Measurement of three-jet production cross-sections in pp collisions at 7 TeV centre-of-mass energy using the ATLAS detector

The ATLAS Collaboration

Double-differential three-jet production cross-sections have been measured in proton-proton collisions at a centre-of-mass energy of $\sqrt{s} = 7$ TeV using the ATLAS detector at the Large Hadron Collider. The measurements are presented as functions of the three-jet invariant mass ($m_{jjj}$) and the sum of absolute rapidity separations between the three leading jets ($|Y^*|$). Invariant masses extending up to 5 TeV are reached for $8 < |Y^*| < 10$. These measurements use a sample of data recorded using the ATLAS detector in 2011, which corresponds to an integrated luminosity of 4.51 fb$^{-1}$. Jets are identified using the anti-$k_t$ algorithm with two different jet radius parameters, $R = 0.4$ and $R = 0.6$. The dominant uncertainty on these measurements comes from the jet energy scale. Next-to-leading order QCD calculations corrected to account for non-perturbative effects are compared to the measurements. A good agreement between the data and the theoretical predictions based on most of the global parton distribution functions is found over the full kinematic range, covering almost seven orders of magnitude in the measured cross-section values.

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1 Introduction

Collimated jets of hadrons are a characteristic feature of high-energy particle interactions. In the theory of strong interactions, Quantum Chromodynamics (QCD), jets can be interpreted as the result of fragmentation of partons produced in a scattering process. In high-energy particle collisions two main phases can be distinguished. In the perturbative phase, high-transverse momentum (\(p_T\)) partons are produced in a hard-scattering process at a scale \(Q\). This phase is described by a perturbative expansion in QCD.

In the transition to the second (non-perturbative) phase these partons emit additional gluons and produce quark-antiquark pairs. The non-perturbative jet evolution is an interplay between the hadronisation process and underlying event. The hadronisation process governs the transition from partons to hadrons and the underlying event represents initial state radiation, multiple parton interactions and color reconnection effects \([1]\). These phenomena lead to highly collimated sprays of particles in the final state, collectively identified as a hadron jet. The effects of both hadronisation and underlying event depend strongly on the jet radius parameter and are most pronounced at low \(p_T\). They are accounted for using phenomenological models that are tuned to the data.

The ATLAS collaboration has measured the inclusive jet cross-sections at 7 TeV \([2]\) and at 2.76 TeV \([3]\) centre-of-mass energy in pp collisions for jets defined by the anti-\(k_t\) algorithm \([4]\) with two jet radius parameters, \(R = 0.4\) and \(R = 0.6\). These measurements test perturbative QCD (pQCD) at very short distances and have provided constraints on the gluon momentum distribution within protons at high momentum fraction. Recent dijet cross-sections measurements at 7 TeV \([5]\) centre-of-mass energy in pp collisions have exploited improved jet energy calibration procedures \([6]\) leading to smaller systematic uncertainties compared to those achieved in Refs. \([2, 3]\). Measurements of the inclusive jet cross-sections ratios of anti-\(k_t\), \(R = 0.5\) and \(R = 0.7\) jets at 7 TeV centre-of-mass energy in pp collisions have been carried out by the CMS Collaboration \([7]\). Previous measurements of three-jet mass cross-sections in pp collisions were performed by D0 \([8]\). The measurements were compared to predictions showing agreement between data and theory within uncertainties.

The analysis presented in this note tests the description of multi-jet events in next-to-leading order (NLO) QCD and uses two different values of jet radius parameter, \(R = 0.4\) and \(R = 0.6\), since three-jet cross-sections depend on the jet radius already in the leading-order (LO) of perturbative expansion. NLO QCD calculations corrected to account for non-perturbative effects are compared to the measured cross-sections. These measurements also provide constraints on the proton PDFs beyond those from inclusive and dijet cross-sections, since they probe a different region of phase space in momentum fraction and squared momentum transfer (\(x, Q^2\)). In this note, measurements of double-differential three-jet production cross-sections are presented as a function of the three-jet mass (\(m_{jjj}\)) and the sum of absolute rapidity separation between three leading jets (\(|Y^*|\)). The measurements are corrected for experimental effects and reported at the particle level. The three-jet mass distributions test the dynamics of the underlying 2 \(\rightarrow\) 3 scattering process. The distributions are sensitive to both the \(p_T\) spectra of the three leading jets and their angular correlations, since a massive three-jet system can be built either from high-\(p_T\) jets or from jets with large rapidity separation. Binning in \(|Y^*|\) allows events with \(m_{jjj}\) originating from these different regions of phase space to be separated.

