MEASUREMENT OF THE STRUCTURE FUNCTIONS $F_2$ AND $x F_3$ AND COMPARISON WITH QCD PREDICTIONS INCLUDING KINEMATICAL AND DYNAMICAL HIGHER TWIST EFFECTS

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The isoscalar nucleon structure functions $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ are measured in the range $0 < Q^2 < 64$ GeV$^2$, $1.7 < W^2 < 250$ GeV$^2$, $x < 0.7$ using $\nu$ and $\bar{\nu}$ interactions on neon in BEBC. The data are used to evaluate possible higher twist contributions and to determine their impact on the evaluation of the QCD parameter $\Lambda$. In contrast to previous analyses reaching to such low $W^2$ values, it is found that a low $\Lambda_{\overline{MS}}$ value in the neighbourhood of 100 MeV describes the data adequately and that the contribution of dynamical higher twist effects is small and negative.
1. **INTRODUCTION**

The first analyses of deep inelastic scattering data in terms of the $\log Q^2$ scaling deviations predicted by QCD yielded values of the strong interaction parameter $\Lambda$ in the range 500-700 MeV [1,2,3]. Subsequent more sophisticated analyses taking into account second order corrections and the effect of the $W$-propagator gave values of $\Lambda_{\overline{MS}}$ around 300-400 MeV [4,5,6]. It came as a surprise when the higher Q$^2$ muon scattering experiments produced $\Lambda$ values around 100 MeV, for both second order and leading order fits [7,8,9].

However, the early data had been analysed down to very low $Q^2$ and $W^2$; for example, the BEBC+GGM analysis [1,4] included elastic events. The theory of QCD predicts that leading twist operators giving rise to scaling deviations of the form $1/\ln(Q^2/\Lambda^2)$ are dominant at high $Q^2$, but that at low $Q^2$ there are sub-dominant higher twist operators which give rise to stronger scaling violations of the form $1/Q^2$, $1/Q^4$, etc (dynamical higher twist effects). Furthermore the theory, as usually applied in terms of the Bjorken scaling variable $x$, is strictly correct only for zero target mass. Finite target mass corrections also induce $1/Q^2$ scaling violations (kinematic higher twist effects) which are important at low $W^2$ [6]. There was also some debate as to the applicability of the theory at low $W^2$ because of resonance effects in the hadron final state [10,11].

Thus it seemed that the higher $\Lambda$ values obtained in the earlier analyses were likely to be a reflection of the positive contribution of additional scaling deviations at low $Q^2$. Most subsequent analyses of deep inelastic scattering data have therefore restricted fitting for $\Lambda$ to $Q^2 \gtrsim 4$ GeV$^2$, $W^2 \gtrsim 10$ GeV$^2$ in order to minimise such effects [12-16].

However, the theoretical situation is not so clear cut. Whereas kinematic higher twist effects can be accounted for exactly, dynamical higher twist calculations involve estimating hadron matrix elements non perturbatively. Despite this difficulty, extensive work has been done in recent years which suggests that higher twist effects are likely to make a negative contribution to the structure function at low $Q^2$ and low $W^2$ [17,18,19], thus contradicting the simple picture given above.

There have been a few attempts to investigate higher twist contributions
experimentally, but these have mostly involved combining the high $Q^2$ data of one experiment with the low $Q^2$ data of another [4,8,20,21]. The early analyses of BEBC+GGM data [4] and CDHS+SLAC data [20] combined data from different targets; it is now evident that such analyses may reveal the EMC effect [22] rather than the effects of higher twist operators. It is also interesting to note that higher twist effects in electron and neutrino induced processes need not be the same [17,23]. The analyses of EMC+SLAC data [8,21] do not have these problems; but small systematic differences between the experiments may nevertheless seriously bias the results.

This paper presents and analyses data on $F_2$ and $xF_3$ from a single experiment covering the range $1.7 < W^2 < 250$ GeV$^2$, $0 < Q^2 < 64$ GeV$^2$, $x < 0.7$. The experiment has four times the statistics of previous comparable data [24], and hence we can set stringent limits on the size and sign of the higher twist contribution.

2. EXPERIMENTAL DETAILS

The data were taken by exposing the Big European Bubble Chamber (BEBC) to the CERN-SPS wide band neutrino and antineutrino beams. BEBC was filled with a heavy neon-hydrogen mixture (75 mole-% neon, density 0.704 gm/cc, radiation length 42 cm) to give good detection efficiency for neutral secondaries. Muons were identified using the two-plane external muon identifier (EMI) [25]. The neutrino and antineutrino beams were produced by using the high energy version of the 2-horn focusing system to direct towards BEBC the charged secondaries from interactions of 400 GeV protons in a beryllium target. A large array of solid state detectors in the shielding in front of BEBC, the neutrino flux monitoring (NFM) system [26], continuously monitored the muon beam produced by the decay of these secondaries in the 400 metre decay path available; it thus indirectly monitored the neutrino/antineutrino beam which is produced by the same decays.

The film from BEBC was scanned twice in the normal fashion. The antineutrino film was then scanned two further times by following all leaving tracks back to where they either originated in an interaction or entered the chamber; this enabled 1-prong and 2-prong charged current interactions to be detected efficiently. Finally, 10% of the pictures distributed throughout the film were scanned painstakingly two more times in a restricted fiducial volume.
where visibility was particularly good; this final scan was highly efficient and was used to determine the overall efficiency of the previous scans as a function of multiplicity. The results were extended outside the restricted volume by assuming constancy of the true multiplicity distributions at given neutrino or antineutrino energy.

For every charged current interaction candidate found inside the 16.4 m³ fiducial volume, every charged secondary was measured (apart from stopping nuclear fragments below 4 cm in length). If the secondary interacted before its momentum could be determined to better than ±30%, its interaction products were also measured in order to establish a minimum value for its momentum; this minimum value was then used in all subsequent calculations. All converted electron pairs and Compton electrons within 80 cm of the main vertex were also measured, together with any outside this distance whose energy exceeded 1 GeV; those due to bremsstrahlung photons were discarded by considering their momentum and emission angle with respect to the parent electron. Finally, every secondary neutral hadron interaction that could be reliably associated with the event was fully measured.

All charged secondaries were assumed to be pions unless otherwise identified. Any that left the visible volume without visibly kinking or interacting were identified as muons if they could be associated simultaneously with hits in both the inner and outer planes of the EMI occurring in the same time slot. In events with two identified muons, the one with the higher transverse momentum with respect to the beam direction was taken to be the one from the lepton vertex. Only events with an identified muon from the lepton vertex of appropriate sign (μ⁻ in neutrino film, μ⁺ in antineutrino film) and of momentum above 5 GeV/c were kept in the data sample. There were 9517 such events in the neutrino film and 16497 in the antineutrino film. After further cuts on the beam conditions and sample scan efficiencies to ensure the uniformity and high quality of the data sample used, 8141 and 13334 of these events respectively, were retained for the present analysis.

