{v_e}} \approx -3 \left( \delta_e - \delta_\tau \right). \]  

(29)

This \( v_e - \bar{v}_e \) asymmetry may be expressed in terms of the "degeneracy parameter" \( \xi = \mu/T \),

\[ L_{v_e} = \frac{n_{v_e} - n_{\bar{v}_e}}{n_{\bar{v}_e}} = \frac{x_I^2}{1.3(3)} \left( \frac{T_{v_e}}{T_e} \right)^3 \left[ 1 + \left( \frac{\xi}{3} \right)^2 \right] \]  

(30)

Since, to sufficient accuracy, \( \xi \ll 1 \) and \( (n_{v_e} + n_{\bar{v}_e})/n_\gamma = 3/4(T_{v_e}/T_e)^3 \),

\[ \xi \approx -\frac{3.7}{4} \left( \frac{\delta_e - \delta_\tau}{\delta_e} \right) \approx -3.3 \left( \delta_e - \delta_\tau \right) \]  

(31)

The neutron-to-proton ratio, for \( T > T_{n/p} \), is modified to

\[ \frac{n}{p} \approx \exp\left( -\frac{m_n - m_p}{T} - \frac{\xi}{3} \right) \]  

(32)

so that the fractional change in \( n/p \) is

\[ \Delta \left( \frac{n}{p} \right) \approx -\xi \approx 3.3 \left( \delta_e - \delta_\tau \right) \]  

(33)

The predicted "He abundance will change, due to this effect of asymmetry, by an amount which may be estimated as
\[ \Delta \gamma^{(2)} \approx (1 - \frac{Y}{2}) \left[ \Delta \frac{(n/p)}{\Delta (n/p)} \right] = 3.3 \left(1 - \frac{Y}{2}\right) \left(\delta_e - \delta_\tau\right) \] (34)

For \( Y = 0.24 \), the total estimated change in the predicted \(^4\)He mass fraction is

\[ \Delta \gamma = \Delta \gamma^{(1)} + \Delta \gamma^{(2)} \approx 0.66 \delta_e - 0.68 \delta_\tau \] (35)

Thus, MSW oscillations between \( \nu_e \) and \( \nu_\tau \) lead to an increase in the predicted \(^4\)He mass fraction. To be at all effective, the oscillations cannot occur too early \((T \lesssim 2 \text{ MeV})\) nor too late \((T \gtrsim T_{n/p} = 0.7 \text{ MeV})\). The maximal effect is for complete conversion at \( T = T_{n/p} \), for which

\[ \Delta \gamma \approx 0.32 \delta_e \lesssim 1.3 \times 10^{-3} \] (36)

Any increase in the predicted abundance of \(^4\)He implies a smaller upper bound on the permitted number of extra flavours of light neutrinos:\(^{20}\):

\[ -\Delta N_\nu \approx \frac{\Delta \gamma}{0.014} \leq 0.09 \] (37)

Now let us consider the conditions for conversion to occur between \( T = 2 \text{ MeV} \) and \( T = 0.7 \text{ MeV} \). As the Universe expands and cools, the density decreases. To estimate the resonance temperature \((T_R)\) at which \( l_r = l_0 \cos 2\theta \), \( E_\nu \) is replaced \(^{23}\) in (1) by the average energy, \( \langle E_\nu \rangle = 3.15 T \), at temperature \( T \). Since the \( \nu_e^- \nu_e^- \) and \( \nu_e^- \nu_e^+ \) scattering amplitudes have opposite signs, (8) is replaced by

\[ \frac{\Delta \gamma}{\ell_0} = \sqrt{2} G_F \sum_{\vec{P}, S} \left[ \langle \nu_e^- | \mathcal{H}_{\text{eff}} | \nu_e^- \rangle - \langle \nu_e^+ | \mathcal{H}_{\text{eff}} | \nu_e^+ \rangle \right] \] (38)

where the matrix elements are summed over the electron and positron momentum distributions and spins at temperature \( T \). As long as the distributions are isotropic and unpolarized, (38) reduces to

\[ \Delta \gamma \approx \sqrt{2} G_F \sum_{\vec{P}, S} \left[ \langle \nu_e^- | \mathcal{H}_{\text{eff}} | \nu_e^- \rangle - \langle \nu_e^+ | \mathcal{H}_{\text{eff}} | \nu_e^+ \rangle \right] \]

---

\(^*\) The energy dispersion \( \Delta E = 0.55 \langle E \rangle \) changes the resonance temperature for individual neutrinos by less than 15\% which is not significant.