The content of this note is structured as follows. The ATLAS detector is briefly described in Section 2, followed by the definition of observables and description of Monte Carlo (MC) samples in Sections 3 and 4, respectively. The trigger, data selection and jet calibration are presented in Section 5. Data unfolding and experimental uncertainties are described in Sections 6 and 7. Section 8 describes the theoretical predictions for the measurements in this note. The cross-section results are presented in Section 9. The conclusions are given in Section 10.
2 The ATLAS experiment

The ATLAS detector is described in detail in Ref. [9]. ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the z-axis pointing along the beam axis. The x-axis points from the IP to the centre of the LHC ring, and the y-axis points upward. Cylindrical coordinates \((r, \phi)\) are used in the transverse plane, \(\phi\) being the azimuthal angle around the beam axis, referred to the x-axis. The transverse momentum \(p_T\) is defined as the component of the momentum transverse to the beam axis.

The inner detector (ID) is used to measure the momenta and trajectories of charged particles. The ID has full coverage in the azimuthal angle \(\phi\) and over the pseudorapidity range \(|\eta| < 2.5\). The ID is immersed in a 2 T magnetic field provided by a solenoid magnet.

The main detector system used for this analysis is the calorimeter. The electromagnetic calorimeters use liquid argon (LAr) as the active detector medium. They consist of accordion-shaped electrodes and lead absorbers, and are divided into one barrel \((|\eta| < 1.475)\) and two end-cap components \((1.375 < |\eta| < 3.2)\). The technology used for the hadronic calorimeters varies with \(\eta\). In the barrel region \((|\eta| < 1.7)\), the detector is made of scintillator tiles with steel absorbers. In the end-cap region \((1.5 < |\eta| < 3.2)\), the detector uses LAr and copper. A forward calorimeter consisting of LAr and tungsten/copper absorbers has both electromagnetic and hadronic sections, and extends the coverage to \(|\eta| < 4.9\).

The muon spectrometer has one barrel and two end-cap air-core toroid magnets. Three layers of precision tracking stations provide muon momentum measurements over the range \(|\eta| < 2.7\).

The ATLAS trigger system consists of three levels of event selection: a first level implemented using custom-made electronics that selects events at a design rate of at most 75 kHz, followed by two successive software-based levels. The level-2 trigger uses fast online algorithms, and the final trigger stage uses reconstruction software with algorithms similar to the offline versions.

3 Cross-section definition

Jets are defined using the anti-\(k_t\) algorithm as implemented in the FastJet [10] package, with two different values of the radius parameter: \(R = 0.4\) and \(R = 0.6\).

Events containing at least three jets within the rapidity range \(|y| < 3.0\) with \(p_T > 50\) GeV are considered. The leading, subleading and sub-subleading jets are required to have \(p_T > 150\) GeV, \(p_T > 100\) GeV and \(p_T > 50\) GeV, respectively. The requirements on \(p_T\) for the three leading jets are asymmetric to improve the stability of the NLO QCD calculations [11].

Three-jet double-differential cross-sections are measured as a function of the three-jet mass

\[
m_{jjj} = \sqrt{(p_1 + p_2 + p_3)^2}
\]

and the summed absolute rapidity separation of the three leading jets

\[
|Y^*| = |y_1 - y_2| + |y_2 - y_3| + |y_1 - y_3|,
\]

where \(p_i(y_i)\) are the four-momenta (rapidities) of the three leading jets. The measurements are made in five ranges of \(|Y^*| < 10.0\), in equal steps of 2.0. In each range of \(|Y^*|\), a lower limit on the three-jet mass is imposed to avoid the region of phase space affected by the jet \(p_T\) cuts. The measurement starts at \(m_{jjj} = 380\) GeV in the \(|Y^*| < 2\) bin, increasing to 1180 GeV for the \(8 < |Y^*| < 10\) bin.

The three-jet mass distributions are corrected for detector effects, and the measured cross-sections are defined at the particle level. Here particle level refers to jets built using generated particles with a proper lifetime longer than 10 ps, including muons and neutrinos from decaying hadrons [12].
4 Monte Carlo samples

The default MC generator used to simulate events is Pythia 6 [13] with the Perugia 2011 tune [14]. It is a LO generator with $2 \rightarrow 2$ matrix element calculations, supplemented by leading-logarithmic parton showers ordered in $p_T$. A simulation of the underlying event, including multiple parton interactions, is also included. The Lund string model [15, 16] is used to simulate the hadronisation process. The signal reconstruction is affected by multiple proton-proton interactions occurring during the same bunch crossing and by remnants of electronic signals from previous bunch crossings in the detectors (pileup). To simulate pileup, inelastic pp events are generated using Pythia 8 [17] with the 4C tune [18] and MRST LO** proton PDF set [19]. The number of minimum bias events overlaid on each signal event is chosen to model the distribution of the average number of simultaneous pp collisions $\langle \mu \rangle$ in an event. Throughout the 2011 data-taking period $\langle \mu \rangle$ changed from 5 to 18 with increasing instantaneous luminosity.

