Appendix A
Large-Time Evolution of the Free Field

Let us consider a negative-frequency free-field solution of the equation of motion for a real scalar field:

\[
F(r, t) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3p}{\sqrt{2E_p}} \phi(p) e^{i(p \cdot r - E_p t)} \equiv \int d^3p \psi(p) e^{i(p \cdot r - E_p t)}, \quad (A.1)
\]

where \(E_p = \sqrt{p^2 + m^2}\), and \(\phi(p)\) is a linear combination of wave packets, assumed to be sufficiently regular (typically, of gaussian shape.) For \(t \to \pm \infty\) and fixed \(r\), the integral vanishes faster than any negative power of \(t\), because the integral over momenta of the phase factor averages to zero. If however the position \(r\) is not held fixed, there is in general a region in the space of momenta where the phase factor is stationary; this region is identified by the condition

\[
\frac{\partial}{\partial p_i}(p \cdot r - E_p t) = r_i - \frac{p_i}{E_p} t = 0, \quad (A.2)
\]

or

\[
p = \frac{mv}{\sqrt{1 - v^2}}; \quad v = \frac{r}{t}, \quad (A.3)
\]

and gives the dominant contribution to the integral in the large-time limit. Thus, we take \(r = vt\) and we consider the limit of Eq. (A.1) for large \(t\) at constant \(v\):

\[
\lim_{t \to \infty} \int d^3p \psi(p) e^{i(p \cdot v - E_p t)}. \quad (A.4)
\]

Expanding the phase in powers of \(q \equiv p - \frac{mv}{\sqrt{1 - v^2}}\) up to second order we get

\[
p \cdot v - E_p = -\sqrt{1 - v^2} \left[ m - \frac{1}{2m} (q^2 - (q \cdot v)^2) \right] + O(q^3). \quad (A.5)
\]
The asymptotic limit of the integral is therefore

\[
\int d^3 p \, \psi(p) \, e^{i(p \cdot v - E_p)t} \sim e^{-im\sqrt{1 - v^2}t} \left( \frac{mv}{\sqrt{1 - v^2}} \right) \int d^3 q \, e^{-i\sqrt{1 - v^2} q^2 - (q \cdot v)^2/2mt} \left( \frac{2\pi m}{it} \right)^{3/2} \frac{1}{(1 - v^2)^{3/4}}.
\]  

(A.6)

By the same technique, we can obtain the following interesting result:

\[
\int d^3 p \, e^{-\frac{(p-k)^2}{2\sigma^2}} e^{\pm iEt} \sim e^{\pm imt - \frac{k^2}{2\sigma^2}} \left( \pm \frac{2\pi im}{t} \right)^{3/2}.
\]  

(A.7)
Appendix B
The S Matrix

In this Appendix, we give a proof of Eq. (3.18),

\[ S_{nm}^{qQ} = T_{qQ}^{nm} - 2\pi i \delta(E_q - E_Q) T_{qQ}^{nm}, \]  

(B.1)

where

\[ S_{qQ}^{nm} = \langle \psi_q^-, \psi_{Q,m}^+ \rangle \]  

(B.2)

and

\[ |\psi_k^{\pm}\rangle = |\varphi_{k,n}\rangle + \frac{1}{E_k - H_0 \pm i\epsilon} V|\psi_{k,n}\rangle. \]  

(B.3)

We will also show that

\[ -2\pi i \delta(E_q - E_Q) T_{qQ}^{nm} = -2\pi i \delta(E_q - E_Q) \langle \varphi_{q,n}|V|\psi_{Q,m}^+\rangle = -2\pi i \delta(E_q - E_Q) \langle \psi_{q,n}^-|V|\varphi_{m}\rangle. \]  

(B.4)

We have

\[ \langle \psi_{q,n}^-|V|\psi_{Q,m}^+\rangle = \langle \varphi_{q,n} + \frac{1}{E_q - H_0 - i\epsilon} V\psi_{q,n}^-|\varphi_{Q,m} + \frac{1}{E_Q - H_0 + i\epsilon} V\psi_{Q,m}^+ \rangle \]

\[ = T_{qQ}^{nm} + \frac{\langle \varphi_{q,n}|V|\psi_{Q,m}^+\rangle}{E_Q - E_q + i\epsilon} - \frac{\langle \psi_{q,n}^-|V|\varphi_{Q,m}\rangle}{E_Q - E_q - i\epsilon} \]

\[ + \langle \psi_{q,n}^-|V\frac{1}{E_q - H_0 + i\epsilon} E_Q - H_0 + i\epsilon V|\psi_{Q,m}^+\rangle, \]  

(B.5)

where we have used \( H_0|\varphi_{q,n} = E_q|\varphi_{q,n} \). We now use the identities

\[ \frac{1}{x + i\epsilon} = \frac{1}{x - i\epsilon} - 2\pi i \delta(x) \]  