Backgrounds to the muon sample arise either from π⁺ or K⁺ punching through the iron of the EMI or decaying in flight, or from accidental correlations between leaving hadron tracks and unassociated EMI hits. The former background was evaluated from the number of events appearing to contain two muons of the same sign in the same time slot, the latter from the number containing two in different time slots. The accidental background was also (and more
precisely) determined by systematically trying to associate leaving hadron tracks with EMI hits registered for other photographs. Corrections were made for both backgrounds, but were found to be very small.

Losses from the muon sample are due (a) to the finite sizes of the EMI planes, (b) to dead wires in them. The former loss was evaluated by Monte Carlo calculations. The dead wires were identified by inspection of cosmic ray data taken throughout the run. The resulting losses were compensated by giving appropriate extra weight to events whose muons would have been lost if particular wires near to the dead ones had also been dead. Additionally there is a uniform 1-2% loss due to electronic inefficiency that is independent of momentum and emission angle.

The energy distributions of the neutrino and antineutrino fluxes were deduced from the events themselves, after correcting for smearing (see Section 4), assuming the values of $\sigma/E$ to be constant in our energy range (approximately 10 to 200 GeV). The effect of a $\pm 5\%$ variation of $\sigma/E$ across this range is negligible [27]. These energy distributions were consistent with those determined from using $\pi^+$ and $K^+$ production data in Beryllium [28], detailed Monte Carlo simulation of the wide band beam focusing system, and the muon fluxes in the shielding measured using the NPM system. The fluxes were normalised using the recently revised $\sigma/E$ values for neutrino and antineutrino charged current interactions in neon [29] determined using the narrow band beam.

3. DETERMINATION OF EVENT VARIABLES

In general, the muon was measured rather precisely (relative momentum error $\sim 1\%$, error in angle $\sim 1$ mrad). The neutrino/antineutrino energy $E_\nu$ was estimated from the momenta of the observed secondaries corrected by one of the methods described below.

The other kinematical variables were then estimated from the lepton variables assuming the neutrino/antineutrino to be in the beam direction and the target to be a nucleon of mass $m$ at rest

$$Q^2 = 2E_\nu(E_\mu - p_\mu) - m_\mu^2$$
$$\nu = E_\nu - E_\mu$$
$$x = Q^2/2m_\nu$$
$$y = \nu/E_\nu - 4$$

(1)
where $m_\mu$, $p^L_\mu$ and $E_\mu$ are the mass, longitudinal momentum and the energy of the muon respectively.

Despite the good efficiency for detecting neutral secondaries afforded by the use of a heavy neon-hydrogen mixture, on average some 25% of the momentum carried by gamma rays, and a larger fraction of that carried by neutrons or by $K_\ell^0$ mesons, remained undetected. From the ratio of the mean transverse momentum of the hadron shower in the lepton plane to that of the muon, $<p^T_{\text{had:shower}}>/<p^T_\mu>$, the average fraction of the total hadron shower momentum actually measured was determined to be $0.84 \pm 0.01$ for neutrino events and $0.81 \pm 0.01$ for antineutrino events. This 16-19% average loss in momentum is the dominant source of systematic uncertainty in our data.

To account for this loss in momentum, three alternative correction methods were employed. The first method was to correct all measured hadron shower momenta by assuming that the above average factors applied to each event individually. In the second method [30a], the fraction $(p^T_{\text{had:shower}}/p^T_\mu)$ was calculated for each event separately and applied to that event only. In the third method [30b], corrections for individual events were calculated following a more complicated prescription which also involved the transverse momenta of individual tracks.

4. UNSMEARING CORRECTION

The imprecision in determining the total hadron shower momentum in individual events distorts the measured values of $x$, $y$ and $Q^2$ from the true values such that the means of the distributions in these variables are shifted and the distributions are broadened. This distortion is different for the three different correction methods used. In each case it has been corrected ('unsmearing correction') using Monte Carlo simulations of these effects. The differences between the results obtained for the three different energy correction methods, after all corrections had been applied, were used as an estimate of part of the systematic uncertainty.

Two Monte Carlo simulations were used. The parameterisation of the structure functions was the same in both Monte Carlo simulations and is given in the Appendix. In both simulations inner bremsstrahlung photons from the muon
were generated following the prescription of de Rujula et al. [32] in order to evaluate the effect of the radiative corrections, including their effect on the energy corrections. The two simulations differed in their treatment of smearing effects.

The first simulation was a detailed one in which full events were generated and all known sources of experimental error were simulated as exactly as possible, including re-interactions inside the nucleus, close secondary interactions, loss of neutrals resulting in incomplete measurement of the hadron shower, measurement errors, particle misidentification, etc. Further details may be found in Reference 31.

In the second simulation the hadron shower was treated as a whole and the distributions of the measured momentum fraction, \( \varepsilon \), and of the transverse momentum loss, \( p_T \), were parameterised as

\[
\frac{d^2\sigma}{d\varepsilon dp_T^2} \sim (f+1)\varepsilon^f e^{-\frac{p_T^2}{0.34}}
\]

(2)

where \( \varepsilon = \frac{p_{\text{meas}}}{p_{\text{true}}} \), the fraction of the true hadron shower momentum that is actually measured, has a mean value \( <\varepsilon> \) which determines the parameter \( f \)

\[
f = (2<\varepsilon>-1)/(1-<\varepsilon>)
\]

(3)

and is given by

\[
<\varepsilon> = 0.85 - 0.51/(p_{\text{had:shower}}^L + 1) \quad \text{for } \bar{\nu}
\]

(4)

\[
= 0.85 - 0.25/(p_{\text{had:shower}}^L + 1) \quad \text{for } \nu
\]

and where \( p_T \) is the transverse momentum of the measured hadron shower relative to the true direction of the momentum transferred to the hadron system. The numerical values given were determined by fitting the transverse momentum distributions of the observed hadron showers in and out of the lepton plane.

The differences between the unsmeared corrections predicted by the two Monte Carlo simulations were used to estimate additional systematic uncertainties.
5. Extractions of the Structure Functions

The differential cross section for scattering of neutrinos or antineutrinos off an isoscalar target is given by

\[
\frac{d^2 \sigma^{\nu \bar{\nu}}}{dx dy} = \frac{G^2 m E}{\pi} \left( \frac{m_n^2}{m_W^2 + Q^2} \right)^2 \left\{ 2x F_1(x, Q^2) \frac{y^2}{2} + F_2(x, Q^2) \left( 1 - \frac{m_X}{2E} \frac{m_Y}{2} \right) \pm x F_3(x, Q^2) y \left( 1 - \frac{y}{2} \right) \right\}
\]

(5)

\[
= \frac{G^2 m E}{\pi} \left( \frac{m_n^2}{m_W^2 + Q^2} \right)^2 \left\{ F_2(x, Q^2) \left( 1 - \frac{m_X}{2E} \frac{m_Y}{2} \right) + \frac{y^2}{2} \frac{(1 + Q^2 / v^2)}{(1 + R(x, Q^2))} \pm x F_3(x, Q^2) y \left( 1 - \frac{y}{2} \right) \right\}
\]

is the ratio of absorption cross sections for longitudinal and transverse W bosons. Hence the number of events in a given bin of \(x\) and \(Q^2\) is given by

\[
n^{\nu \bar{\nu}} = I_2^{\nu \bar{\nu}}(F_2) \pm I_3^{\nu \bar{\nu}}(x F_3)
\]

(7)

in which the 'flux integrals' \(I_2\) and \(I_3\) are defined by

\[
I_2^{\nu \bar{\nu}} = \frac{G^2 m E}{\pi} \int \int \left( \frac{m_n^2}{m_W^2 + Q^2} \right)^2 \left[ 1 - \frac{m_X}{2E} \frac{m_Y}{2} \frac{(1 + Q^2 / v^2)}{(1 + R(x, Q^2))} \right] \phi^{\nu \bar{\nu}}(x) dx dy dE
\]

(8)

\[
I_3^{\nu \bar{\nu}} = \frac{G^2 m E}{\pi} \int \int \left( \frac{m_n^2}{m_W^2 + Q^2} \right)^2 y(1 - y/2) \phi^{\nu \bar{\nu}}(x) dx dy dE
\]

where \(N\) is the number of nucleons in the fiducial volume of the target and \(\phi^{\nu \bar{\nu}}(E)\) is the differential neutrino or antineutrino flux.