To estimate the uncertainties on the hard scattering, hadronisation, underlying event and parton-shower modelling, events are also simulated using ALPGEN [20], a multileg LO MC, with up to six final-state partons in the matrix element calculations, interfaced to Herwig 6.5.10 [21, 22, 23] using the AUET2 tune [24] with the CTEQ6L PDF set [25] for parton showers and Jimmy 4.31 [26] for the underlying event.

The outputs from these event generators are passed to the detector simulation [27], based on Geant4 [28]. Simulated events are digitized [29] to model the detector responses, and then reconstructed using the same software as used to process the data.

5 Data selection and jet calibration

This analysis is based on data collected with the ATLAS detector in the year 2011 during periods with stable pp collisions at $\sqrt{s} = 7$ TeV in which all relevant detector components were operational. The resulting data sample corresponds to an integrated luminosity of $4.51 \pm 0.08 \text{ fb}^{-1}$ [30].

The presence of at least one primary vertex, reconstructed using two or more tracks with $p_T > 500$ MeV, is required to reject cosmic ray events and beam-related backgrounds. The primary vertex with the largest sum of squared transverse momenta of associated tracks is used as a reference point for the analysis.

Due to the high instantaneous luminosity and a limited detector readout bandwidth, a set of single-jet triggers with increasing $p_T$ thresholds is used to collect data events with jets. Only a fraction of the events that fired the trigger are actually recorded. The reciprocal of this fraction is the prescale factor of the trigger considered. The triggers with lower $p_T$ thresholds were prescaled with higher factors and only the trigger with the highest $p_T$ threshold remained unprescaled during the whole data-taking period. The prescale factors are adjusted to keep the jet yield approximately flat as a function of $p_T$.

An event must pass all three levels of the jet trigger system. The trigger is based on the transverse energy ($E_T$) of jet-like objects. Level-1 provides a fast hardware decision based on the summed $E_T$ of calorimeter towers using a sliding-window algorithm. Level-2 performs a simple jet reconstruction in a geometric region close to that which fired the Level-1 trigger. Finally, a full jet reconstruction using the anti-$k_T$ algorithm with $R = 0.4$ is performed over the entire detector by the third trigger level, called the Event Filter (EF).

The trigger efficiencies are determined as a function of $m_{jjj}$ in each bin of $|Y^*|$ separately for $R = 0.4$ and $R = 0.6$ jet radius parameters. They are evaluated using an unbiased sample of events that fired the jet trigger with a $p_T = 30$ GeV threshold at the EF level. This trigger is fully efficient in events with a leading jet passing the three-jet analysis requirements. For every $|Y^*|$ bin, the full range of three-jet mass is divided into subranges, each filled with only one trigger. Triggers are used only where the trigger
efficiency is above 99%. Moreover, the lower $m_{jjj}$ bound for each trigger was shifted up by 15% from the 99% efficiency point to avoid any possible biases from the trigger strategy chosen for this measurement. This shift leads to a negligible, compared to the total uncertainty, increase of the statistical error on the measured cross-sections.

Since the EF reconstructs jets with a radius parameter $R = 0.4$, the $p_T$ threshold at which the trigger for jets defined with $R = 0.6$ becomes fully efficient is significantly higher than that for $R = 0.4$ jets. Using the same trigger subranges for both jet sizes would reduce the number of events with anti-$k_t$, $R = 0.4$ jets. To take advantage of the lower $p_T$ at which triggers are fully efficient for $R = 0.4$ jets, different assignments between triggers and $m_{jjj}$ ranges are considered for these jets and jets reconstructed with $R = 0.6$.

After events are selected by the trigger system, they are fully reconstructed offline. The input objects to the jet algorithm are three-dimensional “topological” clusters [31]. Each cluster is constructed from a seed calorimeter cell with energy $|E_{cell}| > 4\sigma$, where $\sigma$ is the width of the total noise distribution of the cell from both electronics and pileup sources. Neighboring cells are added to the topocluster if they have $|E_{cell}| > 2\sigma$. At the last step, all neighboring cells are added. A local hadron calibration (LC) that accounts for dead material, out-of-cluster losses for pions, and calorimeter response is applied to clusters identified as hadronic by their energy density distribution [32]. The LC improves the topocluster energy resolution and the jet clustering algorithm propagates this improvement to the jet level.