(B.6)
\[
\frac{1}{E_q - H_0 + i\epsilon} \frac{1}{E_Q - H_0 + i\epsilon} = \frac{1}{E_Q - E_q} \left( \frac{1}{E_q - H_0 + i\epsilon} - \frac{1}{E_Q - H_0 + i\epsilon} \right)
\]  
(B.7)

to obtain

\[
\langle \Psi_{q,n}^- | V | \Psi_{Q,m}^+ \rangle = \mathcal{I}_{qQ} - 2\pi i\delta(E_Q - E_q) \langle \varphi_{q,n} | V | \Psi_{Q,m}^+ \rangle
\]  
(B.8)

\[
+ \frac{1}{E_Q - E_q - i\epsilon} \langle \varphi_{q,n} + \Psi_{q}^- | V \frac{1}{E_q - H_0 + i\epsilon} | V \Psi_{Q,m}^+ \rangle
\]

\[
- \frac{1}{E_Q - E_q - i\epsilon} \langle \Psi_{q,n}^- | V \varphi_{Q,m} + V \frac{1}{E_Q - H_0 + i\epsilon} V \Psi_{Q,m}^+ \rangle.
\]

The last two terms cancel against each other, because they are both proportional to

\[
\langle \Psi_{q,n}^- | V | \Psi_{Q,m}^+ \rangle.
\]  
(B.9)

Hence

\[
S_{qQ}^{nm} = \mathcal{I}_{qQ} - 2\pi i\delta(E_Q - E_q) \langle \varphi_{q,n} | V | \Psi_{Q,m}^+ \rangle
\]  
(B.10)

as announced. A similar arguments leads to

\[
S_{QQ} = \mathcal{I}_{qQ} - 2\pi i\delta(E_Q - E_q) \langle \varphi_{q,n} | V | \varphi_{Q,m} \rangle.
\]  
(B.11)

Using

\[
| \Psi_{Q,m}^+ \rangle = | \varphi_{Q,m} \rangle + \frac{1}{E_Q - H_0 + i\epsilon} V | \Psi_{Q,m}^+ \rangle
\]  
(B.12)

recursively, we get

\[
\langle \varphi_{q,n} | V | \Psi_{Q,m}^+ \rangle = \langle \varphi_{q,n} | V | \varphi_{Q,m} \rangle + \langle \varphi_{q,n} | V \frac{1}{E_Q - H_0 + i\epsilon} V | \varphi_{Q,m} \rangle
\]

\[
+ \langle \varphi_{q,n} | V \frac{1}{E_Q - H_0 + i\epsilon} \frac{1}{E_Q - H_0 + i\epsilon} V | \varphi_{Q,m} \rangle
\]

\[
+ \cdots
\]  
(B.13)

The second term in this expansion can be written

\[
\sum_{a} \sum_{b} \langle \varphi_{q,n} | V | \varphi_{p_{a,n_a}} \rangle \frac{\langle \varphi_{p_{b,n_b}} | \varphi_{p_{b,n_b}} \rangle}{E_Q - E_{p_b} + i\epsilon} \langle \varphi_{p_{b,n_b}} | V | \varphi_{Q,m} \rangle
\]

\[
= \frac{\langle \varphi_{q,n} | \langle 0 | V | \varphi_{Q,m} \rangle}{E_Q + i\epsilon} + \sum_{n_a} \int D^{3n_a} p_a \frac{\langle \varphi_{q,n} | V | \varphi_{p_{a,n_a}} \rangle \langle \varphi_{p_{a,n_a}} | V | \varphi_{Q,m} \rangle}{E_Q - E_{p_a} + i\epsilon}
\]  
(B.14)
Appendix C
Spectral Representation for the S Matrix

An alternative approach to the $S$ matrix, often found in the literature, is based on a spectral representation of the time-evolution operator. Let us consider the quantity

$$\lim_{t \to \infty} \lim_{t' \to -\infty} \langle \varphi_f(t) | e^{-\frac{i}{\hbar} H(t-t')} | \varphi_g(t') \rangle = \lim_{t \to \infty} \lim_{t' \to -\infty} \langle \varphi_f(t) | e^{\frac{i}{\hbar} H_0 t} e^{-\frac{i}{\hbar} H(t-t')} e^{-\frac{i}{\hbar} H_0 t'} | \varphi_g(t') \rangle, \quad (C.1)$$

where the free-particle wave packets $| \varphi_{g,f} \rangle$ are defined in Eqs. (3.13) and (3.16).

One can construct a spectral representation the time-evolution operator $e^{-\frac{i}{\hbar} H(t-t')}$ in terms of the incoming (outgoing) states in the equivalent forms

$$e^{-\frac{i}{\hbar} H(t-t')} = \sum_{m} \frac{1}{m!} \int D^3 Q e^{-\frac{i}{\hbar} E_Q (t-t')} | \psi_{Q,m}^+ \rangle \langle \psi_{Q,m}^+ | \quad (C.2)$$

Using the definition Eq. (3.2) we find, in analogy with Eqs. (3.12) and (3.14),

$$e^{-\frac{i}{\hbar} E_q t} \langle \varphi_f(t) | e^{\frac{i}{\hbar} H_0 t} | \psi_{q,n}^- \rangle = \int D^3 \ell \int_{-\infty}^{t} d\tau \int D^3 p f^*(p) e^{\frac{i}{\hbar} (E_p - E_q) \tau} \langle \varphi_{p,\ell} | V | \psi_{q,n}^- \rangle e^{\frac{i}{\hbar} (E_p - E_q) t} \quad (C.3)$$

$$e^{\frac{i}{\hbar} E_q t'} \langle \psi_{Q,m}^+ | e^{-\frac{i}{\hbar} H_0 t'} | \varphi_g(t') \rangle = \int D^3 \ell \int_{-\infty}^{t'} d\tau \int D^3 p g(p) e^{\frac{i}{\hbar} (E_q - E_p) \tau} \langle \psi_{Q,m}^+ | V | \varphi_{p,\ell} \rangle e^{\frac{i}{\hbar} (E_q - E_p) t'} \quad (C.4)$$
As a consequence,

$$\lim_{t \to \infty} \lim_{t' \to -\infty} \langle \varphi_f | e^{\frac{i}{\hbar} H_0 t} e^{-\frac{i}{\hbar} H(t-t')} e^{-\frac{i}{\hbar} H_0 t'} | \varphi_g \rangle = \int D^3n_i QD^3n_f q f^*(q) g(Q) S^{n_i n_f}_{Q}.$$  \hspace{1cm} (C.5)