\(^{**}\) Although the neutrino temperatures differ from the photon temperature, this difference has a negligibly small effect on the neutrino oscillations. Therefore here we take \( T_{\nu_e} = T_{\nu_\tau} = T_\gamma = T \).
\[
\frac{\Delta r}{\ell^2} = \sqrt{2} \, G_F \, \Delta n_e
\]  
(39)

where \(\Delta n_e\) is the excess density of electrons over positrons. Since charge conservation ensures that \(\Delta n_e\) is equal to the proton density, \(\Delta n_e\) may be related to the photon density \(n_\gamma\) by

\[
\frac{\Delta n_e}{n_\gamma} = X_p \left( \frac{\beta}{N_{\gamma_0}} \right) \left( \frac{N_{\gamma_0}}{N_{\gamma}} \right) = \frac{2}{4} \eta \, X_p \left( T \right)
\]  
(40)

where \(\eta = B/N_{\gamma_0}\) is the present ratio of baryons to photons, \(N_{\gamma_0}/N_\gamma = 11/4\) is the ratio of photons (per comoving volume) at present to those at \(T > T_{n/p}\), and \(X_p = n_p (n_p + n_n)^{-1}\) is the proton mass fraction. Although \(\Delta n_e/n_\gamma\) changes because of neutron-to-proton (plus electron) conversions, the variation in \(X_p\) for \(2 \text{ MeV} > T > T_{n/p}\) is not large. The condition for resonance in the early Universe follows from (1), (39) and (40)

\[-\Delta m^2 \cos \Delta \theta_\nu = 3.9 \times 10^{-8} \left( \frac{\eta}{5 \times 10^{-10}} \right) \frac{X_p}{T_R} \frac{T}{T_R} \]  
(41)

In (41), \(\Delta m^2\) is in eV\(^2\), all temperatures are in MeV, and we have assumed \(E_\nu = E_\nu\) and \(T_\nu = T\); nucleosynthesis in the standard model suggests that \(\eta = (5 \pm 2) \times 10^{-10}\).

For total \(v_e - v_\tau\) conversion to occur, at resonance, the adiabatic condition, \((\Delta t/\ell^2)_R > 1\), should be satisfied, where

\[
\frac{\Delta t}{\ell^2}_R = \frac{\ell v_e}{\ell v_\tau} \Delta \left( \frac{v_e}{\ell v_\tau} \right) \ll \left( \frac{\ell v_e}{\ell v_\tau} \right) \frac{d}{dt} \left( \frac{\ell v_e}{\ell v_\tau} \right) \ll \frac{\sin^2 \Delta \theta_\nu}{\ell v} \frac{d}{dt} \left( \frac{\ell v_e}{\ell v_\tau} \right)
\]  
(42)

At resonance, we have:

\[
\ell v = \left( \frac{\sqrt{2} \, \frac{1}{4} \cos \Delta \theta_\nu}{G_F \, \Delta n_e} \right)_R
\]  
(43)

and

\[
\left| \frac{d}{dt} \left( \frac{\ell v}{\ell v_\nu} \right) \right|_R = \frac{2 \cos \Delta \theta_\nu}{t \left( T_{R} \right)}
\]  
(44)

In (44) the logarithmic derivative (with respect to temperature) of \(X_p\) has been neglected. For \(T = \text{ a few } \text{ MeV}\), when there are 43/4 relativistic degrees of freedom, \(t^{-1} = 10.85 \, T^2 / \ell^2_{\text{Planck}}\), and we obtain
\[
\left( \frac{\Delta t}{\ell_m} \right)_R = 0.54 \left( \tan^2 \theta \right) \left( \frac{\eta}{5 \times 10^{-10}} \right) X_p(T_R) T_R
\]  
(45)

Numerical studies show that a significant conversion is possible even if the adiabatic condition \((\Delta t > 1^{\text{Res}} / \ell_m)\) is relaxed significantly. For example, for \(\Delta t = 0.08 / \ell_m\), 50% conversion occurs.