Each topocluster is considered as a massless particle with an energy $E = \sum E_{cell}$, and a direction given by the energy-weighted barycenter of the cells in the cluster with respect to the geometrical centre of the ATLAS detector. The four-momentum of an uncalibrated jet is defined as the sum of the four-momenta of the clusters making up the jet. The jet is then calibrated in four steps:

1. Additional energy due to pileup is subtracted using a correction derived from MC simulation and validated in-situ as a function of the average number of pp collisions in the same bunch crossing, $\langle \mu \rangle$, the number of primary vertices, $N_{PV}$, and jet $\eta$ [33].

2. The direction of the jet is corrected such that the jet originates from the selected hard-scatter vertex of the event instead of the geometrical centre of ATLAS.

3. The energy and the position of the jet are corrected for instrumental effects (calorimeter non-compensation, additional dead material, effects due to the magnetic field) using MC simulation. The jet energy scale is restored on average to that of the particle-level jet. For the calibration, the particle-level jet does not include muons and non-interacting particles.

4. An additional in-situ calibration is applied to correct for residual differences between MC simulation and data, derived by combining the results of dijet, $\gamma$-jet, $Z$-jet, and multijet momentum balance techniques.

The full calibration procedure is described in detail in Ref. [6].

Data-taking in the year 2011 was affected by a read-out problem in a region of the LAr calorimeter, causing jets in this region to be poorly reconstructed. In order to avoid a bias in the spectra, events with any of the three leading jets falling in the region $-0.88 < \phi < -0.5$ were rejected. Approximately 15% of events are removed by this requirement. This inefficiency is corrected for using MC simulation (c.f. Section 6).

The three leading jets are required to pass the “medium” quality criteria as described in Ref. [34], designed to reject cosmic rays, beam halo, and detector noise. More than $5.3(2.5) \times 10^3$ three-jet events are selected when jets with radius parameter $R = 0.4(0.6)$ are considered.
6 Data unfolding

The three-jet cross-sections as a function of $m_{jjj}$ are obtained by unfolding the data distributions, correcting for detector resolutions and inefficiencies. This procedure includes a correction for the presence of muons and neutrinos from hadron decays that cannot be measured in calorimeter jets. The unfolding procedure is based on the iterative, dynamically stabilised (IDS) unfolding method [35] and it is further detailed in Ref. [2]. To account for bin-to-bin migrations, a transfer matrix is built from the MC simulation, relating the particle-level and reconstruction-level three-jet masses. The reconstruction-level to particle-level event association is done in the $m_{jjj}$–$|Y^*|$ plane, such that only a requirement on the presence of a three-jet system is made. Since bin-to-bin migrations predominantly occur due to jet energy resolution smearing of the three-jet mass, and less frequently due to jet angular resolution, the migrations across $|Y^*|$ bins are negligible and the unfolding is performed separately in each $|Y^*|$ bin.

The data are unfolded to the particle level using a three-step procedure

$$N_i^P = \frac{1}{\epsilon_i^P} \sum_j N_j^R \cdot \epsilon_j^R A_{ij},$$

where $i (j)$ is the particle-level (reconstruction-level) bin index, and $N_i^P$ ($N_i^R$) is the number of particle-level (reconstruction-level) events in bin $i$. The quantities $\epsilon_i^R$ ($\epsilon_i^P$) are the fractions of reconstruction-level (particle-level) events matched (associated) to particle-level (reconstruction-level) events in each bin $i$. These efficiencies are used to correct for matching inefficiency at the reconstruction- and particle-level, respectively. The element $A_{ij}$ of the transfer matrix provides the probability for a reconstruction-level event in bin $j$ to be associated with a particle-level event in bin $i$. It is used to unfold the reconstruction-level spectrum for detector effects.

A data-driven closure test is used to evaluate the bias of the unfolded data spectrum shape due to mis-modelling of the reconstruction-level spectrum shape in the MC simulation. The transfer matrix is improved through a series of iterations, where the particle-level distribution from simulation is re-weighted such that the reconstruction-level distribution from simulation matches data distribution. The modified reconstruction-level MC simulation is unfolded using the original transfer matrix, and the result is compared with the modified particle-level spectrum. The resulting bias is considered as a systematic uncertainty. For the analyses in this note, the number of iterations is stopped at one, which leads to a bias in closure tests of less than one percent.