This proves the well known result that the operator

$$U(t, t') = e^{\frac{i}{\hbar} H_0 t} e^{-\frac{i}{\hbar} H(t-t')} e^{-\frac{i}{\hbar} H_0 t'}$$  \hspace{1cm} (C.6)

converges weakly to the $S$ matrix when $t \to \infty$ and $t' \to -\infty$. 
Appendix D

Transition Amplitudes in the High Resolution Limit

In this Appendix, we will give an expression for the amplitude Eq. (3.23) for a generic scattering process which is more suited for its calculation in the semi-classical approximation. To this purpose, we observe that $S$-matrix elements can be written in terms of asymptotic particle creation operators, Eq. (3.4):

$$S_{qQ}^{n2} = \langle \Omega | \prod_{i=1}^{n} A_{\text{out}}(q_i) \prod_{j=1}^{2} A_{\text{in}}^\dagger(Q_j) | \Omega \rangle. \quad (D.1)$$

Using the explicit expressions for the wave packets, and taking for simplicity the same momentum resolution $\delta$ for both initial-state and final-state wave packets, we find

$$A_{kp}^{n2} = \left( \frac{1}{\sqrt{\pi \delta}} \right)^{\frac{1}{2}(n+2)} \prod_{i=1}^{n} \int d^3 q_i \, e^{-\frac{(k_i-q_i)^2}{2\delta^2}} \prod_{j=1}^{2} \int d^3 Q_j \, e^{-\frac{(p_j-Q_j)^2}{2\delta^2}} \langle \Omega | \prod_{i=1}^{n} A_{\text{out}}(q_i) \prod_{j=1}^{2} A_{\text{in}}^\dagger(Q_j) | \Omega \rangle, \quad (D.2)$$

with

$$\sum_{i=1}^{n} k_i = \sum_{j=1}^{2} p_j, \quad \sum_{i=1}^{n} E_{k_i} = \sum_{j=1}^{2} E_{p_j}. \quad (D.3)$$

In order to study the the small-$\delta$ behaviour of the amplitude, under the assumption that all particle momenta are different, it is convenient to change the integration variables as

$$q_i = k_i + \hat{q}_i \delta; \quad Q_j = p_j + \hat{Q}_j \delta, \quad (D.4)$$
so that

\[ A_{kp}^2 = \left( \frac{\delta}{\sqrt{\pi}} \right)^{3(n+2)} \prod_{i=1}^{n} \int d^3 \hat{q}_i e^{-\frac{\hat{q}_i^2}{2}} \prod_{j=1}^{2} \int d^3 \hat{Q}_j e^{-\frac{\hat{Q}_j^2}{2}} \]

\begin{align}
\langle \Omega | \prod_{i=1}^{n} A_{\text{out}}(k_i + \hat{q}_i \delta) \prod_{j=1}^{2} A_{\text{in}}^{\dagger}(p_j + \hat{Q}_j \delta) | \Omega \rangle.
\end{align}

(D.5)

The residual dependence of the vacuum expectation value on \( \delta \) can be made explicit, recalling that it is proportional to an energy-momentum conservation delta function

\begin{align}
\delta \left( \sum_{i=1}^{n} (k_i + \hat{q}_i \delta) - \sum_{j=1}^{2} (p_j + \hat{Q}_j \delta) \right) \delta \left( \sum_{i=1}^{n} E_{k_i + \hat{q}_i \delta} - \sum_{j=1}^{2} E_{p_j + \hat{Q}_j \delta} \right)
\end{align}

\begin{align}
= 2 \delta^4 \delta \left( \sum_{i=1}^{n} \hat{q}_i - \sum_{j=1}^{2} \hat{Q}_j \right) \delta \left( \sum_{i=1}^{n} v_i \cdot \hat{q}_i - \sum_{j=1}^{2} V_j \cdot \hat{Q}_j + O(\delta) \right),
\end{align}

(D.6)

where

\[ v_i = \frac{k_i}{E_{k_i}}; \quad V_j = \frac{p_j}{E_{p_j}} \]

and we have used Eq. (D.3). Hence

\[ A_{kp}^2 \sim \delta^{2n-1}, \]

(D.8)

which is consistent with our previous result Eq. (3.44).