The numerical integrations extend over all \(x\), \(y\) and \(E\) such that \(x\) and \(Q^2\) lie within the bin and all other kinematic variables lie within the imposed cuts (e.g., the muon momentum exceeds 5 GeV/c). Correspondingly, the 'average' \(F_2\) and \(x F_3\) values in the bin can be determined from the corrected numbers of neutrino and antineutrino events, \(n^{\nu \bar{\nu}}\), in the bin:
\[ F_2 = (I_3^\nu n^\nu + I_3 n^\nu)/(I_3^\nu n^\nu + I_3 n^\nu) \]

\[ xF_3 = (I_2 n^\nu - I_2 n^\nu)/(I_2 n^\nu + I_2 n^\nu) \]

To extract values of the structure function \( F_2 \), some assumption has to be made concerning \( R(x,Q^2) \). For our central values we have used:

\[
R(x,Q^2) = R_{QPM} + \Delta R_{QCD} = \frac{4m^2x^2}{Q^2} + 0.73 \frac{1-x}{x} \frac{3.7}{\ln(Q^2/0.24^2\text{GeV}^2)}
\]

where \( R_{QPM} = Q^2/\nu^2 = 4m^2x^2/Q^2 \) is the prediction of the simple quark parton model and the parameterisation of \( \Delta R_{QCD} \), the QCD correction to \( R \) [33], is taken from Reference 13. This form for \( R(x,Q^2) \) is in reasonable agreement with both high and low \( Q^2 \) data, and with the somewhat more phenomenological parameterisations given by Bodek et al [34].

The above equations would be precisely correct in the case of perfect SU(2) symmetry, i.e. if the target were perfectly isoscalar and had equal strange and charmed quark content and if the strange and charmed quarks had equal mass; otherwise the structure functions in \( n \) and \( \bar{n} \) scattering are different. In addition the high mass of the charmed quark gives kinematic threshold effects: the resulting \( Q^2 \) dependence must be allowed for, if scaling deviations arising from QCD are to be evaluated correctly.

Following recent convention [13,35], corrections for all of these effects have been applied. The isoscalar correction assumed \( d_\nu/u_\nu = 0.57*(1-x) \) [36]; the charmed quark content of the nucleon was assumed to be zero, the strange quark content was assumed to be equal to half the \( \bar{u} \) and \( \bar{d} \) content [37]; and the kinematic suppression of \( d\to c \) and \( s\to c \) transitions was accounted for assuming slow rescaling [38] and a charmed quark mass of 1.5 GeV. The corrections were applied using the parameterisation of the structure functions given in the appendix. No correction has been made for fermi motion or nuclear effects, since these are regarded as part of the EMC effect; we choose to quote the results appropriate for the heavy neon nucleus.

The results are shown in Table 1 and Fig 1. Only points for which the systematic error is less than or comparable to the statistical error are given, unreliable data at the edges of the \( x, Q^2 \) plane are excluded. This restricts
the range of our measurement in $W^2$ to $1.7 < W^2 < 250$ GeV$^2$. The values quoted have been interpolated to apply to the $x$ and $Q^2$ values given. The systematic errors quoted have been estimated from the spreads of smooth curves fitted to the results obtained using the different energy correction and unsmearing procedures discussed above. Also shown in Table 1 is the $\langle y \rangle$ value, the effect of varying the assumption made about $R$, and the effects of applying the largest corrections to the data, namely the unsmearing correction, the slow rescaling and the radiative correction. The errors shown in Fig 1 represent the total error, systematic and statistical errors have been folded in quadrature. The curves shown in Fig 1 are discussed below.

6. ANALYSIS OF THE STRUCTURE FUNCTIONS

(i) Method

The predictions of QCD have been fitted to the measured structure functions given in Table 1 using the program of Devoto, Duke, Owens and Roberts [16]. This program solves the Altarelli-Parisi equations [39] to a very high degree of accuracy, to second order in the $\overline{MS}$ scheme [40]; it includes target mass corrections (kinematical higher twist effects), following the Georgi-Politzer formalism [41]. In order to investigate the contribution of dynamical higher twist effects at low $Q^2$ and low $W^2$, we have also incorporated a few specific predictions for the form of the higher twist terms as options in our fits.

The program requires a parameterisation of $F_2$, $xF_3$ and $G$, the gluon distribution, at a particular $Q^2 = Q_0^2$. It then uses the evolution equations to predict the values of $F_2$ and $xF_3$ at any higher $Q^2$. The following parameterisation was used:

$$
xF_3(x, Q_0^2) = A_v x^\alpha (1-x)^\beta (1+\gamma x) \\
F_2(x, Q_0^2) = xF_3(x, Q_0^2) + S(x, Q_0^2) \\
S(x, Q_0^2) = A_s (1-x)^s + B_s (1-x)^\eta \\
G(x, Q_0^2) = A_g (1-x)^\chi (1+\gamma x)
$$

(11)

with $Q_0^2$ set to 0.5 GeV$^2$ to allow fits to data down to $Q^2 = 1$ GeV$^2$ (the fits are insensitive to this choice); with $\gamma$ set to 4.0 and $\gamma$ to 0.0 giving a
hard gluon distribution in agreement with analyses of gluon fusion data [42] and of CHARM, CDHS and EMC data [43]; and with \( \gamma \) set to 0.0 and \( s_8 \) to 5.3 to correspond, at \( Q^2 = Q^2_0 \), to the parameterisation of our data given in the Appendix. The effect of changing these choices is discussed below. The value of \( A_g \) is determined by the GLS sum rule at second order [44]; the value of \( A_g \) is given by the momentum sum rule. Thus, in general, the free parameters in the fits are \( \alpha, \beta, A_s \) and \( \Lambda^{MS}_{\overline{MS}} \).

Direct analysis of the structure functions has an advantage over analysis of their moments in that it allows experimentally and theoretically motivated kinematic cuts to be introduced without undue sensitivity to the contributions of the excluded regions [45].