The statistical uncertainties on the unfolded results are estimated using pseudo-experiments. Each event in the data and in the MC simulation is counted $n$ times, where $n$ is sampled from a Poisson distribution with a mean of one. A fluctuated transfer matrix and efficiency corrections are calculated as the average over these pseudo-experiments in MC simulation. Then, each resulting pseudo-experiment of the data spectrum is unfolded using the fluctuated transfer matrix and efficiency corrections. Finally, the covariance matrix between bins of measured $m_{jjj}$ cross-section is calculated using the set of unfolded pseudo-experiments of the data. The random numbers for the pseudo-experiments are generated using unique seeds. The dijet [5] and inclusive jet cross-section measurements use the same unique seeds to evaluate the statistical uncertainties. In this way, the statistical uncertainty and bin-to-bin correlations for both the data and the MC simulation are encoded in the covariance matrix and the statistical correlation between different measurements can be taken into account in combined fits.

7 Experimental uncertainties

The uncertainty on the jet energy scale (JES) calibration is the dominant uncertainty for this measurement. The uncertainties in the central region are determined using a combination of the transverse momentum balance techniques, such as Z-jet, $\gamma$-jet and multijet balance measurements performed in-situ.
In each of the methods the uncertainties on the energy of the well-measured objects, e.g. electron/photon or low-\(p_T\) jets, are propagated to the energy of the balancing jet. The JES uncertainty in the central region is propagated to the forward region using transverse momentum balance between a central and a forward jet in events with two jets. The difference in the balance observed between MC simulation samples generated with \textsc{Pythia} and \textsc{Herwig} is treated as an additional uncertainty in the forward region. Complete details of the JES derivation can be found in Ref. [6].

The uncertainty due to the JES calibration on each individual jet is between 1 and 4% in the central region (\(|\eta| < 1.8\)), and it increases up to 5% in the forward region (1.8 < |\(\eta\)| < 4.5).

The uncertainties due to the JES calibration are propagated to the measured cross-sections using the MC simulation. Energy and \(p_T\) of each jet in the three-jet sample is scaled up or down by one standard deviation of a given uncertainty component, after which the luminosity-normalised three-jet event yield is measured from the resulting sample. The yields from the nominal sample and the samples where all jets were scaled up and down are unfolded, and the difference between each of these variations and the nominal result is taken as the uncertainty due to that JES uncertainty nuisance parameter. Since the sources of JES calibration uncertainty are taken as uncorrelated with each other, the corresponding uncertainty components on the cross-section are also taken as uncorrelated.

Each jet is affected by the additional energy deposited in the calorimeters due to pileup effects. Additional energies due to pileup are subtracted during the jet energy calibration procedure [6]. To check any residual pileup effects in the measured cross-sections, the luminosity-normalised three-jet yields in all three-jet mass and rapidity separation bins are split into bins of different pileup conditions under which the data were collected. No statistically significant deviation from the nominal result is observed.

The jet energy resolution (JER) is measured in the data using the bisector method in dijet events [36], where good agreement with the MC simulation is observed. The uncertainty on the JER is determined by the selection parameters for jets, such as the amount of nearby jet activity, and depends on both jet \(p_T\) and jet \(\eta\).

Jet angular resolution (JAR) is studied by matching particle-level jets to reconstruction-level jets in simulation. Jets are matched by requiring that the angular distance \(\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}\) between the particle-level and reconstruction-level is less than the jet radius parameter. The angular resolution is obtained from a Gaussian fit to the distribution of the difference of reconstruction-level and particle-level jet rapidity.

The difference between the JAR determined from the nominal MC simulation and that from the ALPGEN sample is taken as a systematic uncertainty. The resolution varies between 0.005 radians and 0.03 radians depending on the jet \(\eta\) and \(p_T\) values. The JAR uncertainty is about 10 – 15% for \(p_T < 150\) GeV and decreases to ~ 1% for \(p_T > 400\) GeV. The jet angular bias is found to be negligible.

The JER and JAR uncertainties are propagated to the measured cross-section through the unfolding transfer matrix. The energy and angle of all jets in the MC sample is smeared according to its uncertainty. To avoid being limited by statistical fluctuations this procedure is repeated 1000 times in each event. The average transfer matrix derived from these pseudo-experiments is used to unfold the three-jet yields, and the deviation from the three-jet yield unfolded using the nominal transfer matrix is taken as a systematic uncertainty.