We now introduce the initial and final creation operators

\[ A_{j}^{\dagger} = \frac{1}{\sqrt{2}} \sum_{j=1}^{2} A_{j,\text{in}}^{\dagger}, \quad A_{F}^{\dagger} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} A_{i,\text{out}}^{\dagger}, \]

(D.9)

where

\[ A_{j,\text{in}}^{\dagger} = \int d^3 Q \psi_{p_j}(Q)A_{\text{in}}^{\dagger}(Q); \quad A_{i,\text{out}}^{\dagger} = \int d^3 q \psi_{k_i}(q)A_{\text{out}}^{\dagger}(q) \]

(D.10)

so that

\[ A_{kp}^2 = \langle \Omega | \prod_{i=1}^{n} A_{i,\text{out}}^{\dagger} \prod_{j=1}^{2} A_{j,\text{in}}^{\dagger} | \Omega \rangle. \]

(D.11)
The operators $A_I, A_F$ are normalized so that

$$[A_I, A_I^\dagger] = [A_F, A_F^\dagger] = 1.$$ (D.12)

In the limit of infinite momentum resolution of the wave packets, $\delta \to 0$, the amplitude Eq. (D.11) can be written in terms of the (non normalized) coherent states

$$|I\rangle \equiv e^{\sqrt{2}A_I^\dagger}|\Omega\rangle, \quad |F\rangle \equiv e^{\sqrt{n}A_F^\dagger}|\Omega\rangle$$ (D.13)

as

$$\langle F|I \rangle - 1 = A_{kp}^{n^2} \left[ 1 + O(\delta^{3n-1}) \right].$$ (D.14)

Here is the proof. Using the definitions in Eqs. (D.9) and (D.10) we find

$$\langle F|I \rangle = \langle \Omega|e^{\sum_{i=1}^n A_{i,\text{out}}} e^{\sum_{j=1}^2 A_{j,\text{in}}} |\Omega\rangle$$ (D.15)

$$= \langle \Omega| \prod_{i=1}^n e^{A_{i,\text{out}}} \prod_{j=1}^2 e^{A_{j,\text{in}}} |\Omega\rangle$$

$$= \sum_{\nu_1,\ldots,\nu_n=0}^{\infty} \sum_{\mu_1,\mu_2=0}^{\infty} \frac{1}{\nu_1! \mu_1! \nu_2! \mu_2!} \langle \Omega| \prod_{i=1}^n (A_{i,\text{out}})^{\nu_i} \prod_{j=1}^2 (A_{j,\text{in}})^{\mu_j} |\Omega\rangle.$$ (D.16)

Due to vacuum translation invariance, and for sufficiently small $\delta$, the vacuum expectation values (vev’s) in the right-hand side of Eq. (D.15) are non-zero only for

$$\sum_{i=1}^n \nu_i k_i = \sum_{j=1}^2 \mu_j p_j, \quad \sum_{i=1}^n \nu_i E_k_i = \sum_{j=1}^2 \mu_j E_p_j.$$ (D.16)

Now, the constraints Eq. (D.16) are compatible with energy-momentum conservation, Eqs. (D.3), only if the $\nu_i$’s and $\mu_j$’s are equal to the same integer $K$. Thus

$$\langle F|I \rangle - 1 = \sum_{K=1}^{\infty} \frac{1}{(K!)^{n+2}} \langle \Omega| \prod_{i=1}^n (A_{i,\text{out}})^K \prod_{j=1}^2 (A_{j,\text{in}})^K |\Omega\rangle.$$ (D.17)

In quantum field theory, vacuum expectations values of products of operators $\langle \Omega| \prod_{j=1}^n O_j |\Omega\rangle$ are recursively decomposed into truncated parts, defined iteratively by the following construction:

---

1 In the general case, when the $O_j$’s are local operators in a massive theory, the truncated vev’s satisfy a cluster property, that is, they vanish exponentially when any space-like distance between the operator points diverges.
\[
\langle \Omega | O_i | \Omega \rangle = \langle \Omega | O_i | \Omega \rangle_T \\
\langle \Omega | O_i O_j | \Omega \rangle = \langle \Omega | O_i O_j | \Omega \rangle_T + \langle \Omega | O_i | \Omega \rangle_T \langle \Omega | O_j | \Omega \rangle_T \\
\langle \Omega | O_i O_j O_k | \Omega \rangle = \langle \Omega | O_i O_j O_k | \Omega \rangle_T \\
+ \langle \Omega | O_i O_k | \Omega \rangle_T \langle \Omega | O_j | \Omega \rangle_T + \langle \Omega | O_i | \Omega \rangle_T \langle \Omega | O_j | \Omega \rangle_T \langle \Omega | O_k | \Omega \rangle_T \\
\ldots 
\]

(D.18)

If the \(O_i\)'s are asymptotic creation or annihilation operators, each truncated vev is proportional to an energy-momentum conservation delta function, since the vacuum state is space-time translation invariant. For a generic choice of particle momenta, only constrained by total energy-momentum conservation, the coefficients of these delta functions are regular, although non-analytic, functions.

However, if the energy-momentum constraint is also satisfied by subsets of the \(q\)’s and \(Q\)’s (as is the case for all terms with \(K > 1\) in the sum of Eq. (D.17)), singularities in the coefficient functions appear due to the vanishing of some \(E_k - H_0 \pm i\epsilon\) denominator in the recursive expansion of Eq. (3.2). In a Lippman-Schwinger approach these singularities correspond to intermediate states whose particles are on their mass-shell. That is, the momenta of all the intermediate particles are fixed by the momentum conservation and the total energy is degenerate with the total energy of the process.