Equation (45) may be solved for the value of \(\sin^2 2\theta_v\), which, for fixed \(\eta\) and \(T_R\), corresponds to a specific value of \((\Delta t / \ell_m)_R\):

\[
\sin^2 2\theta_v = \left( \frac{\Delta t}{\ell_m} \right)_R \left[ \left( \frac{\Delta t}{\ell_m} \right)_R + 0.54 \left( \frac{\eta}{5 \times 10^{-10}} \right) T_R X_p(T_R) \right]^{-1}
\]  
(46)

For each value of \(\eta\) and \(T_R\), there is a minimum value of \(\theta_v\) determined by setting \((\Delta t / \ell_m)_R = (\Delta t / \ell_m)_{R\text{min}} = 0.08\). Then, corresponding to this choice of \(\theta_v = \theta_{v\text{min}}\), the minimum value of \(\Delta m^2\) is found by substituting in (41):

\[
-\Delta m^2_{\text{min}} = 7.1 \times 10^{-3} \left[ 0.54 \left( \frac{\eta}{5 \times 10^{-10}} \right) T_R X_p(T_R) (0.08 + 0.54 \left( \frac{\eta}{5 \times 10^{-10}} \right) T_R X_p(T_R)) \right]^{1/2}
\]  
(47)

It must be checked that the conversion of \(\nu_e\) to \(\nu_\tau\) occurs sufficiently rapidly to affect the neutron-proton ratio. If \(\Delta t\) is compared with \(T_R = t(T_R)\), it follows that

\[
\left( \frac{\Delta t}{t} \right)_R = \frac{4}{5} \tan 2\theta_v
\]  
(48)

If \(T_\tau\) is defined by \(t(T_\tau) = t(t) + \Delta t\), then \(T_\tau / T_R = \left[ 1 + (\Delta t / t) \right]^{-1/2}\). For \(T_\tau \geq T_{n/p} = 0.7 \text{ MeV}\), we are restricted to \(T_R \geq 0.8 \text{ MeV}\).

Finally, we note that there is an upper bound to the vacuum mixing angle of interest here. Large vacuum mixing angles reduce the effects on nucleosynthesis described above because they reduce the \(\nu_e - \nu_\tau\) asymmetry and, also, because they lead to \(\bar{\nu}_e - \bar{\nu}_\tau\) oscillations. However, the presence of matter reduces the mixing angles for neutrinos before resonance and for antineutrinos at all \(t\) [see Eq. (10)]. As a result, we limit our considerations to \(\sin^2 2\theta_v \leq \frac{1}{2}\), for which vacuum oscillations are small though not entirely negligible.
Our results are summarized graphically in the Figure where the region in the $\Delta m^2 - \sin^2 2\theta_\nu$ plane is shown for $\eta = 5 \times 10^{-10}$ and $0.8 \text{ MeV} < T_R < 2 \text{ MeV}$. For each value of $T_R$ (and $\eta$), $\sin^2 2\theta_\nu$ lies between $\sin^2 2\theta_\nu^{\text{min}}$ [Eq. (46)] with $\langle \Delta t/\Delta t \rangle = 0.08$ and $\sin^2 2\theta_\nu^{\text{max}} = \frac{1}{2}$. Also shown in the Figure is the "solution" to the solar neutrino problem: $(-\Delta m^2)\sin^2 2\theta_\nu = 10^{-7.5} \text{ eV}^2$; there is overlap with early Universe oscillations for $1.1 \text{ MeV} < T_R < 1.8 \text{ MeV}$.