The uncertainty due to the jet reconstruction inefficiency as a function of jet \(p_T\) is estimated by comparing the efficiency for reconstructing a calorimeter jet, given the presence of an independently measured track-jet of the same radius, in data and in MC simulation [37, 6]. Here, a track-jet refers to a jet reconstructed using the anti-\(k_t\) algorithm taking as input all tracks with \(p_T > 500\) MeV and \(|\eta| < 2.5\) in the event assuming they have the mass of a pion. Since this method relies on tracking, its application is restricted to the acceptance of the tracker for jets of \(|\eta| < 1.9\) to have the full jet contained in the tracker acceptance for both \(R = 0.4\) and \(R = 0.6\) jets. For jets with \(p_T > 50\) GeV, relevant for this analysis, the
Figure 1: Total systematic uncertainty on the three-jet cross-section as a function of $m_{jjj}$ for $R = 0.6$ jets. The orange/yellow/green band shows the uncertainty due to jet energy scale/jet energy resolution/jet angular resolution. The cyan band denotes the combined uncertainty due to jet quality selection and unfolding. The violet band represents the total experimental uncertainty.

reconstruction efficiency in both the data and the MC simulation is found to be 100% for this rapidity region, leading to no additional uncertainty. The same efficiency is assumed for the forward region, where jets of a given $p_T$ are more energetic and, therefore, their reconstruction efficiency is expected to be at least as good as that of jets in the central region.

Comparing the single jet quality selection efficiency for jets passing the “medium” quality criteria in data and MC simulation, agreement of the efficiency within 0.25% is found [34]. Because three jets are considered for each event selected for the analysis, a 0.75% systematic uncertainty on the cross-section is assigned.

The impact of a possible mis-modelling of the shape of $m_{jjj}$ spectra in MC simulation, introduced through the unfolding as described in Section 6, is also included. The luminosity uncertainty is 1.8% [30] and is fully correlated between all data points.

The total experimental uncertainty on the three-jet cross-section is summarised in Fig. 1. The total uncertainty ranges from 8 − 10% at low three-jet mass up to 28% at high three-jet mass for the range $|Y^*| < 6$, and slightly increases for larger $|Y^*|$ bins. In the $8 < |Y^*| < 10$ bin the total uncertainty changes from 18% to 38%, where it is dominated by the jet energy scale uncertainty component for forward jets.

8 Theoretical predictions and uncertainties

The NLO QCD predictions by NLOJET++ [38], corrected for hadronisation effects and the underlying event activity using Monte Carlo simulations with Pythia 6 with Perugia 2011 tune [14] are compared to the measured three-jet cross-sections.

8.1 Fixed order predictions

The fixed-order QCD calculations are performed with the NLOJET++ program interfaced to APPLGRID [39] for fast convolution with various PDF sets. The renormalisation ($Q_R$) and factorisation ($Q_F$) scales are set to the mass of the three-jet system, $Q = Q_R = Q_F = m_{jjj}$. The following proton PDF sets
are considered for the theoretical predictions: CT 10 [40], HERAPDF 1.5 [41], MSTW 2008 [42], NNPDF 2.3 [43], GJR 08 [44] and ABM 11 [45].

To estimate the uncertainty due to missing higher-order terms in the fixed-order perturbative expansion, the renormalisation scale is varied up and down by a factor of two. The uncertainty due to the dependence of the theoretical predictions on the factorisation scale, which specifies the separation between the short-distance hard scattering and long-distance non-perturbative dynamics, is estimated by varying the factorisation scale up and down by a factor of two. All permutations of these two scale choices are considered, except the cases where both scales are shifted in opposite directions. The maximal deviations from the nominal prediction are taken as the scale uncertainty. The scale uncertainty is generally within $10 - 20\%$ and increases up to $40\%$ for $m_{jjj}$ above $3 - 4\text{ TeV}$.

The multiple uncorrelated uncertainty components of each PDF set, as provided by the various PDF groups, are also propagated through the theoretical calculations. The PDF groups generally derive these from the experimental uncertainties on the data used in the fits. For the results shown in Section 9, the standard Hessian sum in quadrature [46] of the various independent components is calculated taking into account asymmetries of the uncertainty components. The NNPDF 2.3 PDF set is exception, where uncertainties are expressed in terms of replicas instead of by independent components. These replicas represent a collection of equally likely PDF sets, where the data used in the PDF fit were fluctuated within their experimental uncertainties. For the plots shown in Section 9, the uncertainties on the NNPDF 2.3 PDF set are evaluated as the RMS of the replicas in each observable bin, producing equivalent PDF uncertainties on the theoretical predictions. These uncertainties are symmetric by construction. Where needed, the uncertainties of PDF sets are rescaled to the 68% confidence level (CL). HERAPDF provide three types of uncertainties: experimental, model and parametrisation. The three uncertainty sources were added in quadrature to get a total PDF uncertainty.