We make simultaneous singlet and non-singlet fits to \( F_2 \) and \( xF_3 \) respectively (the accuracy of the \( F_3 \) data is insufficient for non-singlet fits alone to give much information, such fits give results in agreement with the combined fits). The \( xF_3 \) data are fitted in the \( x \) range \( 0.0 < x < 0.4 \) and the \( F_2 \) data in the \( x \) range \( 0.05 < x < 0.7 \). The \( F_2 \) data for \( x < 0.05 \) are excluded in order to limit the reliance on the assumptions concerning \( R \), the strange sea, the charm threshold behaviour, and the radiative corrections. The statistical and systematic errors on the structure functions have been combined in quadrature before making the fits.

(ii) Results

Fitting the data in the region \( Q^2 > 4 \) GeV\(^2\), \( W^2 > 10 \) GeV\(^2\), where scaling violations due to the effects of finite target mass and dynamical higher twist effects are expected to be negligible [6,16], gives

\[
\Lambda^{MS}_{\overline{MS}} = 100^{+110}_{-85} \text{ MeV} \quad \chi^2 = 27.4 \text{ for (36-4) degrees of freedom}
\]

\[
\alpha = 0.86 \pm 0.08 \quad \beta = 3.6 \pm 0.2 \quad A_s = 1.10 \pm 0.10
\]

Table 2 shows the effect of relaxing the cuts on the data sample to include data at low \( Q^2 \) and \( W^2 \); there is no significant change in the value of \( \Lambda^{MS}_{\overline{MS}} \). A rise would be expected if substantial positive higher twist terms contributed to the scaling deviations in this kinematic region. With the most relaxed cuts, \( Q^2 > 1 \) GeV\(^2\) and no explicit \( W \) cut, there is instead a slight fall; the fit gives

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\[
\frac{A_{\text{MS}}}{MeV} = 52^{+36}_{-27} \quad \chi^2 = 56.3 \text{ for (63-4) degrees of freedom} \quad (13)
\]
\[\alpha = 0.87 \pm 0.04 \quad \beta = 3.8 \pm 0.1 \quad A_s = 0.95 \pm 0.06\]

The table also gives for each set of cuts the value obtained for \(A_{\text{MS}}\) when target mass corrections are not included. These corrections are largest at low \(W^2\), however they do not change the results substantially. They are included in all the fits discussed below.

To investigate the contribution of the more interesting dynamical higher twist terms, we introduced a parameterisation for such terms into the fits such that

\[F(x,Q^2) = F_{\text{LT}}(x,Q^2) \ast (1 + \text{HT}(x,Q^2)) \quad (14)\]

where \(F_{\text{LT}}(x,Q^2)\) is the leading twist QCD prediction for the structure function and \(\text{HT}(x,Q^2)\) is the higher twist contribution.

Table 3 gives the results for various forms of \(\text{HT}(x,Q^2)\) suggested in the literature [4,11,20, 21,24,46,47], fitted to the data with \(Q^2 = 1 \text{ GeV}^2\) and no cut on \(W^2\). It shows that the value of \(A_{\text{MS}}\) is largely insensitive to the assumed form of the higher twist term, ranging between 95 and 200 MeV consistent with the value found when excluding the data with low \(Q^2\) and \(W^2\). The higher twist term itself is found to be small and negative in all cases.

Whereas assuming no higher twist contribution gives an acceptable fit \((\chi^2/\text{NDF} = 56.3/59)\), adding a negative higher twist term reduces the \(\chi^2\) by between 3 and 10 according to the form assumed, corresponding to a 2 to 3 standard deviation negative higher twist signal. The envelope of all these fits is shown in Fig 1.

The complex forms of higher twist contribution calculated by Nason, Gunion and Blanckenbecler [46] contain the term \(509 \mu^2 x^7 / W^2\) which is dominant and positive by definition, consequently the fit gives a meaningless negative value for the square of the spectator quark mass \(\mu^2\). Keeping only the term \(-7 \mu^2 x / W^2 (1-x)\) gives the physically reasonable value of \(\sim 220 \text{ MeV}\) for the spectator quark mass. However, our data do not probe the \(x > 0.9\) region where their calculations should be most reliable.

Using an additive form for the higher twist contribution \(F(x,Q^2) = F_{\text{LT}}(x,Q^2) + \mu^2 x / (1-x) Q^2\), as suggested in [20,21], gives a good fit (see last entry in
Table 3). However the multiplicative form of the same term gives a better fit (see 3rd entry of Table 3).

It is perhaps surprising that the simple term \( \mu^2/Q^2 \) independent of \( x \) gives the best fit of all, reducing \( \chi^2 \) by 10 (see 2nd entry of Table 3). We have attempted to extract the form of the higher twist term as a function of \( x \) by fixing the parameters \( A_{\overline{MS}} \), \( \alpha \), \( \beta \), \( \gamma \) and \( A_s \) at the values obtained for \( Q^2 > 4 \) GeV\(^2\), \( W^2 > 10 \) GeV\(^2\) (see Eqn 12), where higher twist effects are not important, and fitting the form

\[
F_{LT}(x,Q^2) (1 + \mu^2(x)/Q^2)
\]

in each \( x \) bin separately, including the low \( Q^2 \) and low \( W^2 \) data. The result is shown in Figure 2, where the errors include the errors on \( A_{\overline{MS}} \) as given in Eqn 12. The two best fits from Table 3, \( \mu^2(x) = -0.16 \) GeV\(^2\) and \( \mu^2(x) = -0.41 \) x/(1-x) GeV\(^2\) are superposed for comparison.

Table 4 shows the effect on the preferred fit of varying the parameterisation used at \( Q^2 = Q_0^2 \). The fits are almost completely insensitive to the parameterisations of the sea and gluon distributions. This lack of sensitivity is general to all fits, both with and without higher twist terms and with and without low \( Q^2 \) and low \( W^2 \) data.

A more significant change in parameterisation is to allow \( \gamma \neq 0 \) in the parameterisation of \( xF_3 \), as has been done in some QCD analyses [12,16]. This choice lowers \( \Lambda \) by 60 MeV and gives a marginally less negative higher twist term. However in this case there is no true minimum in \( \chi^2 \); for increasing \( \gamma \), \( \chi^2 \) falls monotonically approaching a constant as \( \gamma \) and \( \alpha \) become completely anti-correlated. The correlated change in \( \mu^2 \) and \( \Lambda \) due to allowing \( \gamma \neq 0 \) is only one standard deviation and our conclusions remain essentially unchanged.

All errors quoted represent the total error. The contribution of the systematic error to the total error on \( \Lambda \) and \( \mu^2 \) ranges from 35% for \( W^2 > 10 \) GeV\(^2\), to 39% for \( W^2 > 1.7 \) GeV\(^2\). The largest systematic error in our analysis comes from the unsmeared corrections discussed in Section 4. The sizes of these corrections are given in Table 1. If all points for which this correction is larger than 25% are dropped from the fits, the results remain essentially unchanged.
In all fits, \( \mu^2 \) and \( \Lambda \) are anti-correlated. This is illustrated in Figure 3 for the fit to the preferred higher twist term \( HT(x,Q^2) = \mu^2/Q^2 \). There is no significant correlation between \( \mu^2 \) and the other fitted variables. The second and third standard deviation contours illustrate the increasing asymmetry in the errors on \( \Lambda \), as the lower bound on \( \Lambda \) tends to zero. This is a general feature of our fits.