**GENERAL PROPERTIES**

We would like to conclude with a comparison between the general properties of the neutrino oscillations in vacuum and in matter. If the charged and neutral current weak interactions are described by the standard model, then $\nu_\mu$ and $\nu_e$ interact with matter in the same way while the interaction of $\nu_\tau$ is different from that of $\nu_\mu$ and $\nu_e$. If CPT invariance holds, the probabilities of the transitions of interest $\nu_e \rightarrow \nu_\ell$ and $\nu_\ell \rightarrow \nu_e$ ($\ell = e, \mu, \tau$) in vacuum $P^V(\nu_e \rightarrow \nu_\ell)$ and $P^V(\nu_\ell \rightarrow \nu_e)$ satisfy the following conditions:

$$P^V(\nu_e \rightarrow \nu_\ell) = P^V(\bar{\nu}_e \rightarrow \bar{\nu}_\ell),$$

$$P^V(\nu_\ell \rightarrow \nu_e) = P^V(\bar{\nu}_e \rightarrow \bar{\nu}_\ell),$$

$$\ell = e, \mu, \tau.$$  \hspace{1cm} (49)

In particular, it follows from CPT invariance that

$$P^V(\nu_e \rightarrow \nu_\ell) = P^V(\bar{\nu}_e \rightarrow \bar{\nu}_\ell)$$  \hspace{1cm} (50)

Further, if CP invariance holds in the leptonic sector, one has:

$$P^V(\nu_e \rightarrow \nu_\ell) = P^V(\bar{\nu}_e \rightarrow \bar{\nu}_\ell),$$  \hspace{1cm} (51)

$$P^V(\nu_\ell \rightarrow \nu_e) = P^V(\bar{\nu}_e \rightarrow \bar{\nu}_\ell),$$

$$\ell = e, \mu, \tau.$$
Equation (50) implies, however, that the study of the transitions $\nu_e \rightarrow \nu_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ in vacuum cannot give information about possible CP violation in the weak interaction of leptons.

In contrast, none of the relations (49) to (51) are valid for the probabilities of transitions of neutrinos propagating through matter. This is true even if the interactions of neutrinos are both CPT and CP conserving. In particular, as we have seen, in the case of two flavours,

$$P^{\nu_e}(\nu_e \rightarrow \nu_e) \neq P^{\nu_e}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

because $\nu_e$ and $\bar{\nu}_e$ scatter differently off electrons. This drastic difference between the properties of the probabilities of neutrino transitions in vacuum and of neutrino transitions in matter is a consequence of the fact that, due to the absence of positrons and antineutrons, ordinary matter is neither CPT nor CP symmetric, while the vacuum is (at least to a good approximation).

Let us suppose further that CP invariance does not hold in the leptonic sector and therefore the lepton mixing matrix contains non-trivial CP violating phases. The possible number of these depends on the type of massive neutrinos. In particular, if neutrinos do not take part in weak interaction with right-handed currents, which shall be assumed further, for a lepton flavours there may be $(n-1)(n-2)/2$ CP violating phases if the neutrinos with definite mass are Dirac particles and $n(n-1)/2$ CP violating phases in the case of Majorana mass eigenstate neutrinos.\textsuperscript{27,28} We shall show next that the probabilities of the neutrino transitions in matter, like the transition probabilities in vacuum,\textsuperscript{27,28} do not depend on the additional $(n-1)$ CP violating phases which are characteristic for the existence of massive Majorana neutrinos. For brevity we shall call these $(n-1)$ phases Majorana phases, while the term Dirac phases shall be used for the other $(n-1)(n-2)$ CP violating phases since they may be present in the lepton mixing matrix also in the case of massive Dirac neutrinos.

For the state of the left-handed flavour neutrino $\nu_{\ell}$ produced at the moment $t = 0$ in some weak process we have in the case of interest

$$|\nu_{\ell}> = \sum_{k=1}^{n} U_{\ell k} |\chi_k(p_k;L)>,$$

Here $U$ is an $n \times n$ unitary matrix - the lepton mixing matrix in vacuum [it is the complex conjugate of the matrix $U^\dagger$ defined in (24)], $|\chi_k(p_k;L)>$ is the state of

\textsuperscript{27,28}
the Majorana neutrino $\chi_k$ with mass $m_k$ and definite four-momentum $p_k$. In particular,

$$H_0 \left| \chi_k(p;L) \right> = E_k \left| \chi_k(p;L) \right>$$

(53)

where $H_0$ is the free neutrino Hamiltonian and $E_k = \sqrt{p^2 + m_k^2}$. In the limit $m_k = 0$ the state $\left| \chi_k(p;L) \right>$ describes a left-handed neutrino.

