INTRODUCTION TO STRING GROUND STATE CONSTRUCTION

E. Rabinovici

CERN-Geneva

ABSTRACT

A review of the construction of ground states in string theory is given. Simple mathematical and physical aspects of the compactified dimensions such as tori and orbifolds are discussed.

*) Permanent address: Racah Institute of Physics, Jerusalem.
+ Invited talk at XXIIth Rencontres de Moriond Les Arcs (March 1987).
Introduction

There was a period when it was clear that the electromagnetic interactions were well described by a field theory. However, it was not at all clear how field theory could describe any other of the observed interactions. Phenomena such as the Higgs mechanism and confinement were yet to be discovered. At that period attempts were made to derive results following from algebraic properties of the systems which would be independent of the particular dynamics. Such were the current algebra and dispersion relation studies and such were the investigations of the impact of symmetries including Lie algebras. Many possible Lie algebras underwent study, starting from the familiar SU(2) and ending with excursions into the properties of the exceptional groups.

Amid many difficulties, researchers in that period had one advantage, once they were interested in studying all Lie groups they could find their complete classification in the mathematical literature. Many of the insights obtained by these methods are still with us, indeed, string theory was born out of the Veneziano\(^1\) model. It also turned out that for many years it was enough to master SU(2) and, through it, SU(3).

In later years the focus shifted towards analyzing all relevant interactions and uncovering their dynamical properties, many of which were manifested by the ground state structure. There existed a well-defined principle by which the ground state was chosen, it was the minimum energy state. In string theory attempts to discover all possible forms of string interactions have just started\(^2a\) and the main results are in classifying all possible ground states of the string theory. Each ground state is described by a two-dimensional field theory with very special properties. Let us call them conformal-string field theories. The classification of such theories was turned into an algebraic problem. The dynamical issue of inventing a criterion to choose between the various ground states is yet unresolved.

After presenting general properties of string ground states, we study in more detail various examples, including the simplest orbifold compactification. We end with a discussion on the nature of compactifications.
Conformal field theories in two-dimensions Generalities\textsuperscript{2)}

Consider a field theory $L(\psi_i, g_j)$ where $\psi_i$ denotes a set of fields and $g_j$ a set of couplings.

For the fixed values of the coupling constants, i.e., for those couplings for which the beta-functions are zero (up to a field redefinition), the theory is invariant under global scale transformations. In two-dimensions the additional requirements of translational and rotational invariance lead to an invariance under a large class of local scale transformations which will be made more precise momentarily. These are the conformal field theories. An example of such a theory is free field theory in the plane.

\[
\int d^2 \sigma L = \int d^2 \sigma \, g^{\alpha \beta} \partial_\alpha X^\beta \phi X_\beta
\]

or, a little more generally as discussed in Callan's talk\textsuperscript{2a)}, theories of the form

\[
L = \mathcal{G}^{\mu \nu}(x) g^{\alpha \beta} \partial_\alpha X_\beta \partial_\nu X_\nu
\]

where $g^{\alpha \beta}$ is the two-dimensional gravity (which is integrated over) and $\mathcal{G}^{\mu \nu}(x)$ is a given background metric which should satisfy

\[
\mathcal{G}^{\mu \nu} = 0
\]

The classification of all possible such two-dimensional theories was turned into an algebraic one\textsuperscript{3)}.

The underlying algebra is extracted in the following manner. The theory has an energy momentum-tensor $T_{\mu \nu}(x,y)$. Due to the translation and scale invariance

\[
\partial^\alpha T_{\alpha \beta} = 0, \quad T_{\alpha \alpha} = 0
\]

The two, by now, independent components of $T_{\mu \nu}(x,y)$ can be written as

\[
T(\bar{z}) = T_{uu} - T_{11} + i T_{12}, \quad \bar{T}(\bar{z}) = T_{uu} - T_{11} - i T_{12}
\]

with $z = x + yi$ and $\bar{z} = x - iy$. 
New operators \( L_n \) and \( \bar{L}_n \) are defined by:

\[
L_n = \frac{1}{2\pi i} \oint dz \bar{z}^n T(z) \quad ; \quad \bar{L}_n = \frac{1}{2\pi i} \oint \bar{z} \bar{z}^{n+1} \bar{T}(\bar{z})
\]

(6)

These operators are the generators of the conformal transformations

\[
z \rightarrow f(z) \quad ; \quad \bar{z} \rightarrow f(\bar{z})
\]

(7)

In particular \( f(z) = a_n z^{n+1} \) is generated by \( L_n \).