(iii) **Comparison with Previous Low \( W^2 \) Experiments**

Figure 2 also shows the results of an analysis done by the EMC [21] by combining their high \( W^2 \) \( \mu p \) data with low \( W^2 \) \( ep \) data from SLAC. The evident inconsistency between these results and ours may be due to the systematic problems inherent in combining data from different experiments in this way; or it may be due to real differences in the higher twist effects between weak and electromagnetic scattering [17,23] and/or between scattering off bound isoscalar nucleons (our analysis) and scattering off free protons (their analysis).

The CDHS collaboration [20] combined their high \( W^2 \) \( \nu \), \( \bar{\nu} \)Fe data with low \( W^2 \) SLAC \( eD^2 \) data without accounting for the difference in \( F_2 \), then unknown, between heavy and light nuclei. Indeed, the EMC effect [22] predicts the trend of the \( x \) dependence of the discrepancy observed between the CDHS data and the SLAC data, which cannot therefore be confidently attributed to higher twist effects.

There have been three previous structure function analyses using \( \nu \) and \( \bar{\nu} \) data at low \( W^2 \). All analyses were of bubble chamber data.

The B2BC+GGM analysis [1,4] covered a wide kinematic range by combining data from two different experiments using different beams and targets. However, if EMC effects in structure functions are proportional to \( \ln(A) \) [48], the effect of the difference in targets is small in this case. A small positive higher twist effect was found by fitting the form \( F_{LT}(x,Q^2)(1+\mu^2)\frac{x}{(1-x)Q^2} \) to their non-singlet structure function data at leading order. They obtained

\[
\Lambda = 74^{+158}_{-74} \text{ MeV}, \; \mu^2 = 0.25\pm0.14 \text{ GeV}^2, \; \chi^2/\text{NDF} = 118/52
\]
for all data for which $q^2 > 1$ GeV$^2$ (lowest $W^2 = 1.2$ GeV$^2$). Applying our analysis method to their data (both singlet and non-singlet) gives

$$\Lambda = 168\pm77\text{ MeV}, \mu^2 = 0.09\pm0.07, \chi^2/\text{NDF} = 158/63$$

There are three data points with very large $\chi^2$ in this fit. Removing these gives

$$\Lambda = 116\pm77\text{ MeV}, \mu^2 = 0.13\pm0.08 \text{ GeV}^2, \chi^2/\text{NDF} = 97/60$$

Restricting the data fitted to $W^2 > 1.7$ GeV$^2$ in order to exclude the unreliable low $W$ region we obtain

$$\Lambda = 249\pm100\text{ MeV}, \mu^2 = -0.28\pm0.17 \text{ GeV}^2, \chi^2/\text{NDF} = 92/57$$

Although the $\chi^2$ value remains large, these values of $\Lambda$ and $\mu^2$ agree well with our corresponding result (entry 3 of Table 3)

$$\Lambda = 200^{+80}_{-70}\text{ MeV}, \mu^2 = -0.41\pm1.12 \text{ GeV}^2, \chi^2/\text{NDF} = 48/58$$

Thus there is no contradiction between these experiments.

The WA25 collaboration [36] used the same beam as the present experiment and a deuterium target, in order to measure the proton and neutron structure functions separately. Although no quantitative results on higher twist contributions are given, they report that a fit to the isoscalar structure function $xF_3$, using data down to $W^2 = 1.5$ GeV$^2$, yields a higher twist contribution that is small and negative.

The GGM-SPS collaboration also used the same beam as the present experiment, to measure the isoscalar structure function on a heavy nuclear target. Their analysis [24] used data down to $W^2 = 1.45$ GeV$^2$ and yielded predominantly negative higher twist contributions in agreement with our results but with larger errors due to low statistics.

* More significantly positive higher twist effects were obtained in the BEBC+GGM analysis by the moments method, but this method emphasises the unreliable low $W^2$ region even more strongly [45].
We present data on the structure functions $F_2$ and $xF'_3$, in the kinematic range, $0 < Q^2 < 64 \text{ GeV}^2$, $1.7 < W^2 < 250 \text{ GeV}^2$, $0 < x < 0.7$, as extracted from $\nu, \bar{\nu}$ scattering on a neon target.

A value of
\[
A_{\overline{\text{MS}}} = 100^{+110}_{-85} \text{ MeV}, \quad \chi^2/\text{NDF} = 27.4/32
\]  
(16)

gives a good fit to these data at high $Q^2$ and $W^2$ ($Q^2 > 4 \text{ GeV}^2$, $W^2 > 10 \text{ GeV}^2$). Including data at lower $Q^2$ and $W^2$ ($1 < Q^2 < 64 \text{ GeV}^2$, $1.7 < W^2 < 250 \text{ GeV}^2$) in the fit gives a value of
\[
A_{\overline{\text{MS}}} = 52^{+36}_{-27} \text{ MeV}, \quad \chi^2/\text{NDF} = 56.3/59
\]  
(17)

with no higher twist term and a value of $A_{\overline{\text{MS}}}$ ranging from 95 to 200 MeV according to the form of the higher twist term chosen, where the higher twist term itself is small and negative at the 2 to 3 standard deviation level (see Table 3).

In conclusion, a relatively low $A_{\overline{\text{MS}}}$ value in the neighbourhood of 100 MeV suffices to describe all our data, both at high $Q^2$ and $W^2$ and at low $Q^2$ and $W^2$. There is no evidence for the strong positive higher twist contributions at low $Q^2$ and $W^2$ for which evidence has been claimed in previous structure function analyses. Moreover, fits to specific forms for such terms yield contributions which are small and negative.

**ACKNOWLEDGEMENTS**

We thank R.G. Roberts for many useful discussions and advice on the incorporation of higher twist terms in the fitting programme, the BEBC, EMI, NFM, neutrino beam and SPS crews for a very successful run, and our scanning and measuring staffs for their painstaking and skilful work.
APPENDIX

The parameterisation for the differential cross-sections used in both Monte Carlo simulations is:

\[
\sigma_v = q_v(x,Q^2) \cos^2 \theta_c + q_s(x,Q^2) \frac{1}{(2 + \frac{s}{d})} (\cos^2 \theta_c + \frac{s}{d} \sin^2 \theta_c)
\]

\[
+ q_s(x,Q^2) \frac{1}{(2 + \frac{s}{d})} (1 - y)^2
\]

\[
+ R'(x,Q^2)[q_v(x,Q^2) + q_s(x,Q^2)] (1 - y)
\]

\[
+ [\Theta(M_c^2)] (1 - y + \frac{x}{z}) [q_v(z,Q^2) \sin^2 \theta_c + q_s(z,Q^2) \frac{z}{(2 + \frac{s}{d})} (\sin^2 \theta_c + \frac{s}{d} \cos^2 \theta_c)] - A1
\]

\[
\sigma_v = [q_v(x,Q^2) + q_s(x,Q^2) \frac{1}{(2 + \frac{s}{d})}] (1 - y)^2
\]

\[
+ q_s(x,Q^2) \frac{1}{(2 + \frac{s}{d})} (\cos^2 \theta_c + \frac{s}{d} \sin^2 \theta_c)
\]

\[
+ R'(x,Q^2)[q_v(x,Q^2) + q_s(x,Q^2)] (1 - y)
\]

\[
+ [\Theta(M_c^2)] (1 - y + \frac{x}{z}) [q_s(z,Q^2) \frac{1}{(2 + \frac{s}{d})} (\sin^2 \theta_c + \frac{s}{d} \cos^2 \theta_c)] - A2
\]