A simple example is provided by the term \(K = 2\) for \(2 \rightarrow 2\) scattering (\(n = 2\)). This term is proportional to

\[
\langle \Omega | A_{\text{out}}(q_2) A_{\text{out}}(q_2) A_{\text{out}}(q_3) A_{\text{out}}(q_4) A_{\text{in}}^\dagger(Q_1) A_{\text{in}}^\dagger(Q_2) A_{\text{in}}^\dagger(Q_3) A_{\text{in}}^\dagger(Q_4) | \Omega \rangle 
\]

(D.19)

with

\[
q_1 + q_2 \simeq Q_1 + Q_2; \quad E_{q_1} + E_{q_2} \simeq E_{Q_1} + E_{Q_2} \quad (D.20)
\]
\[
q_3 + q_4 \simeq Q_3 + Q_4; \quad E_{q_3} + E_{q_4} \simeq E_{Q_3} + E_{Q_4} \quad (D.21)
\]

up to corrections of order \(\delta\).

It is shown in Appendix B that, if the interaction is given by \(\frac{\lambda}{4!\hbar^4} \phi^4\), the third order term in the expansion of the amplitude given by Eq. (3.2) contains terms proportional to the product

\[
\frac{1}{(E_{Q_1} + E_{Q_2} - E_{q_2} - E_{Q_1 + q_2 - q_2} + i\epsilon)(E_{q_3} + E_{q_4} - E_{Q_3} - E_{q_3 + q_4 - Q_4} + i\epsilon)} 
\]

(D.22)

and analogous ones which apparently diverge if the first two initial particles and the first two final ones fulfill the energy conservation constraint.

It can be checked that, independently of the particular choice of the interaction, a truncated vev of the ordered products of asymptotic creation and annihilation operators \(\langle \Omega | \prod_{i=1}^n A_{\text{out}}(q_i) \prod_{j=1}^m A_{\text{in}}^\dagger(Q_j) | \Omega \rangle_T\) contains a pair of vanishing
denominators for each independent subset of initial and final particles fulfilling the energy-momentum conservation constraint. The divergence is actually regulated by the presence of wave packets with a finite width.

In the general case we have

\[
\langle \Omega | \prod_{i=1}^{n} (A_{i,\text{out}})^2 (A_{j,\text{in}})^2 | \Omega \rangle = \langle \Omega | \prod_{i=1}^{n} (A_{i,\text{out}}) K \prod_{j=1}^{2} (A_{j,\text{in}}) K | \Omega \rangle_T
\]

\[
+ K \langle \Omega | \prod_{i=1}^{n} (A_{i,\text{out}})^{K-1} \prod_{j=1}^{2} (A_{j,\text{in}})^{K-1} | \Omega \rangle_T \langle \Omega | \prod_{i=1}^{n} A_{i,\text{out}} \prod_{j=1}^{2} A_{j,\text{in}} | \Omega \rangle_T + \cdots
\]

\[
+ \left( \langle \Omega | \prod_{i=1}^{n} A_{i,\text{out}} \prod_{j=1}^{2} A_{j,\text{in}} | \Omega \rangle_T \right)^K
\]

(D.23)

all other possible truncated vev’s being zero. By the same procedure that led us to Eq. (D.8), we find

\[
\langle \Omega | \prod_{i=1}^{n} (A_{i,\text{out}})^M \prod_{j=1}^{2} (A_{j,\text{in}})^M | \Omega \rangle_T = \delta^{-4} \left( \frac{\delta}{\sqrt{\pi}} \right)^{\frac{3}{2}M(n+2)}
\]

\[
\prod_{i=1}^{nM} \int d^3 \hat{q}_i e^{-\frac{\hat{q}_i^2}{2}} \prod_{j=1}^{2M} \int d^3 \hat{Q}_j e^{-\frac{\hat{Q}_j^2}{2}}
\]

\[
\delta \left( \sum_{i=1}^{nM} \hat{q}_i + \sum_{j=1}^{2M} \hat{Q}_j \right) \delta \left( \sum_{i=1}^{nM} \hat{q}_i \cdot v_i + \sum_{j=1}^{2M} \hat{Q}_j \cdot V_i + O(\delta) \right)
\]

\[
\sum_{a=0, b \geq 1}^{M} \prod_{i=1}^{a} \Delta_i^{-1}(a, b, \hat{q}, \hat{Q}, k, p) C(a, b, \hat{q}, \hat{Q}, k, p),
\]

(D.24)

where the index \( a \) counts the number of vanishing denominators \( \Delta_i(a, b, \hat{q}, \hat{Q}, k, p) \), \( b \) labels the different terms with the same number of vanishing denominators appearing in the expansion and \( C(a, b, \hat{q}, \hat{Q}, k, p) \) are regular coefficient functions.