This infinite set of Virasoro operators \(^4, ^5\) fulfill an algebra

\[
[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} (m^3 - m) \delta_{m,-n}
\]

(8)

where the number \( c \) is an input describing the short-distance behaviour of the product of the energy momentum tensor.

\[
T(z) T(z') = \frac{c}{2(z-z')^4} + \frac{2 T(z)}{(z-z')^2} + \cdots
\]

(9)

For a given value of \( c \) the properties of the spectrum are fully determined once the "ground state energy" \( h \) is given. \( h \) is the eigenvalue of the operator \( L_0 \).

\[
L_0 |h\rangle = h |h\rangle \quad ; \quad L_n |h\rangle = 0 \quad \text{for} \quad n > 0
\]

(10)

The spectrum is generated by applying \( L_n \) operators on the ground state. All levels are equally spaced (by a unit distance) and the complexity of the system lies in determining the number of independent states at each level.

For \( c > 1 \) all values of \( c \) and \( h \) are allowed, however, it is not known which of these representations (reducible or irreducible) correspond to a spectrum of a field theory. Even without requiring any extra constraints these are of great interest as they represent all possible critical statistical mechanical models in two dimensions (without curvature). In order to qualify as string conformal models, some more requirements are imposed:
1. The representations should be unitary.

2. The value of \( c \) should be either 26 (for the conformal case) or 10 (for the superconformal case). This restriction on the allowed value of \( c \) follows from demanding that the theory has no quantum conformal anomaly. It is a world sheet property, associated with the world sheet metric, \( g_{\alpha\beta} \). This is thus a demand on the string with no particular reference to the realization of the Virasoro algebra (actually the requirement is that finally the total \( c \) vanishes).

3. Another world sheet constraint is the absence of global world sheet gravitational anomalies. In particular, on a torus the partition function should be invariant under the so-called modular transformations.

\[
\mathcal{Z} \rightarrow \frac{m z^{n}}{(z + k)}
\]  

(11)

4. Perhaps one also needs to impose factorization in the target space \(^{6a}\).

5. If one believes that low-energy string theory is related to 1990 high energy physics one may wish to impose additional experimental constraints such as a chiral content for fermions, an \( SU(3) \times SU(2) \times U(1) \) gauge symmetry, a vanishing cosmological constant and more.

The general solution for this more restricted problem is still unknown. Physicists have found "physical" solutions to the mathematical problem. This will be described later on. For \( c < 1 \) it was shown \(^7\) that unitarity restricts the allowed value of \( c \) (in the conformal case) to be of the form

\[
c < 1 \quad c = \frac{6}{m(m+1)} \quad , \quad m = 3, 4, \ldots
\]  

(12)

for each given value of \( c \) only a finite number of values of \( h \) are allowed!

\[
h_{c} = \frac{((m+1)\lambda - m\lambda)^{2} - 1}{4m(m+1)} \quad \lambda = 1, \ldots, m-1
\]  

(13)

One way to construct a conformal-string theory would be to take a tensor product of these models leading to \( c = 22 \) and append it to a four-dimensional free boson realization of the algebra \(^{8},9\).
For \( c > 1 \) many examples are known.

**Conformal theories Examples**

1. The closed bosonic string in D-flat dimensions.

A closed string moving in D-flat dimensions is described by the Lagrangian:

\[
L = \frac{1}{2 \alpha'} \partial^\mu X^\nu \partial^\nu X^\mu \quad \mu, \nu = 1, \ldots, D
\]

\(-\infty < X^\mu < \infty\), all beta-function are zero in this background.

The field \( X(\sigma, \tau) \) is expanded into its vibrational modes:

\[
X = X_0 + \sum_n a_n \text{exp}(i n (\tau - \sigma)) + \bar{a}_n \text{exp}(i n (\sigma - \tau))
\]

The Virasoro operators are given by

\[
L_n = \frac{1}{\alpha'} \sum_n a_n a_{n-m} a_m
\]

The central charge, \( c \), is equal to the number of space-time dimensions, \( c = D \).