These differential cross sections contain terms to account for

(i) the SU(3) asymmetry of the sea [37], \( \frac{s}{d} = 0.5 \);

(ii) the kinematic suppression of d + c and s + c transitions assuming slow rescaling [38]; \( \theta_c \) is the Cabbibo angle, a charmed quark mass \( M_c = 1.5 \text{ GeV} \) is used and \( z = x(1 + \frac{M_c^2}{Q^2}) \);

(iii) the violation of the Callan-Gross relation such that
\[ R'(x,Q^2) = \left( R(x,Q^2) - \frac{Q^2}{\nu^2} \right) / \left( 1 + \frac{Q^2}{\nu^2} \right) \]

where \( R(x,Q^2) \) is given in Equation 10 of the text.

The \( Q^2 \) dependent parameterisations of the valence and sea contributions follow the formalism of Buras & Gaemers [49] as follows.

The valence contribution is given by,

\[ q_v(x,Q^2) = A_v(Q^2) \times \alpha(Q^2) (1-x) \beta(Q^2) \]

where

\[ \alpha(Q^2) = \alpha(Q_0^2) + \alpha's \]
\[ \beta(Q^2) = \beta(Q_0^2) + \beta's \]
\[ s = \frac{\ln(\frac{Q^2}{Q_0^2})}{\Lambda^2} \]

and the normalisation, \( A_v(Q^2) \), is determined such that the GLS sum rule is obeyed for all \( Q^2 \). The parameters \( \alpha(Q_0^2) \), \( \beta(Q_0^2) \), \( \Lambda \) were determined by fitting our structure function data iteratively, as

\[ \alpha(Q_0^2) = 0.70, \beta(Q_0^2) = 3.21, \Lambda = 45 \text{ MeV} \]

for \( Q_0^2 = 0.5 \text{ GeV}^2 \). For these values of \( \alpha(Q_0^2) \) and \( \beta(Q_0^2) \) the \( Q^2 \) dependence required by QCD [49] fixes \( \alpha' \) and \( \beta' \) to be:

\[ \alpha' = -0.19, \beta' = 0.82 \]

Everything is now defined except the sea contribution which is given by

\[ q_s(x,Q^2) = S_2(Q^2) \left( \beta_s(Q^2) + 1 \right) (1-x) \beta_s(Q^2) \]

where

\[ \beta_s(Q^2) = \frac{S_2(Q^2)}{S_3(Q^2)} - 2 \]

and for \( n = 2, 3 \)
\[ S_n(Q^2) = \frac{1}{4} D^2_n(Q^2) + \frac{1}{4} D^1_n(Q^2) \]  

\[ D^2_n(Q^2) = \left( 1 - \alpha_n \right) \left( S_n(Q_0^2) + V_n(Q_0^2) \right) - \beta_n G_n(Q_0^2) e^{-\gamma_n} \]  

\[ + \left[ \alpha_n \left( S_n(Q_0^2) + V_n(Q_0^2) \right) + \beta_n G_n(Q_0^2) \right] e^{-\gamma_n} + V_n(Q_0^2) e^{-\gamma_n} \]  

\[ D^1_n(Q^2) = S_n(Q_0^2) e^{-\gamma_n} \]  

where \( \alpha_2, \beta_2, \alpha_3, \beta_3, \gamma_2^{\perp}, \gamma_2^{\parallel}, \gamma_2^{\perp}, \gamma_2^{\parallel}, \gamma_3^{\perp}, \gamma_3^{\parallel}, \gamma_3^{\perp} \) are given by QCD [49] and

\[ S_n(Q_0^2) = \frac{1}{2} \int_0^1 x^{n-2} q_s(x,Q_0^2) \, dx \]  

\[ V_n(Q_0^2) = \frac{1}{2} \int_0^1 x^{n-2} q_v(x,Q_0^2) \, dx \]  

\[ G_n(Q_0^2) = \frac{1}{2} \int_0^1 x^{n-2} g(x,Q_0^2) \, dx \]  

are the moments of the sea quark, valence quark and gluon distributions at \( Q^2 = Q_0^2 \), respectively. The normalisation \( V_2(Q_0^2) \) is determined by the parameters \( \alpha(Q_0^2) \), \( \beta(Q_0^2) \) such that the GLS sum rule is obeyed at \( Q_0^2 \). The normalisation \( S_2(Q_0^2) \) is determined by an extra parameter \( B(Q_0^2) \) such that \( S_2(Q_0^2) = V_2(Q_0^2) (1-B(Q_0^2))/B(Q_0^2) \), and the normalisation \( G_2(Q_0^2) \) is determined by the momentum sum rule. The third moments of the sea and gluon distributions at \( Q_0^2 \) are given by:

\[ S_3(Q_0^2) = S_2(Q_0^2)/(\beta_s(Q_0^2) + 2) \]  

\[ G_3(Q_0^2) = S_2(Q_0^2)/(\beta_g(Q_0^2) + 2) \]  

in terms of the two new parameters, \( \beta_s(Q_0^2) \), \( \beta_g(Q_0^2) \). The parameter values:

\[ B(Q_0^2) = 0.77, \beta_s(Q_0^2) = 5.3, \beta_g(Q_0^2) = 4 \]  

give a good fit to our structure function data.
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<td>0.545</td>
<td>0.13</td>
<td>0.279±0.030±0.044</td>
<td>0.999</td>
<td>1.084</td>
<td>0.969</td>
<td>1.025</td>
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<tr>
<td></td>
<td>0.649</td>
<td>0.11</td>
<td>0.148±0.029±0.031</td>
<td>0.998</td>
<td>1.518</td>
<td>0.973</td>
<td>1.142</td>
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</table>