The uncertainties on the cross-sections due to the strong coupling, $\alpha_S$, are estimated using two additional proton PDF sets, for which different values of $\alpha_S$ are assumed in the fits, such that the effect of the strong coupling value on the PDFs is included. This follows Ref. [47]. The resulting uncertainty is approximately $3\%$ across all three-jet mass and $|Y^*|$ ranges considered.

The scale uncertainties are dominant in low and intermediate three-jet mass regions, while the PDF uncertainties become dominant at high $m_{jjj}$. The uncertainties on the theoretical predictions due to those on the PDFs range from $5\%$ at low $m_{jjj}$ up to $20\%$ at high three-jet mass for the range of $|Y^*|$ values up to 4. For the values of $|Y^*|$ between 4 and 10, the PDF uncertainties reach $40 - 70\%$ at high three-jet mass, depending on the PDF set.

### 8.2 Non-perturbative effects

Non-perturbative corrections (NPC) are evaluated using leading-logarithmic parton-shower generators, separately for each value of the jet radius parameter. The corrections are calculated as bin-by-bin ratios of the three-jet differential cross-section at the particle level, including hadronisation and underlying event effects, over that at the parton level after the parton shower (before the hadronisation process starts) with the underlying event simulation switched off. The nominal corrections are calculated using Pythia 6 with the Perugia 2011 tune. The non-perturbative corrections as a function of three-jet mass are shown in Fig. 2 for the range $|Y^*| < 2$. The NPC are smaller than $10\%$ in all $m_{jjj}$ bins.

The uncertainties on the non-perturbative corrections, arising from the modelling of the hadronisation process and the underlying event, are estimated as the maximal deviations of the corrections from the nominal using the following configurations: Pythia 8 with the 4C tune [18] and with AU2 [24] tune using the CTEQ6L1 PDF set [25]; Pythia 6 with AUET2B [48] tune with CTEQ6L1; Herwig++ 2.6.3 [49, 50] with the UE-EE-3 tune [51] using CTEQ6L1 set. The uncertainty on the non-perturbative corrections varies up to $\sim 10\%$ depending on the three-jet mass in all $|Y^*|$ bins. The NPC correction for $|Y^*| < 2$ bin is shown in Fig. 2.
Figure 2: Non-perturbative corrections obtained using various MC generators and tunes are shown for the differential three-jet cross-section as a function of three-jet mass in the range \(|Y^*| < 2\) with values of jet radius parameter \(R = 0.4\) (a) and \(R = 0.6\) (b). The non-perturbative correction uncertainty is taken as the envelope of the various tunes.

The total theoretical uncertainty is calculated as a sum in quadrature of PDF, scale, \(\alpha_s\) and NPC uncertainties.

9 Cross-section results

Measurements of the double-differential three-jet cross-sections as a function of the three-jet mass in various ranges of \(|Y^*|\) are shown in Figs. 3 and 4 for anti-\(k_t\) jets with values of the radius parameter \(R = 0.4\) and \(R = 0.6\), respectively. The cross-section decreases rapidly as a function of the three-jet mass. The NLO QCD calculations by NLOJET++ using the CT10 PDF set corrected for non-perturbative effects are compared to the measured cross-sections. Good agreement between the data and the theoretical predictions is found over the full kinematic range, covering almost seven orders of magnitude in the measured cross-section values.

The ratios of the theoretical predictions calculated with different PDF to the measured cross-sections with \(R = 0.4\) jets are presented in Figs. 5 and 7 and in Figs. 6 and 8 for \(R = 0.6\) jets. Theoretical calculations based on CT 10, MSTW 2008 and HERAPDF 1.5 are compared to data in Figs. 5 and 6 and comparisons to other global PDFs, namely GJR 08, ABM 11 and NNPDF 2.3 are presented in Figs. 7 and 8.

Three-jet cross-sections are well described by the calculations based on CT 10, NNPDF 2.3, GJR 08, MSTW and HERAPDF 1.5 PDFs. A tension between data and ABM 11 PDFs is observed for most of the cross-sections measured with both jet radius parameters.