Defining a number operator \( N \) by

\[
N = \sum_n a_n a_n^\dagger
\]

one obtains the mass spectrum of the system:

\[
\frac{M^2}{\alpha'} = N + \bar{N} - 2
\]

All states are subject to momentum conservation on the world sheet which implies

\[
N = \bar{N} = 0
\]

For \( D = 26 \), the low-lying states are given by: \( |N = \bar{N} = 0 \) with \( M^2 = -2 \) the
infamous tachyon and $|N = \tilde{N} = 1\rangle$ with $B^2 = 0$ the 26-dimensional graviton ($G_{\mu\nu}$), antisymmetric tensor ($B_{\mu\nu}$) and the scalar dilaton ($\phi$).

2. A string moving on a space with one dimension compactified on a torus.

Another case with the same value of $c$ is given by requiring that some (or all) of the "dimensions" $X^\mu$ are actually tori.

Let us discuss here the case where $0 < X^{24} < 2\pi R$, that is the string has "compactified" in one dimension. The mathematics of this case is simple. Expanding the string co-ordinate $X^{24}$ into:

$$X^{24} = X_0 + \frac{1}{2} t^2 + 2 \pi RL \theta + \sum \alpha_n e^{it \theta} \left( \sin(n \theta + \pi \gamma) + \sum \beta_n e^{i \theta} \right)$$

one obtains an $R$-dependent spectrum

$$\frac{H^2}{\gamma} = (N - \tilde{N})^2 + \frac{M^2}{\gamma R^2} + L^2 R^2$$

subject to an $R$-independent constraint

$$N = \tilde{N} + ML$$

The integer $M$ appeared through the quantization conditions on $p$.

The probability $|\psi(X_0)|^2$ to measure the string's centre of mass at the point $X_0$ should be the same as $|\psi(X_0 + 2\pi R)|^2$, thus $p = M/R$. $M$ makes its appearance in the usual Kaluza-Klein compactifications. The integer $L$ appeared as a "stringy" effect. A string extending from $X_0$ to $X + 2\pi RL$ is an open-string on the line but is a closed string on a torus and must be accounted for.

Defining left ($p_L$) and right ($p_R$) momenta

$$p_L = \frac{M}{2R} - LR, \quad p_R = \frac{M}{2R} + LR$$

one can rewrite the mass and momentum conditions as:

$$N = \tilde{N} + \frac{1}{2} \left( p_R^2 - p_L^2 \right)$$

$$\frac{H^2}{\gamma} = (N - \tilde{N})^2 + \frac{1}{2} \left( p_L^2 - \frac{1}{L} p_R^2 \right)$$
For a general value of $R$, the theory has a $U_L(1) \times U_R(1)$ gauge symmetry, one
gauge field is $g_{24 \mu}$ coming from the original $g_{\mu \nu}$ and the other is $B_{24 \mu}$ arising
from the antisymmetric tensor $B_{\mu \nu}$. Another massless state is the scalar $g_{24,24}$.  
It is an extra dilaton associated with the flat potential in the $R$ direction.
The radius of compactification is undetermined leading to a one parameter family  
of $c = D$ solutions. In general $d$ radii and the angles between them lead to a $d^2$
parameter family of $c = D$ field theories. In this example we follow some  
properties of the $d = 1$ family. At large $R$ the model has a $U(1) \times U(1)$ symmetry,  
$R = 3$ the model has non-linearly realized supersymmetry, at $R = 1$ the model  
is equivalent to a free Dirac fermion, at $R = 1/\sqrt{2}$ the symmetry is enhanced to  
$SU_L(2) \times SU_R(2)$. 

At that value of $R$ the lattice momenta form an $SU(2)$ lattice.

$$\frac{\beta^2}{\phi} = \frac{2\mu - 2}{\mu} \left( M - L \right)^2 \quad \bar{N} = \bar{\phi} + \phi L$$  \hspace{2cm} (26)

The additional massless states are given by  

\begin{align*}
\bar{N} = 0 & \quad \bar{\phi} = 1 & M = L = \pm 1 & \quad R = 1, \ldots, 23  \\
\bar{N} = 1 & \quad \bar{\phi} = 0 & M = -L = \pm 1  
\end{align*}  \hspace{2cm} (27)