- 22 -
<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>x</th>
<th>&lt;y&gt;</th>
<th>$F_2$ ±stat ±syst</th>
<th>Factor changes in $F_2$ for</th>
<th>$x F_3$ ±stat ±syst</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>5.73</td>
<td>0.028</td>
<td>0.70</td>
<td>1.43±0.174±0.069</td>
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<td>0.076</td>
<td>0.59</td>
<td>1.60±0.086±0.135</td>
<td>1.010</td>
<td>1.078</td>
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<td>0.149</td>
<td>0.49</td>
<td>1.389±0.064±0.022</td>
<td>1.010</td>
<td>0.951</td>
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<td>0.248</td>
<td>0.39</td>
<td>1.085±0.038±0.060</td>
<td>1.008</td>
<td>0.900</td>
</tr>
<tr>
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<td>0.347</td>
<td>0.34</td>
<td>0.837±0.035±0.025</td>
<td>1.004</td>
<td>0.904</td>
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<tr>
<td></td>
<td>0.447</td>
<td>0.27</td>
<td>0.645±0.027±0.026</td>
<td>1.001</td>
<td>0.932</td>
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<td>0.545</td>
<td>0.24</td>
<td>0.267±0.024±0.019</td>
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<td>1.036</td>
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<td>0.649</td>
<td>0.21</td>
<td>0.127±0.019±0.032</td>
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<td>1.263</td>
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<tr>
<td>11.28</td>
<td>0.076</td>
<td>0.72</td>
<td>1.626±0.127±0.113</td>
<td>0.997</td>
<td>1.118</td>
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<tr>
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<td>0.149</td>
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<td>1.412±0.058±0.097</td>
<td>1.004</td>
<td>1.030</td>
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<tr>
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<td>0.248</td>
<td>0.51</td>
<td>1.071±0.042±0.017</td>
<td>1.006</td>
<td>0.950</td>
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<tr>
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<td>0.347</td>
<td>0.43</td>
<td>0.674±0.031±0.062</td>
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<td>0.922</td>
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<tr>
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<td>0.447</td>
<td>0.40</td>
<td>0.420±0.025±0.013</td>
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<td>0.911</td>
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<td>0.545</td>
<td>0.38</td>
<td>0.246±0.020±0.025</td>
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<td>0.976</td>
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<tr>
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<td>0.649</td>
<td>0.32</td>
<td>0.122±0.014±0.022</td>
<td>0.996</td>
<td>1.063</td>
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<tr>
<td>22.32</td>
<td>0.149</td>
<td>0.69</td>
<td>1.391±0.083±0.119</td>
<td>0.993</td>
<td>1.038</td>
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<tr>
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<td>0.248</td>
<td>0.61</td>
<td>1.039±0.054±0.018</td>
<td>1.000</td>
<td>0.998</td>
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<td>0.347</td>
<td>0.55</td>
<td>0.763±0.041±0.011</td>
<td>1.002</td>
<td>0.950</td>
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<tr>
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<td>0.447</td>
<td>0.51</td>
<td>0.435±0.028±0.020</td>
<td>1.002</td>
<td>0.958</td>
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<td>0.545</td>
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<td>0.193±0.018±0.017</td>
<td>0.999</td>
<td>0.942</td>
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<td>0.649</td>
<td>0.40</td>
<td>0.094±0.013±0.006</td>
<td>0.997</td>
<td>1.029</td>
</tr>
<tr>
<td>43.54</td>
<td>0.149</td>
<td>0.82</td>
<td>1.502±0.218±0.087</td>
<td>0.983</td>
<td>1.169</td>
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<td>0.248</td>
<td>0.72</td>
<td>0.994±0.081±0.060</td>
<td>0.993</td>
<td>1.099</td>
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<td>0.347</td>
<td>0.67</td>
<td>0.748±0.054±0.029</td>
<td>0.998</td>
<td>0.960</td>
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<tr>
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<td>0.447</td>
<td>0.64</td>
<td>0.401±0.036±0.015</td>
<td>1.000</td>
<td>0.968</td>
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<td>0.56</td>
<td>0.202±0.024±0.021</td>
<td>0.999</td>
<td>0.982</td>
</tr>
<tr>
<td></td>
<td>0.649</td>
<td>0.51</td>
<td>0.072±0.014±0.017</td>
<td>0.997</td>
<td>1.080</td>
</tr>
</tbody>
</table>

The measured values of $F_2$ and $x F_3$ for $R=R_{Qar{Q}N}+R_{QCD}$ (Equation 10) at the given $x, Q^2$ values; the bin edges used were at $x=0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ and 0.7; $Q^2=0.0, 0.3, 0.6, 1.0, 2.0, 4.0, 8.0, 16.0, 32.0$ and 64.0 GeV$^2$. Also given are the mean value of $y$ in each bin and, for $F_2$, the factor by which the result would have been changed if (a) $R=1.281 Q^2/(Q^2+1.158)^2$ (Ref 34) had been assumed, and (b) the unsmeareding correction, (c) the slow rescaling correction (including the strange sea correction) and (d) the radiative correction had not been made.
### Table 2

Variation of $\Lambda_{\text{MS}}$ and of the Parameters Describing the Structure Functions According to the Cuts Applied to the Data Sample When no Higher Twist Term is Included in the Fit

<table>
<thead>
<tr>
<th>$\chi^2$/NDF</th>
<th>Cuts (Applied in GeV$^2$)</th>
<th>$\Lambda_{\text{MS}}$ (MeV)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$A_s$</th>
<th>$\Lambda$ (MeV) No Target Mass Corrn</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.4/36-4</td>
<td>$Q^2 &gt; 4$ \atop $W^2 &gt; 10$</td>
<td>$100^{+110}_{-85}$</td>
<td>0.86±0.08</td>
<td>3.6±0.2</td>
<td>1.10±0.10</td>
<td>95±90</td>
</tr>
<tr>
<td>27.8/40-4</td>
<td>$Q^2 &gt; 4$ \atop no W cut \atop (ie $W^2 &gt; 1.7$)</td>
<td>$51^{+62}_{-44}$</td>
<td>0.82±0.08</td>
<td>3.7±0.2</td>
<td>1.10±0.10</td>
<td>70±46</td>
</tr>
<tr>
<td>38.1/44-4</td>
<td>$Q^2 &gt; 1$ \atop $W^2 &gt; 0$</td>
<td>$140^{+90}_{-80}$</td>
<td>0.95±0.09</td>
<td>3.7±0.2</td>
<td>0.99±0.07</td>
<td>135±87</td>
</tr>
<tr>
<td>56.3/63-4</td>
<td>$Q^2 &gt; 1$ \atop no W Cut \atop (ie $W^2 &gt; 1.7$)</td>
<td>$52^{+36}_{-27}$</td>
<td>0.87±0.04</td>
<td>3.8±0.1</td>
<td>0.95±0.06</td>
<td>70±41</td>
</tr>
</tbody>
</table>
### TABLE 3

Fits of the Data with $Q^2 > 1$ GeV$^2$ and no $W$ Cut to Different Forms of Higher Twist Term