For all PDF sets, except ABM 11, the predictions for \(R = 0.4\) jets agree well with measured cross-sections, while for \(R = 0.6\) jets theory underestimates data by approximately 15% across the full \(m_{jjj}-|Y^*|\) plane. The jet radius dependence of theory-to-data ratios is similar for all PDF sets considered, demonstrating that this disagreement is independent of the assumptions made in different PDF determinations.
Figure 3: Three-jet double-differential cross-section as a function of $m_{jjj}$, binned in $|Y^*|$. The results are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.4$. For convenience, the cross-sections are multiplied by the factors indicated in the legend. The data are compared to the NLOJET++ prediction with the CT10 PDF set corrected for non-perturbative effects. The experimental error bars, although hardly visible, are included on the data points and indicate the statistical-only uncertainty on the measurement and the total experimental uncertainty (quadratic sum of the statistical and experimental systematic uncertainties).
Figure 4: Three-jet double-differential cross-section as a function of $m_{jjj}$, binned in $|Y^*|$. The results are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.6$. For convenience, the cross-sections are multiplied by the factors indicated in the legend. The data are compared to the NLOJET++ prediction with the CT10 PDF set corrected for non-perturbative effects. The experimental error bars, although hardly visible, are included on the data points and indicate the statistical-only uncertainty on the measurement and the total experimental uncertainty (quadratic sum of the statistical and experimental systematic uncertainties).
Figure 5: The ratio of NLO QCD predictions, obtained by using NLOJET++ with different PDF sets (CT 10, MSTW 2008, HERAPDF 1.5) and corrected for non-perturbative effects, to data as a function of $m_{jjj}$, binned in $|Y^*|$. The ratios are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.4$. Data points are always at 1. The thick line shows the central values and thin lines represent the total theory uncertainty. The experimental error bands designate the relative statistical-only and total (statistical and systematic added in quadrature) experimental uncertainties.
Figure 6: The ratio of NLO QCD predictions, obtained by using NLOJET++ with different PDF sets (CT 10, MSTW 2008, HERAPDF 1.5) and corrected for non-perturbative effects, to data as a function of $m_{jjj}$, binned in $|Y^*|$. The ratios are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.6$. Data points are always at 1. The thick line shows the central values and thin lines represent the total theory uncertainty. The experimental error bands designate the relative statistical-only and total (statistical and systematic added in quadrature) experimental uncertainties.
Figure 7: The ratio of NLO QCD predictions, obtained by using NLOJET++ with different PDF sets (GJR 08, NNPDF 2.3, ABM 11) and corrected for non-perturbative effects, to data as a function of $m_{jjj}$, binned in $|Y^*|$. The ratios are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.4$. Data points are always at 1. The thick line shows the central values and thin lines represent the total theory uncertainty. The experimental error bands designate the relative statistical-only and total (statistical and systematic added in quadrature) experimental uncertainties.
Figure 8: The ratio of NLO QCD predictions, obtained by using NLOJET++ with different PDF sets (GJR 08, NNPDF 2.3, ABM 11) and corrected for non-perturbative effects, to data as a function of $m_{jjj}$, binned in $|Y^*|$. The ratios are shown for jets identified using the anti-$k_t$ algorithm with $R = 0.6$. Data points are always at 1. The thick line shows the central values and thin lines represent the total theory uncertainty. The experimental error bands designate the relative statistical-only and total (statistical and systematic added in quadrature) experimental uncertainties.
10 Conclusions

Cross-section measurements of the three-jet production in pp collisions at 7 TeV centre-of-mass energy as functions of the three-jet mass and the sum of absolute rapidity separation between three leading jets are presented. Jets are reconstructed with the anti-$k_t$ algorithm using two values of the radius parameter, $R = 0.4$ and $R = 0.6$. The measurements are based on the full data set collected with the ATLAS detector during 2011 data-taking at the LHC, corresponding to the integrated luminosity of 4.51 fb$^{-1}$. The measurements are corrected for detector effects and reported at the particle level. The total experimental uncertainty in these measurements is dominated by the jet energy scale calibration uncertainty. The measurement uncertainties are smaller than, or similar to, those in the theoretical predictions.

The measurements probe three-jet masses up to $\sim 5$ TeV and are well described by perturbative QCD in NLO accuracy across the full $m_{jjj}-|Y^*|$ plane. The comparison of NLO QCD predictions corrected for non-perturbative effects to the measured cross-sections is performed using several modern PDF sets. A good quality of data description is observed when using CT 10, NNPDF 2.3, HERAPDF 1.5, GJR 08 and MSTW 2008 PDFs. Tension is observed between measurements and theoretical calculations based on ABM 11 PDF.

Comparison of measured cross-sections to theoretical predictions for two different jet radius parameters shows a good agreement for $R = 0.4$ jets and shifted theory-to-data ratios for $R = 0.6$ jets. This shift has only a minor dependence on the PDF set used.

References


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