These are $O(23)$ Lorentz vectors. Their $SU_L(2) \times SU_R(2)$ quantum numbers are $(1,0)$  
and $(0,1)$. The $(1_L)^* \times (1_L)$ quantum number are $\left[ (M + L) / 2, (M - L) / 2 \right]$. There are also  
nine massless scalars which form a $(1,1)$ representation of $SU(2) \times SU(2)$. Looking
"backward" from $R = 1/\sqrt{2}$ to $R > 1/\sqrt{2}$ one sees that $SU(2) \times SU(2)$ was
spontaneously broken by the $(1,1)$ Higgs to $U(1) \times U(1)$. $\langle \phi^2 \rangle \sim (R - 1/\sqrt{2})^2$ looking
"forward" for $R < 1/\sqrt{2}$ an amusing property emerges. The theory in an
$R$-background is equivalent to that in a $1/2R$ background. This can be seen by
exchanging $M$ with $L$ after changing $R$ into $1/2R$ in Eqs. (23) and (25).

The duality effect is coming from beyond a particle theory; it is what is
called a "stringy" effect. It results in this case from the discrete lattice
symmetry under the "parity" transformation

$$I_L \rightarrow -I_L \quad ; \quad P_L \rightarrow P_L$$  \hspace{2cm} (28)

This underscores the as yet unclear space meaning the compactified dimensions
may have in string theory. As mentioned, the full compactified spectrum
information appears in the momentum lattice.
\[ P_L = \frac{n}{2\alpha} - RL \]
\[ P_R = \frac{n}{2\alpha} + RL \]
\[ P_R^2 - P_L^2 = 2ML \]

(P\(L\), P\(R\)) is the even O(1,1) Lorentzian lattice, parametrized by R. The Lorentz boost flipping the sign of the P\(L\) part of the lattice, maintaining the P\(R\) part (N + L) is a symmetry.

3. A string moving on a space with one dimension compactified on an orbifold\(^{13}\).

Another case where the central charge \(c\) equals D is given by a string moving on a space where one dimension, \(x^{24}\), is compactified:
\[ 0 \leq x^{24} \leq 2\pi R^{24} \]
on a circle, \(S^{13},8\), where the points lying at angles \(\pm \theta\) are identified.

The points (0) and (\(\pi R\)) are fixed points of this identification. A string moving on such a manifold \(S^1/Z_2\) has two types of closed configurations.

Strings which were closed on the circle as well. For such strings the propagation on \(S^1/Z_2\) manifests itself in the requirement that
\[ \psi(x) = \psi(-x) \]
That is the amplitude to find some closed string passing through a set of points \(x\) should be equal to the amplitude of those strings to pass through the set \((-x)\). Through this requirement \(M, L\) cease to be good quantum numbers and only states of the form \(|M, L > \pm |M, -L >\) (the sign depends on the occupation numbers \(N, \tilde{N}\)) are permitted. This projection halves the Hilbert space of a string moving on a circle \(S^1\). In particular, the gauge states are projected out and the generic \(S^1\) symmetry \(U(1) \times U(1)\) is broken by the vacuum structure.

The value of \(c\) is unaffected by this massive projection because new states appear, those of strings open on \(S^1\) but closed on \(S^1/Z_2\). Those are the open strings on \(S^1\) whose end points are \(X\) and \(-X\). These strings utilize the structure of \(S^1/Z_2\). In this so-called twisted sector all oscillators \(N\) are quantized in half-integer units. There are two such twisted strings, those whose centre of mass can be associated by either of the fixed points. Both the twisted and
untwisted sectors mix and thus should be both kept. A generic $S^1/Z_2$ orbifold has a spontaneously broken $U(1) \times U(1)$ symmetry. At $R = 1$, it is equivalent to the tensor product of two ($c = 1/2$) modular invariant Ising-models. For $R = 1/\sqrt{2}$, the $U(1) \times U(1)$ symmetry is restored, beyond that point the model is dual again.