<table>
<thead>
<tr>
<th>$x^2$ /NDF</th>
<th>Form of Higher Twist Term HT(x,Q$^2$)</th>
<th>Ref</th>
<th>$A_{NS}$ (MeV)</th>
<th>$\mu^2$ (in GeV$^2$ or GeV$^4$ as appropriate)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$A_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>56.3/63-4</td>
<td>0</td>
<td>52$^{+36}_{-27}$</td>
<td>0</td>
<td>0.87±0.04</td>
<td>3.8±0.1</td>
<td>0.95±0.06</td>
<td></td>
</tr>
<tr>
<td>46.4/63-5</td>
<td>$\mu^2/Q^2$</td>
<td>110$^{+50}_{-45}$</td>
<td>-0.16 ±0.05</td>
<td>0.94±0.006</td>
<td>3.8±0.1</td>
<td>1.10±0.10</td>
<td></td>
</tr>
<tr>
<td>47.7/63-5</td>
<td>$\mu^2 x/Q^2 (1-x) = \mu^2/(W^2-m^2)$ [4,20,23,48]</td>
<td>200$^{+80}_{-70}$</td>
<td>-0.41 ±0.12</td>
<td>1.05±0.09</td>
<td>3.7±0.1</td>
<td>1.03±0.09</td>
<td></td>
</tr>
<tr>
<td>53.0/63-5</td>
<td>$\mu^2 x^2/Q^4 (1-x)^2$</td>
<td>[20]</td>
<td>97±53</td>
<td>-0.27 ±0.14</td>
<td>0.91±0.06</td>
<td>3.7±0.1</td>
<td>0.95±0.06</td>
</tr>
<tr>
<td>53.0/63-5</td>
<td>$\mu^2 x^3/\mu^2$</td>
<td>[11]</td>
<td>101±55</td>
<td>-2.2 ±1.1</td>
<td>0.90±0.05</td>
<td>3.6±0.1</td>
<td>0.97±0.06</td>
</tr>
<tr>
<td>53.2/63-5</td>
<td>$\mu^2 x^2/\mu^4$</td>
<td>[11]</td>
<td>96±52</td>
<td>-2.6 ±1.3</td>
<td>0.90±0.05</td>
<td>3.7±0.1</td>
<td>0.96±0.06</td>
</tr>
<tr>
<td>52.7/63-5</td>
<td>$\mu^2 x^3/Q^2 (1-x)$</td>
<td>[21]</td>
<td>108±57</td>
<td>-0.79 ±0.37</td>
<td>0.91±0.06</td>
<td>3.7±0.1</td>
<td>0.96±0.06</td>
</tr>
<tr>
<td>51.8/63-5</td>
<td>$-7\mu^2 x/W^2 (1-x)$</td>
<td>[46]</td>
<td>128±61</td>
<td>-0.0032±0.0011</td>
<td>0.92±0.06</td>
<td>3.6±0.1</td>
<td>0.97±0.06</td>
</tr>
<tr>
<td></td>
<td>+509$\mu^2 x^2/W^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+70($\mu^2 x/W^2 (1-x))^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.1/63-5</td>
<td>$-7\mu^2 x/W^2 (1-x)$</td>
<td>[46]</td>
<td>97±53</td>
<td>-0.0044±0.0016</td>
<td>0.89±0.05</td>
<td>3.6±0.1</td>
<td>0.97±0.06</td>
</tr>
<tr>
<td></td>
<td>+509$\mu^2 x^3/W^2$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>+70($\mu^2 x/W^2 (1-x))^2$</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>51.8/63-5</td>
<td>$-\mu^2 x/W^2 (1-x)$</td>
<td>[46]</td>
<td>129±63</td>
<td>0.050 ±0.022</td>
<td>0.93±0.06</td>
<td>3.6±0.1</td>
<td>0.98±0.07</td>
</tr>
<tr>
<td>53.1/63-5</td>
<td>$\mu^2 x/Q^2 (1-x)$</td>
<td>[20,21]</td>
<td>95±53</td>
<td>-0.066 ±0.037</td>
<td>0.89±0.05</td>
<td>3.6±0.1</td>
<td>0.97±0.07</td>
</tr>
<tr>
<td></td>
<td>but additive</td>
<td></td>
<td></td>
<td></td>
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</table>
TABLE 4

Variation of $\Lambda_{\text{MS}}$ and $\mu^2$ with Change of the Parameterisation at $Q^2=Q_0^2$ when Fitting a Higher Twist Term of the Form $\mu^2/Q^2$ to the data with $Q^2 > 1$ GeV$^2$ and No $W$ Cut

<table>
<thead>
<tr>
<th>$\mu^2$ (GeV$^2$)</th>
<th>$\Lambda_{\text{MS}}$ (MeV)</th>
<th>$\chi^2$/NDF</th>
<th>Comment</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$A_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.16 \pm 0.05$</td>
<td>$110^{+50}_{-45}$</td>
<td>46.4/63-5</td>
<td>standard</td>
<td>0.94±0.06</td>
<td>3.8±0.1</td>
<td>1.10±0.10</td>
</tr>
<tr>
<td>$-0.17 \pm 0.05$</td>
<td>$104 \pm 52$</td>
<td>46.3/63-5</td>
<td>$\beta_g = 8$</td>
<td>0.94±0.06</td>
<td>3.8±0.1</td>
<td>1.20±0.10</td>
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<tr>
<td>$-0.14 \pm 0.05$</td>
<td>$113 \pm 54$</td>
<td>45.3/63-6</td>
<td>$\gamma_g = 9$</td>
<td>0.91±0.07</td>
<td>3.8±0.1</td>
<td>-1.6±3.0</td>
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<tr>
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<td></td>
<td>$B_s$ free = 2.6±2.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.15 \pm 0.05$</td>
<td>$103 \pm 51$</td>
<td>45.3/63-6</td>
<td>$B_s$ free = 3.9±2.9</td>
<td>0.91±0.06</td>
<td>3.8±0.1</td>
<td>-2.9±3.0</td>
</tr>
<tr>
<td>$-0.15 \pm 0.05$</td>
<td>$103 \pm 51$</td>
<td>47.2/63-5</td>
<td>$\beta_s = 3.7$</td>
<td>0.80±0.06</td>
<td>3.9±0.1</td>
<td>0.91±0.08</td>
</tr>
<tr>
<td>$-0.16 \pm 0.05$</td>
<td>$124 \pm 61$</td>
<td>53.0/63-5</td>
<td>$\beta_s = 6.9$</td>
<td>1.00±0.06</td>
<td>3.9±0.1</td>
<td>1.30±0.12</td>
</tr>
<tr>
<td>$-0.10 \pm 0.05$</td>
<td>$52 \pm 48$</td>
<td>41.3/63-6</td>
<td>$\gamma$ free = 11±16</td>
<td>0.5±0.3</td>
<td>4.5±0.2</td>
<td>1.23±0.09</td>
</tr>
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</table>
FIGURE CAPTIONS

Fig 1: Plots of the measured structure functions $F_2$ (Fig 1(a)) and $xF_3$ (Fig 1(b)). Statistical and systematic errors have been summed in quadrature. The envelope of the fits described in Table 3 is superposed on the data. Points depicted as open triangles were not used in the fits, but the fit envelope is extrapolated outside the fitted region to indicate that the fits are not sensitive to the exact kinematic region chosen.

Fig 2: Results of a fit to the $x$ dependence of a general higher twist term of the form $HT(x,Q^2) = \mu^2(x)/Q^2$; for this experiment ($\Delta$) and a combination of EMC ($\nu p$) and SLAC-MIT ($e p$) data ($x$) [21]. The inner error bars correspond to $\Lambda_{\overline{MS}} = 100$ MeV and the total errors include the errors on $\Lambda_{\overline{MS}}$ (see Equation 12) summed in quadrature. The dashed line represents $\mu^2(x) = -0.16$ GeV$^2$ and the dotted line represents $\mu^2(x) = -0.41 x/(1-x)$ GeV$^2$.

Fig 3: The correlation between the parameters $\Lambda_{\overline{MS}}$ and $\mu^2$ for the higher twist term $HT(x,Q^2) = \mu^2/Q^2$. The lines show the one, two and three standard deviation contours.
Figure 1(b)

\[ xF_3(x, Q^2) \]

\[ Q^2 \text{ (GeV/c)}^2 \]