As we have seen in the example given above, a generic vanishing denominator has the form

\[
\Delta_i(a, b, \hat{q}', \hat{Q}', k, p) = \left( \sum_{i=1}^{n} v_{a,b,i} \cdot \hat{q}_i + \sum_{j=1}^{n} V_{a,b,j} \cdot \hat{Q}_j \right) \delta + O(\delta^2).
\]

(D.25)
Hence

\[ \langle \Omega \prod_{i=1}^{n} (A_{i,\text{out}})^{M} (\prod_{j=1}^{m} A_{j,\text{in}}^\dagger)^{M} | \Omega \rangle_T \sim \delta^{-4-2(M-1)} \delta^{\frac{3}{2}M(n+2)} = \delta^{\frac{3}{2}Mn+M-2}. \]  

(D.26)

This result implies that in the \( \delta \to 0 \) limit each term in the sum Eq. (D.23) vanishes as

\[ \delta^{\frac{3}{2}Mn+M-2} \delta^{(K-M)(\frac{3}{2}n-1)} = \delta^{\frac{3}{2}Kn-K+2M-2} \]  

(D.27)

The sum is therefore dominated by the term \( M = 0 \):

\[ \langle \Omega \prod_{i=1}^{n} (A_{i,\text{out}})^{K} (\prod_{j=1}^{2} A_{j,\text{in}}^\dagger)^{K} | \Omega \rangle = \left( \langle \Omega \prod_{i=1}^{n} A_{i,\text{out}} (\prod_{j=1}^{2} A_{j,\text{in}}^\dagger) | \Omega \rangle \right)^{K} [1 + O(\delta^{2})] \]

\[ \sim \delta^{(\frac{3}{2}n-1)K} [1 + O(\delta^{2})] \]  

(D.28)

(note that the result Eq. (D.8) is recovered for \( K = 1 \)). As a consequence

\[ \langle F | I \rangle - 1 = \langle \Omega \prod_{i=1}^{n} A_{i,\text{out}} (\prod_{j=1}^{2} A_{j,\text{in}}^\dagger) | \Omega \rangle [1 + O(\delta^{2})] = A_{kp}^{n2}[1 + O(\delta^{\frac{3}{2}n-1})]. \]  

(D.29)

This completes the proof.

The representation Eq. (D.29) of the generic transition amplitude will prove particularly suited to identify the asymptotic properties of the field operator, which is a crucial step in the construction of the semiclassical approximation to the scattering amplitude.
Appendix E
Scattering from an External Density

A simple and interesting application of our formulae Eqs. (3.124) and (3.129) is the computation of scattering amplitudes in a model with interaction given by

$$\mathcal{L}_I = \frac{g(x)}{2} \phi^2(x).$$  \hspace{1cm} (E.1)

In this model, the field equations are linear, and the function $g(x)$ plays a role which is similar to that of a potential in the Schrödinger equation. The solution of Eq. (3.129) is

$$S[\phi^{(\text{as})}] = -\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \int d^4x \, g(x) \phi^{(\text{as})}(x) \left( (\Delta \ast g)^n \phi^{(\text{as})} \right) (x)$$
$$= -\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \int d^4x \, \prod_{i=1}^{n} dy_i \phi^{(\text{as})}(x) g(x) \Delta(x - y_1) g(y_1) \Delta(y_1 - y_2) g(y_2) \cdots \Delta(y_{n-1} - y_n) g(y_n) \phi^{(\text{as})}(y_n).$$  \hspace{1cm} (E.2)

Thus, $S$ is quadratic in $\phi^{(\text{as})}(x)$, and describes three possible processes: two-particle annihilation, pair production, and single-particle scattering. To first order in $g$, one has

$$S[\phi^{(\text{as})}] = -\frac{1}{2} \int d^4x \, g(x) \left( \phi^{(\text{as})}(x) \right)^2,$$  \hspace{1cm} (E.3)

and therefore the scattering amplitude

$$S_{p \rightarrow k}[g] = -\int d^4x \, g(x) \frac{e^{i(k-p) \cdot x}}{2(2\pi)^3 \sqrt{E_p E_k}} \equiv -\frac{\tilde{g}(p - k)}{\sqrt{E_p E_k}}$$  \hspace{1cm} (E.4)

while the annihilation amplitude for a pair of particles with momenta $p_1$ and $p_2$ is...
\[ S_{p_1,p_2 \to 0}[g] = -\pi \frac{\tilde{g}(p_1 + p_2)}{\sqrt{E(p_1)E(p_2)}}. \]  

(E.5)

Finally, the pair production amplitude is

\[ S_{0 \to k_1,k_2}[g] = -\pi \frac{\tilde{g}(-k_1 - k_2)}{\sqrt{E_{k_1}E_{k_2}}}. \]  

(E.6)

We now consider the case

\[ g(x) = g_1(x) + g_2(x), \]  

(E.7)

with

\[ \theta(x_0^0 - x_1^0)g_1(x_1)g_2(x_2) = g_1(x_1)g_2(x_2), \]  

(E.8)

that is, \( g_1 \) acts before \( g_2 \). Selecting in the scattering amplitude due to \( g(x) \) the first order terms in \( g_1 \) and \( g_2 \), one has

\[ S_{1,2} = \int d^4x \ g_1(x)\phi^{(as)}(x) \left( \Delta \circ g_2\phi^{(as)} \right)(x) \]
\[ = \int d^4x_1 d^4x_2 \ g_1(x_1)\phi^{(as)}(x_1)\Delta(x_1 - x_2)g_2(x_2)\phi^{(as)}(x_2). \]  

(E.9)

Replacing

\[ \phi^{(as)}(x) = \frac{e^{ik \cdot x}}{\sqrt{2E_k(2\pi)^3}} + \frac{e^{-ip \cdot x}}{\sqrt{2E_p(2\pi)^3}}, \]  