At $R = 1/\sqrt{2}$, in the untwisted sector

$$\frac{\mu^2}{\nu} = \nu - L + \frac{1}{2} (\mu - L)^2$$

$$\nu = \bar{\nu} + mL$$

(32)

The massless vector states are

$$\bar{\mu} = 1, \quad \nu = 0$$

$$\frac{1}{m_L} \left( |n = L = 0 \rangle - |n = L = -1 \rangle \right)$$

$$\bar{\nu} = 0, \quad \nu = 1$$

$$\frac{1}{m_L} \left( |n = L = 1 \rangle - |n = L = -1 \rangle \right)$$

(33)

These are superpositions of the $L_z = 0$ states of $SU(2) \times SU(2)$. In fact, the full spectrum of the string moving on the $S^1/Z_2$ orbifold of radius $R = 1/\sqrt{2}$ is equivalent to that of a string moving on a circle $S^1$ with $R = \sqrt{2}$. At this point the string does not feel the difference between propagating on an orbifold or a circle. This background $R$ from which a branch of broken $U(1) \times U(1)$ grows allows a smooth interpolation between different target space topologies. This is another "stringy" effect not arising in field theoretical gravity. For completeness the more formal definitions of an orbifold are given on the $d$-dimensional Euclidean plane $\mathbb{R}^d$ one defines (as one does in solid states physics) a space group $g$. Each element of $g$ is defined by a $d$-dimensional rotation $R$ and translation $v$. The action of $g$ on a point $X$ in the plane is given by:

$$gX = RX + v$$

(34)

The substructure $\Lambda = (1, v)$ maps $\mathbb{R}^d$ into $\mathbb{R}^d/\Lambda \equiv \mathbb{T}^d$ and is called the lattice group.

The group $R$ is called the point group. For any symmetry operation $R$, there is an associated translation $v_R$, the pairs $(r, v_R)$ define the group $P$. The manifold $\overline{\Omega}$ resulting from the identification of points $X$ and $gX$ in the plane is given by:

$$\overline{\Omega} = \mathbb{R}^d/\mathbb{Z} = \mathbb{R}^d/\Lambda \equiv \mathbb{T}^d$$

(35)

The manifold $\overline{\Omega}$ is called an orbifold if it has fixed points under $P$ ($0$, and $\pi R$ in the example $S^1/Z_2$).
4. More \( c = D \) models of the bosonic string.

The \( d \)-compactified dimensions are parametrized by the \( d^2 \) data\(^ {10} \) describing the \( d \) vectors \( e^i \) along which the identification:

\[
X_i = x_i + 2\pi n^i e^i
\]  

(36)

is performed.

The momentum lattice formed are \( O(d,d) \) Lorentzian lattices of which (up to discrete symmetries) \( O(d,d)/O(d) \times O(d) \) are different\(^ {10} \). This \( d^2 \) parameter family is given equivalently by the \( d^2 \) constant values of \( g_{ij} \), \( b_{ij} \)\(^ {11} \) \((i,j = 1, \ldots d)\).

The string propagates in the background

\[
L = g^{i\mu} g^{j\nu} \partial_{\mu} X_i \partial_{\nu} X_j + \varepsilon^{i\mu i\nu} \partial_{\mu} X_i \partial_{\nu} X_i
\]

(37)

For \( 2g_{ij} \) = Cartan matrix of a group \( G \) of rank \( d \) and \( b_{ij} \) given by:

\[
b_{ij} = \begin{pmatrix}
\alpha_{i} & g_{ij} \\
-g_{ij} & \alpha_{j}
\end{pmatrix}
\]

(38)

In that case, the Lorentzian momentum lattice describes the lattices of the simply laced lie group \( G \) (non simply laced groups can also be reproduced)\(^{8} \).

The motion of strings in compactified backgrounds is a physicist's way of constructing representations of the Virasoro algebra having a field theoretic realization. Closed bosonic strings have tachyons, although we shall return to tachyons in a while most research focuses on string theories with no tachyons.

5. \( c = D \) models of heterotic strings\(^ {15},^{6} \).

In the belief that heterotic-like models have a low-energy spectrum which is related to phenomenology, physicists\(^ {16} \) use them as a starting point for construction of string representations of the Virasoro algebra.

Heterotic string theories in ten-dimensions come in several varieties; the "classical" one has a left-handed sector consisting of an eight-component bosonic
field and an eight-component fermionic (Majorana-Weyl) field (the left-handed sector has no tachyon). The right-handed sector has a different structure, in addition to eight-bosonic fields it has another sixteen-bosonic field whose momenta are fixed on the root lattice of $E_8 \times E_8$ or $SO(32)/Z(2)$. The zero mass sector has a supersymmetric gauge multiplet in addition to the graviton supermultiplet.