The matrix $U$ can be represented in the form $^{29)-31)}$

$$U = U^D P$$

(54)

Here $U^D$ is an $n \times n$ unitary matrix which contains the $\frac{1}{2}(n-1)(n-2)$ Dirac CP violating phases, while $P$ is an $n \times n$ diagonal unitary matrix

$$p_{jk} = \delta_{jk} e^{i\alpha_k}, \quad \alpha_n = 0, \quad j, k = 1, 2, ..., n$$

(55)

where $\alpha_k$, $k = 1, 2, ..., n-1$ are $(n-1)$ CP violating phases associated with the Majorana character of the massive neutrinos.

After a time $t$ one has:

$$\left| \nu_\ell \right>_t = \sum_{\ell' = e, \mu, \tau} X_{\ell \ell'}(t) \left| \nu_{\ell'} \right>_t$$

(56)

where $X_{\ell \ell'}(t)$ is the probability amplitude of the transition $\nu_\ell \leftrightarrow \nu_{\ell'}$ at time $t$. The time evolution of the state $\left| \nu_\ell \right>_t$ is determined by the Schrödinger equation:

$$i \frac{d}{dt} \left| \nu_\ell \right>_t = H \left| \nu_\ell \right>_t$$

(57)

Here $H = H_0 + H_{\text{int}}$, where $H_{\text{int}}$ is the neutrino interaction Hamiltonian. In the case of neutrinos propagating through matter, one has:

$$\left< \nu_\ell, H_{\text{int}} \nu_{\ell'} \right> = \delta_{\ell \ell'} H_{\ell}, \quad H_{\ell} \neq H_{\mu} = H_{\tau}$$

(58)

(the validity of the standard model for the weak interaction of neutrinos has been assumed). From (57), using (52)-(54), (56) and (58), we obtain the following equation for the probability amplitude $X_{\ell \ell'}(t)$:
\[
\begin{align*}
\frac{d}{dt} X_{\ell e'}(t) &= \sum_{\ell''} X_{\ell e''}(t) \left< \frac{e''}{\ell''}, H, \frac{e'}{\ell'} \right> \\
&= X_{\ell e'}(t) \delta_{\ell e'} + \sum_{\ell''} X_{\ell e''}(t) \sum_j (U^D P)_{\ell'' j} E_j (P^+ U^D)_{j e'} \tag{59}
\end{align*}
\]

Obviously, it follows from (55) that

\[
\sum_j (U^D P)_{\ell'' j} E_j (P^+ U^D)_{j e'} = \sum_j U^D_{\ell'' j} E_j U^D_{j e'} \tag{60}
\]

Consequently, the amplitude \(X_{\ell e}(t)\) of the transition \(\nu_1 \rightarrow \nu_1\), in matter does not depend on the \((n-1)\) CP violating Majorana phases. It is not difficult to shown that the same result is valid for the amplitude \(X_{\ell e'}(t)\) of the antineutrino transition \(\bar{\nu}_1 \rightarrow \bar{\nu}_1\), in matter. In such a way, we arrive at the conclusion that it is impossible to determine the type of massive neutrinos by studying the oscillations of neutrinos propagating through matter. An analogous result has been previously obtained for the case of oscillations of neutrinos in vacuum\(^{27,28}\).

**CONCLUSIONS**

Mikheyev and Smirnov have proposed a novel and plausible solution to the solar neutrino problem, based on the resonant amplification of neutrino oscillations in matter. In particular, the results of the \(^{37}\text{Cl}\) experiment can be accounted for if \(\nu_e = \nu_1\) is the lightest neutrino with \((-\Delta m^2) = m_2 - m_1 = 5 \times 10^{-5} \text{ eV}^2\) even for very tiny vacuum mixing angle, or for \((-\Delta m^2) \sin^2 2\theta_V = 10^{-7} - 10^{-5} \text{ eV}^2\).