(E.10)

and recalling Eqs. (E.8) and (3.95), we find

\[ S_{1,2} = \frac{\pi^2}{\sqrt{E_pE_k}} \int \frac{dq}{E_q} \left[ \tilde{g}_1(p - q)\tilde{g}_2(q - k) + \tilde{g}_1(-k - q)\tilde{g}_2(q + p) \right] \]
\[ = \int dq \left[ S_{p \to q}[g_1]S_{q \to k}[g_2] + S_{0 \to q,k}[g_1]S_{p,q \to 0}[g_2] \right]. \]  

(E.11)

This shows that the scattering amplitude appears as the sum of two terms. The first describes the scattering due to \( g_1 \) from \( p \) to \( q \), followed by the scattering due to \( g_2 \) from \( q \) to \( k \). The second term describes creation from the vacuum of a pair with momenta \( k \) and \( q \), followed by the annihilation of the initial particle with momentum \( p \) with that with momentum \( q \). Thus, the scattering process factorizes into the contributions due to \( g_1 \) and \( g_2 \) consistently with the causal order.
Appendix F
Dirac Matrices

Most calculations involving Dirac matrices can be performed using only their general properties,
$$\{\gamma_\mu, \gamma_\nu\} = 2Ig_{\mu\nu}; \quad \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!} \epsilon^{\mu\nu\rho\sigma} \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma,$$
$$\gamma^\dagger_\mu = \gamma_0\gamma_\mu\gamma_0,$$ (F.1)
with no reference to a specific representation. Some immediate consequences of Eq. (F.1) are
$$\text{Tr} \gamma_\mu\gamma_\nu = 4g_{\mu\nu}; \quad \{\gamma_\mu, \gamma_5\} = 0; \quad \gamma_5^2 = I,$$ (F.2)
and the identities
$$\gamma^\mu\gamma^\alpha\gamma_\mu = -2\gamma^\alpha$$ (F.3)
$$\gamma^\mu\gamma^\alpha\gamma^\beta\gamma_\mu = 4g^{\alpha\beta}$$ (F.4)
$$\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma_\mu = -2\gamma^\gamma\gamma^\beta\gamma^\alpha.$$ (F.5)

The trace of the product of an odd number of $\gamma$ vanishes. Indeed,
$$\gamma_\mu = -\gamma_5\gamma_\mu\gamma_5,$$ (F.6)
and therefore
$$\text{Tr} \gamma_{\mu_1}\cdots\gamma_{\mu_{2n+1}} = (-1)^{2n+1}\text{Tr} \left(\gamma_5\gamma_{\mu_1}\gamma_5\right)\cdots\left(\gamma_5\gamma_{\mu_{2n+1}}\gamma_5\right)$$
$$= -\text{Tr} \gamma_{\mu_1}\cdots\gamma_{\mu_{2n+1}},$$ (F.7)
where we have used the circular property of the trace. It is easy to prove that
\[
\text{Tr} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) \quad (F.8)
\]

\[
\text{Tr} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^5 = 4i\epsilon^{\mu\nu\rho\sigma}. \quad (F.9)
\]

In Sect. 6.1 we have introduced a particular representation of the Dirac matrices:

\[
\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}. \quad (F.10)
\]

Different representations of the \( \gamma \) matrices are related by similarity transformations on spinor fields. A representation which is often used (especially in applications that involve the non-relativistic limit) is the so-called standard representation:

\[
\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (F.11)
\]
Appendix G
Violation of Unitarity in the Fermi Theory

In this Appendix, we show that unitarity of the $S$ matrix is violated in the Fermi theory of weak interactions. We rewrite the unitarity constraint, Eq. (4.49), for $i = j$:  

$$
\sum_f \int d\phi_{n_f}(P_i; k^f_1, \ldots, k^f_{n_f}) \left|\mathcal{M}_{if}\right|^2 = -2 \text{Im} \mathcal{M}_{ii}, \quad (G.1)
$$

which is the so-called optical theorem: the total cross section for the process $i \to f$, summed over all possible final states $f$, is proportional to the imaginary part of the forward invariant amplitude $\mathcal{M}_{ii}$.

Let us now assume that $i$ is a state of two massless particles with momenta $p_1, p_2$; furthermore, let us assume that only $2 \to 2$ processes are allowed. Under these conditions, the states $f$ are also two-particle states, and the amplitudes $\mathcal{M}_{if}$ depend on the initial and final states through the two independent Mandelstam variables $s, t$:  

$$
\mathcal{M}_{if} = \mathcal{M}(s, t), \quad (G.2)
$$

where

$$
s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2. \quad (G.3)
$$

In the center-of-mass frame,

$$
t = -\frac{s}{2}(1 - \cos \theta) \rightarrow \cos \theta = 1 + \frac{2t}{s}, \quad (G.4)
$$

where $\theta$ is the scattering angle. Thus, for a given value of the center-of-mass squared energy $s$, the amplitude $\mathcal{M}(s, t)$ is a function of $\cos \theta$ only, and can be expanded on the basis of the Legendre polynomials

$$
P_J(z) = \frac{1}{J!2^J} \frac{d^J}{dz^J}(z^2 - 1)^J. \quad (G.5)
$$

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The Legendre polynomials obey the orthogonality conditions

\[ \int_{-1}^{1} dz \, P_J(z) \, P_K(z) = \frac{2}{2J+1} \delta_{JK} \]  
\[(G.6)\]

and the normalization conditions

\[ P_J(1) = 1. \]  
\[(G.7)\]