These models can be modified without changing the central charge. The sixteen right-handed bosonic sector can be twisted and correlated with the left-handed fermions, this results in breaking for example $E_8 \times E_8$ to $O(16) \times O(16)$\(^6\),\(^13\). Various background fields (analogous to the bosonic string case) can be turned on as some of the ten dimensions are compactified on tori. They may also be compactified on symmetric or asymmetric orbifolds. Many four-dimensional models occur\(^16\). In some of them one may even control the number of chiral families\(^13\),\(^17\). Many of the models have some realistic features, none as yet passed all the stringent tests of a realistic $SU(3) \times SU(2) \times U(1)$ model.

A seemingly orthogonal construction\(^9\) uses as building blocks statistical mechanical models. Calabi-Yau like manifolds\(^18\) are more complicated string ground state solutions.

The complete classification of all possible ground states is not known, it is unclear whether they are all ground states of the same string field theory or of different types of string field theories. A new criterion, replacing the minimal energy, is yet to be found so that a selection can be made between the various ground state. Perhaps all of them can appear, forming various domains in space-time through which only a few fortunate particles can travel. Such a mechanism in which a "small" gauge group results was envisaged by Nilsson\(^19\) for chaotic systems.

**To compactify or not to compactify**

In the previous section I have stressed the rather mathematical role that the extra-space dimensions play in four-dimensional string theories. Quarks, of course, were also treated at their infancy as mathematical tools. If a more physical role should be ascribed to the extra-dimensions one will eventually need to develop an understanding of the dynamics of compactification.

---

\(^*)\) Work discussed in this section was done in collaboration with S. Elitzur and A. Forge.
It may well be that the problem will be solved in the context of string cosmology in which it will transmute into the problem of understanding the different expansion rates of the different dimensions. It may also be solved by a ground state that is conformal only after all perturbative and non-perturbative string effects are accounted for. In this section, however, I shall search for a mechanism leading to compactification.

In many of the models described in the former section the extra dimensions were characterized by the lattice formed by their momenta. There exist systems in our very low-energy world which exhibit such a structure, those are the many formation patterns of matter lattices.

The effective potential describing solidifications of liquids was given by Landau\textsuperscript{20}. He has written down a Lagrangian in which a spontaneous breaking of both translational and rotational invariance occurs. The field (order parameter) appearing in the Landau potential is $\rho$ the difference in density between the translational and the rotational invariant liquid density and the density of the solid. Near the melting point an expansion in terms of powers of $\rho$ may be justified, in any case it is done.

The quadratic term in the effective potential is of the form

$$V_2 = \int d^4 \mathcal{q} \ A(q^2) \rho(\mathcal{q}) \rho(-\mathcal{q})$$

(39)

$\rho(\mathcal{q})$ is the Fourier transform of $\rho(\mathbf{x})$. In free-field theory with a non-negative value of the mass parameter $m^2$, $A(q^2) = \sqrt{q^2 + m^2}$ which is minimal for the translational invariant $\mathcal{q} = 0$ mode. Landau suggested to study the case of a dispersion $A(q^2)$ possessing a minimum at a non-zero value of $\mathcal{q}$. A Goldstone-like phenomenon in $\mathcal{q}$ space leading to a spontaneous breaking of translational invariance. The breakdown of rotational invariance is achieved when considering a third-order term in the effective action

$$V_3 = g \int \int \int \int d\mathcal{q}_1 d\mathcal{q}_2 d\mathcal{q}_3 \delta(\mathcal{q}_1 + \mathcal{q}_2 + \mathcal{q}_3) \rho(\mathcal{q}_1) \rho(\mathcal{q}_2) \rho(\mathcal{q}_3)$$

(40)

where $\mathcal{q}$ is already restricted on a sphere with a fixed radius. The term $V_3$ shapes $\mathcal{q}_i$ into an equilateral triangular form. For a small value of $\rho(\mathcal{q})$ a fourth order term serves only to stabilize the potential and allow the absolute value $|\rho(\mathcal{q})|$ to have itself a non-zero value. This line of argument was used to explain the abundance of bcc crystals near the melting line by Alexander and McTague\textsuperscript{21} and a microscopic theory led to the desired form of $A(q^2)$ in a neutron star\textsuperscript{22}. 
Now we attempt to point some analogy between this theory of melting and string theory. To that end we utilize the tachyon the villain of bosonic string theory\textsuperscript{23}).