We have considered several aspects of the MSW effect. In particular, the masses and the mixing angles suggested by the \(^{37}\text{Cl}\) results are consistent with the predictions of the seesaw model, with the large mass scale given by the typical GUT scale \(M_{\text{GUT}} \sim 10^{16} \text{ GeV}\) and the neutrino and charged-lepton Dirac masses related to the quark masses according to the simple \(SO(10)\) GUT prediction (21). Typical predictions for the neutrino masses are \(m_1 = 10^{-11} \text{ eV}, m_2 = 8 \times 10^{-7} \text{ eV}, m_3 = 10^{-3} \text{ eV}\), where \(\nu_1, \nu_2\) and \(\nu_3\) massive neutrinos are the dominant components of the \(\nu_e, \nu_\mu\) and \(\nu_\tau\) neutrinos, respectively. These values would be changed somewhat by reasonable modifications in the corresponding assumptions and should be considered as illustrative only. Nevertheless, they suggest that the relevant transition process for the solar neutrinos may be \(\nu_e \rightarrow \nu_\tau\) rather than...
\( \nu_e + \nu_\mu \) (for which the adiabatic condition is badly violated). For a wide class of heavy Majorana neutrino mass matrices, the order of magnitude of the effective mixing angle is predicted to be \( |\theta| \sim |V_{bt}| \sim 0.005 - 0.02 \), where \( V \) is the Cabibbo-Kobayashi-Maskawa matrix. This is within the range that can account for the \(^{37}\text{Cl}\) results, and for which the expected suppression of the \(^{71}\text{Ga}\) counting rate is small.

We have also considered cosmological implications of the MSW effect, concentrating on oscillations between the same two species that are relevant to the solar neutrino problem. There is a small range of parameters, shown in the Figure, for which resonant conversions between \( \nu_e \) and \( \nu_\tau \) (or \( \nu_e \) and \( \nu_\mu \)) can occur between the time of neutrino decoupling and freeze-out of the neutron-to-proton ratio. This range overlaps with the region of values of the parameters, which lead to a solution of the solar neutrino problem, but requires somewhat larger vacuum mixing angles than those predicted by most versions of the seesaw model. The \( \nu_e - \nu_\tau \) conversions can affect nucleosynthesis because they induce a \( \nu_e - \nu_e \) asymmetry (since resonance only affects neutrinos) and also because the \( \nu_e - \nu_\tau \) interchange modifies the freeze-out temperature. However, the effect turns out to be very small: the \(^4\text{He}\) abundance is predicted to increase by less than \( 1.3 \times 10^{-3} \), which is equivalent in its effect to the existence of less than 0.09 new neutrino species.

Finally, we have discussed some general properties of neutrino oscillations in matter. We have shown, in particular, that (just as in the case of vacuum oscillations) the probabilities of neutrino transitions in matter do not depend on the type (Dirac or Majorana) of the neutrinos with definite mass.

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The region of the $\Delta m^2 \, \sin^2 2\theta$ plane where significant $\nu_e - \nu_x$ conversions occur in the early Universe for $0.8 \text{ MeV} \lesssim T_R \lesssim 2 \text{ MeV}$ (and $\eta = 5 \times 10^{-10}$). Also shown is the "solution" to the solar neutrino problem: $(-\Delta m^2) \sin^2 2\theta = 10^{-7.5} \text{ eV}^2$. The region includes larger vacuum mixing angles than are predicted by most versions of the seesaw model.
\[ \eta = 5 \times 10^{-10} ; \sin^2 2\theta_V \leq 0.5 ; \left( \frac{\Delta t}{\langle m \rangle} \right)_R \geq 0.08 \]

\[ \log(-\Delta m^2) \]

\[ \sin^2 2\theta_V \]

- Diagram showing the relationship between \( \sin^2 2\theta_V \) and \( \log(-\Delta m^2) \) with different values of \( T_R \) (1.0, 1.2, 1.5, 1.8, 2.0).