We find

\[ M(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos \theta), \]  
\[(G.8)\]

where the partial-wave amplitudes \( a_J(s) \) are given by

\[ a_J(s) = \frac{1}{32\pi} \int_{-1}^{1} d\cos \theta \, P_J(\cos \theta) \, M(s,t). \]  
\[(G.9)\]

Replacing Eq. (G.8) in the l.h.s. of Eq. (G.1) we get

\[ \int \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \frac{d^3k_2}{(2\pi)^3 2E_{k_2}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) |M(s,t)|^2 \]
\[ = \frac{1}{16\pi} \int_{-1}^{1} d\cos \theta \]
\[ \left[ 16\pi \sum_J (2J+1) a_J(s) P_J(\cos \theta) \right] \left[ 16\pi \sum_K (2K+1) a_K^*(s) P_K(\cos \theta) \right] \]
\[ = 32\pi \sum_J (2J+1) |a_J(s)|^2, \]  
\[(G.10)\]

while the r.h.s. is given by

\[ -2 \text{ Im } M(s,0) = -32\pi \sum_J (2J+1) \text{ Im } a_J(s), \]  
\[(G.11)\]

where we have set \( t = 0 \), or equivalently \( \cos \theta = 1 \), as appropriate for a forward amplitude, and we have used the normalization condition (G.7). Therefore, the unitarity constraint Eq. (G.1) requires

\[ |a_J(s)|^2 = -\text{ Im } a_J(s) \]  
\[(G.12)\]
Appendix G: Violation of Unitarity in the Fermi Theory

for all partial amplitudes. Equation (G.12) provides the unitarity bound

\[ |a_J(s)| \leq 1. \]  

(G.13)

Let us now consider a specific process, namely

\[ e^{-}(p_1) + \nu_{\mu}(p_2) \rightarrow \mu^{-}(k_1) + \nu_{e}(k_2) \]  

(G.14)

within the Fermi theory. The relevant amplitude is

\[ M(s, t) = -\frac{G_F}{\sqrt{2}} \bar{u}(k_2) \gamma_\alpha (1 - \gamma_5) u(p_1) \bar{u}(k_1) \gamma_\alpha (1 - \gamma_5) u(p_2), \]  

(G.15)

where all lepton masses have been neglected. This gives

\[ \sum_{\text{pol}} |M(s, t)|^2 = 2G_F^2 \text{Tr} \left[ \gamma_\alpha \not{p}_1 \gamma_\beta (1 - \gamma_5) \not{k}_2 \right] \text{Tr} \left[ \gamma_\alpha \not{p}_2 \gamma_\beta (1 - \gamma_5) \not{k}_1 \right] = 32G_F^2 s^2. \]  

(G.16)

We see that only the partial amplitude \( a_0(s) \) is nonzero, since there is no \( t \) dependence at all. Using the definition Eq. (G.9) we obtain

\[ |a_0(s)| = \frac{G_F s}{2\sqrt{2\pi}}. \]  

(G.17)

The unitarity bound Eq. (G.13) is therefore violated for

\[ \sqrt{s} \geq \sqrt{\frac{2\sqrt{2\pi}}{G_F}} \simeq 875 \text{ GeV}. \]  

(G.18)

The total cross section obtained from Eq. (G.16),

\[ \sigma = \frac{G_F^2 s}{2\pi}, \]  

(G.19)

grows linearly with the squared center-of-mass energy \( s \).

In the standard model, the same amplitude involves the exchange of a virtual \( W \) boson with mass \( m_W \) and coupling \( g/(2\sqrt{2}) \) to left-handed fermions. The standard model squared amplitude is obtained from the result in Eq. (G.16) by the replacement

\[ -\frac{G_F}{\sqrt{2}} \rightarrow \frac{g^2}{8} \frac{1}{t - m_W^2} = \frac{G_F}{\sqrt{2}} \frac{m_W^2}{2(t - m_W^2)}. \]  

(G.20)
We get
\[ \sum_{\text{pol}} \left| \mathcal{M}^{\text{SM}}(s, t) \right|^2 = 64 G_F^2 s^2 \left( \frac{m_W^2}{t - m_W^2} \right)^2. \] (G.21)

The total cross section is now given by
\[ \sigma^{\text{SM}} = \frac{G_F^2 s m_W^2}{2\pi (s + m_W^2)}, \] (G.22)
that reduces to the result obtained in the Fermi theory, Eq. (G.19), for \( s \ll m_W^2 \). In this case, however, the linear growth of the cross section with \( s \) is cut off at \( s \sim m_W^2 \).

At very large energy,
\[ \sigma^{\text{SM}} \rightarrow \frac{G_F^2 m_W^2}{2\pi}. \] (G.23)

The value of \( m_W \) is related to the size of the coupling \( g \) through \( G_F / \sqrt{2} = g^2 / (8 m_W^2) \).

If \( m_W \) were close to the energy at which the Fermi theory breaks down, about 900 GeV, then \( g \) would take a value close to 10, far from the perturbative domain. The fact that the measured value \( m_W \) is instead much smaller, \( m_W \simeq 80 \text{ GeV} \), is a signal of the fact that a theory of weak interactions with an intermediate vector boson can be treated perturbatively: indeed, in this case we get \( g \sim 0.7 \).
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