A string ground state is formed if the string propagates in a conformal background. A naive dimensional counting allows as backgrounds the coefficients of the marginal, dimension-two, operators of the system. These correspond to the massless states in target space, the graviton $G_{\mu \nu}$, the antisymmetric tensor $B_{\mu \nu}$ and the dilaton $\phi$. The propagation of the bosonic string in this background is described by:

$$
\mathcal{L} = g^{\mu \nu} \partial_{\mu} X \partial_{\nu} X - \xi \Phi B^{\mu \nu} \partial_{\mu} X \partial_{\nu} X + \phi R^{(2)}
$$

(41)

$R^{(2)}$ is the two-dimensional curvature tachyon. The backgrounds are subject to the conformal conditions

$$
\Phi G_{\mu \nu} = 0 \quad \beta B_{\mu \nu} = 0 \quad \beta \phi = 0
$$

(42)

which are equivalent to the equations of motion of these particles in the target space. The tachyon being the lowest mass state should also couple to these equations of motion. However, the tachyon $T(X)$ background is a coefficient of a perturbatively relevant operator (of zero dimension), the two-dimensional cosmological constant and as such does not perturbatively contribute to $\beta G_{\mu \nu}$. A more detailed analysis\textsuperscript{24}) shows that the tachyon is a coefficient of an operator which has an anomalous dimension two and thus contributes to $\beta G_{\mu \nu}$. The tachyon also possesses its own beta-function $\beta_T$, the beta-functions are

$$
\begin{align*}
\beta G_{\mu \nu} &= \alpha' \left[ R_{(\mu \nu)} + 2 \nabla_\mu \nabla_\nu \phi + \nabla_\mu T \nabla_\nu T \right] \\
\beta B_{\mu \nu} &= \alpha' \left[ \mathcal{L}_{\mathcal{F}_{\mu \nu}} - 2 \partial_\lambda \phi \ H_{\lambda \mu \nu} \right] \\
\beta \phi &= \frac{\alpha'}{2} \left[ (\partial \phi)^2 - \nabla^2 \phi - \frac{\lambda}{\alpha'_2} + \nabla_\mu T \nabla_\nu T + \frac{1}{8} (\nabla T)^2 + \frac{T^2}{\alpha'_2} + O(T^3) \right] \\
\beta T &= \frac{\alpha'}{2} \left[ -\frac{\lambda}{\alpha'_2} \ T - \nabla^2 T + 2 \nabla_\mu T \nabla_\nu T + O(T^3) \right]
\end{align*}
$$

(43)

A vanishing beta-function for $T$ has somewhat similar properties to the stationary points of the Landau potential, the anomalous dimension of the tachyon vertex operator leads to a non-zero value of (Euclidean) $k^2$ and the three-point coupling which appears in the effective target space theory are the result of the three-point string coupling which is basic to string interactions. These ingredients may lead again to a breakdown of translational and rotational invariance. The existence of a fixed point where $k \neq 0$ would provide a crack in
flat 24-dimensional space. It is a far cry from a consistent dynamical picture of compactification in which on top fermions are formed and all tachyons (identity operators) are banished. If one does not wish to form a 24-dimensional lattice, one may go on to conjecture that another phase discussed by Landau, that of a liquid crystal, is formed. Four dimensions are flat and the rest latticised (in momentum space?) on a nice group lattice. Tensorial order parameters describe these phases in statistical physics.

Well, by now it is high time to ski down!

ACKNOWLEDGEMENTS

We wish to thank S. Alexander for many discussions on the theory of solidification.
REFERENCES


2a) C. Callan, these Proceedings.

2b) For a review see Lectures by D. Friedan and S. Schenker published in
Proceedings of the Workshop on Unified String Theories, Santa Barbara
(1985);

3) A.M. Polyakov, JETP Lett. 12 (1970) 381.
A.A. Belavin, A.M. Polyakov and A.B. Zamolchikov, Nucl. Phys. B241


(1971) 12;
J. Weiss note in the above.


(1986) 155.


8) S. Elitzur, E. Gross, E. Rabinovici and N. Seiberg, Nucl. Phys. B283
(1987) 413.


P. Goddard and D. Olive in Vertex Operators in Mathematics and Physics
MSRI (Springler Verlag, 1985).


13) L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys B261 (1985) 651;

14) S. Elitzur, A. Giveon and E. Rabinovici, unpublished.

253; B267 (1986) 75.


